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DEPARTMENT OF PHYSICS

II MSC PHYSICS

NUCLEAR AND PARTICLE PHYSICS

CODE:18KP3P10

NUCLEAR SIZE

In 1919 Rutherford found that the deviation from pure coulomb's scattering, anomalous scattering was observable When ∝ particles were scattered by the lightest elements. In these light elements, the closest distance of approach was of the order of 5×1^{15} m . The distance of closest approach at which anomalous scattering begins was identified as the first measure of nuclear radius .

It is evident from experiments that the nucleus is composed of proton and neutrons only known as nucleons. The shape of the nucleus is taken spherical, because for a given volume this shape possesses the least surface area and thus provides the maximum short range binding force between the nucleons in the nucleus. Small asymmetries of the distribution of negative charge are present in some nuclei as these nuclei exhibit high electric quadrupole moments . In most nuclei the ellipticity is only of the order of one percent , thus we may suppose that protons are uniformly distributed inside the spherical nucleus.

There is a experimental evidence that nuclear density P remains approximately constant over most of the nucleus volume and then decreases rapidly to zero . This means that the nuclear volume is approximately proportional to the number of nucleons, i.e., mass number A.

$$\mathbf{V=4/3\,\frac{4}{3}\pi R^3 \propto A} \quad \textbf{or} \text{ nuclear radius} \quad \mathbf{R \propto A^{1/3}}$$

$$\mathbf{R = R_0\, A^{1/3}}$$

Where R_0 is the nuclear unit radius . Thus our purpose is to find R_0.

The methods of measuring nuclear radius are divided into two main categories. One group of method is based on the study of the range of nuclear forces, the nucleus is probed by the a nucleon or light nucleus. Other group of methods studies the electric field and charge distribution of the nucleus, the nucleus is probed by electron or muon.

NUCLEAR MASS

In a nuclear physics we are concerned with the mass of nucleus of but experimental determination with mass Spectrograph provide the atomic mass (M). The nuclear mass $M_{nucleus}$ is obtained by subtracting mass of Z orbital electrons from the atomic mass.

$$M_{nucleus} = M - Zm_e$$

Where m_e is mass of an electron. This expression is not exact because it does not take into account the binding energies of electrons in the atom but the error involved in very small because binding energy of electron is of the order of few electron –Volts .

The exact atomic number is determined by a small spectrometer which gives an accuracy of a few parts in 10^7. Now mass spectrograph are used to determine the isotopic masses of all elements of periodic table.

NUCLEAR CHARGE

Charge of A nucleus is due to the proton contained in it. Proton has a positive charge of 1.6×10^{-19} C. The nuclear Charge Ze where Z is the atomic number of the nucleus. The value of Z is know from X-ray scattering experiment, from the nuclear scattering of ∝ particle and form the X-ray spectrum

Spin angular momentum

Both the proton and neutron, like the electron have an intrinsic spin. The spin angular momentum is completed by $L = \sqrt{(l+1)}\ h/2\pi$ where the quantum number l, commonly called the spin, is equal to 1/2. The spin angular momentum , then has a value $L = \sqrt{3}/2.\ h\pi/2$.

SEMI-EMPIRICAL MASS FORMULA

Von-Weizsacker in 1935 expressed the atomic mass of nuclide in terms of a series of binding energy correction terms to the main mass combination from protons and neutrons. The expression is called semi-empirical mass formula or Wcizsacker mass formula. Accordingly the mass M of a neutral atom whose nucleons contains Z-protons and N-neutrons can be expressed as

$$M\ (Z,\ A) = Z\ m_p + (A - Z)\ m_n - \frac{B}{c^2}$$

Where B binding energy B constitutes a series of terms, each representing some general characteristic of nuclide.

i)Volume Energy

Nucleus is a sphere containing A nucleons and the nucleons are held together by the strong attracting forces, therefore reduction in mass of nucleus should be proportional to A. so the binding energy should be proportional to A. Since the volume of the nucleus is proportional to A, therefore this term is referred as volume energy and is denoted by E_{vol}

$$\mathbf{E_{VOL}=-C_{vol}A}$$

where $\mathbf{C}_{vol}$ is constant. This term is important for large A-nuclei

ii)Surface Energy.

In volume energy term it is assumed that all nucleons are surrounded by other nucleons; it will be true in the case of infinite nuclear matter. But in fact the nucleons on the surface of the nucleus will be attracted only nucleons on one side. Hence there is a positive surface energy term analogous to surface energy of a liquid droplet.

This effect gives rise to surface tension in liquids. The surface energy of nucleus is proportional to surface area, which in turn is proportional to square of radius, hence $A^{2/3}$. Thus surface energy term can be written as

$E_{sur}=C_{sur}A^{2/3}$, where C_{sur} is a constant.

iii)Coulomb Energy.

In writing the volume energy term, the charge of nucleons was ignored. Actually a nucleus consists of Z-protons and (A —Z) neutrons. The protons experience Coulomb's repulsion, hence the binding energy will be less than that of a neutron.

The energy due to Coulomb repulsion must be subtracted from volume energy term. The Coulomb energy term will be equal to the electrostatic potential energy of Z-protons packed together In a spherically symmetric assembly of mean nuclear radius $R = r_0 A^{1/3}$. For a spherical volume of uniform charge density, the Coulomb energy is given by $\mathbf{E_{coul}}$

$$= \frac{3}{5}\,\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

AS $R = r_o A^{1/3}$, this term may be expressed as

$E_{coul} = C_{coul}\, \underline{Z(Z-1)} = C_{coul}\, \underline{Z^2 A^{-1/3}} / A^{1/3}$ (for large Z)

where C_{coul} is a constant.

iv) **Asymmetry Energy**.

It has been observed that for light nuclei N = Z is most favoured stable system, while for heavy nuclei there is excess of neutrons over protons, i.e., N > Z. It has been found that whether (N — Z) increases or decreases, the nuclei are found to be less stable.

The difference in number of protons and number of neutrons in nuclei causes asymmetric configuration. Therefore a term in the binding energy signifying this asymmetric effect must be included. For a nucleus of mass number A, atomic number Z, the number of neutrons will be N= A —Z,

so excess of neutrons over protons is (N-Z). This causes decrease in binding energy because they are out of reach of other nucleons. The fraction of nuclear volume so affected is $\frac{(N-Z)}{A}$ therefore the net decrease in binding energy is proportional to (N-Z) $\frac{(N-Z)}{A}$ or $\frac{(N-Z)^2}{A}$

Therefore the asymmetric binding energy term is expressed as

$$E_{asy} = C_{asy}\,\frac{(N-Z)^2}{A} = C_{asy}\,\frac{(N-2Z)^2}{A} \quad \text{(since N = A-Z)}$$

C_{asy} is proportionality constant.

v) **Pairing Energy.**

It has been observed that the nuclei containing even number of protons and even number of neutrons, known as even-even nuclei are most stable, while those having odd number of protons and odd number of neutrons, known as odd-odd nuclei are least stable. The nuclei having even number of protons and odd number of neutrons and vice-versa, known as even-odd nuclei have intermediate degree of instability.

This effect is referred as pairing effect. This arises mainly from the fact that the nucleons tend to pair up (align with spins up and down) in order to get lowest energy-state. It is to be mentioned that the neutrons and protons occupy different energy states. Obviously this effect causes an increase in binding energy when both N and Z are even and decrease in binding energy when both N and Z are odd.

The pairing energy term Epair may be expressed as

The pairing energy term E_{pair} may be expressed as

$$E_{pair} = \begin{cases} 0 & \text{for odd } A \\ -\delta & \text{for odd } N \text{ and odd } Z \\ +\delta & \text{for even } N \text{ and even } Z \end{cases}$$

It is found empirically that the pairing energy term is

$$\delta(A, Z) = \mp C_{pair} A^{-3/4} \quad \ldots(6)$$

where C_{pair} is a constant.

With all above corrections, the binding energy expression becomes

$$B = C_{vol} A - C_{sur} A^{2/3} - C_{coul} Z^2 A^{-1/3} - C_{asy} (A - 2Z)^2 A^{-1} \pm C_{pair} A^{-3/4}$$

Thus the semi-empirical mass formula (1) becomes

$$M(Z, A) = Zm_p + (A - Z)m_n - [C_{vol} A - C_{sur} A^{2/3} - C_{coul} Z^2 A^{-1/3} - C_{asy} (A - 2Z)^2 A^{-1} \pm C_{pair} A^{-3/4}] \quad \ldots(7)$$

This expression is known as *Weizsäcker mass formula.* The constants of this formula that fit the experimental data best are as follows :

$$\left.\begin{aligned} &C_{vol} = 16{\cdot}11\ MeV,\ C_{sur} = 20{\cdot}21\ MeV \\ &C_{coul} = 0{\cdot}583\ MeV,\ C_{asy} = 20{\cdot}65\ MeV,\ C_{pair} = 33{\cdot}5\ MeV \end{aligned}\right\} \quad \ldots(8)$$

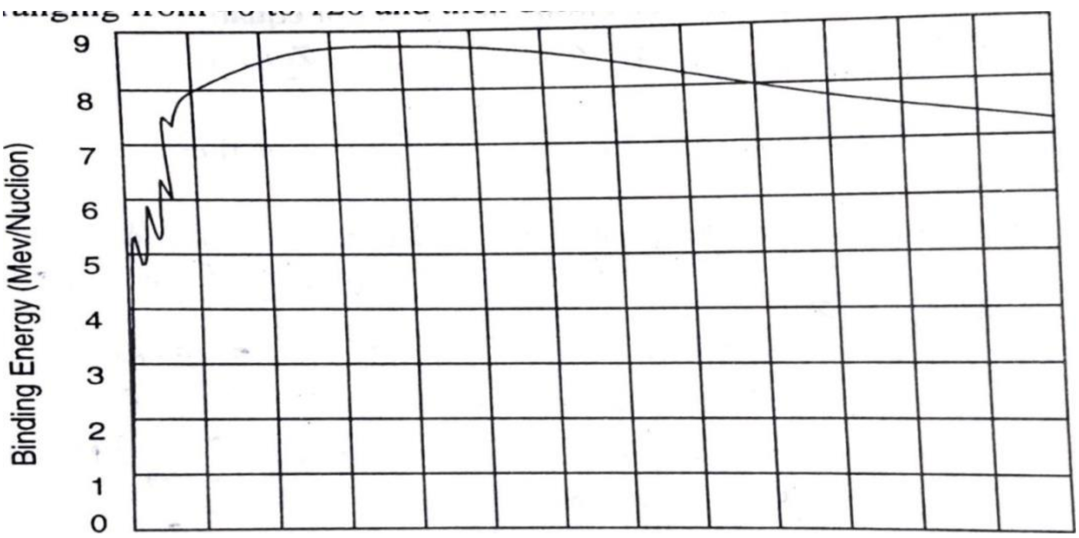
STABILITY OF NUCLEI

The studies of binding energies help to measure the stability of nuclei. The calculations are in general made for atoms rather than atomic nuclei. The reason is that the atomic masses can be accurately known from mass-spectrographic analysis. Thus while calculating mass defect the masses of electrons along with those of protons and neutrons are also considered.

Such calculations have been performed for a large number of atoms in periodic table and the average binding energy per nucleon has been estimated.

Fig represents the binding energy per nucleon plotted against mass number A. The noticeable features of the graph are :

l. For lighter nuclides (A < 30), the curve shows periodicity with maxima occurring at mass numbers which are integral multiples of 4 and which contain equal number of Protons and neutrons. This suggests that higher nuclide have a tendency to group themselves into particle subgroups and that $2He^4$, $4Be^8$, $6C^{12}$, $8o^{16}$ etc. possess considerably greater energies than their neighbours.



20 40 60 80 10 120 140 160 180 200 220 240 260

Mass Number A ⟶

The nuclear binding energies per nucleon

2. The binding energy per nucleon is relatively small for lighter elements, increases with mass number and attains maximum value = MeV per nucleon for moderate nuclei having mass number ranging from 40 to 120 and then decreases steadily for still more massive nuclei.

3. The very light elements and the very heavy elements possess smaller binding energies per nucleon indicating that these elements are unstable. It is this fact that leads to the possibility of developing atomic energy by nuclear fusion (combination of lighter elements into a heavier one) or by nuclear fusion (splitting of a very heavy element into smaller fragments).

NEUTRON SCATTERING AT LOW ENERGIES

Neutron-proton scattering experiments below 10 MeV reveal that the differential cross-section in the centre of mass system is practically isotropic (i.e., independent of angle).

The experimental arrangement of Storrs and Frisch is sketched in Fig. (2·12). In this experiment the protons accelerated from a VandeGraff generator were bombarded on a lithium target. As a result the neutron of energy 1·315 MeV are emitted in all directions. The counter '1' called the monitor counter determines the length of exposure, while counter 2 detects the collimated beam of neutron emitted in the forward direction. A thin scatterer of either graphite (C) or polyethrylene (CH_2) in the form of a cylinder is placed in front of slit S, which allows a narrow beam of neutrons to be incident on the scatterer. The number of neutron counts in the narrow 2 was counted in the presence and absence of scatterer for a given number of counts in the monitor counter 1.

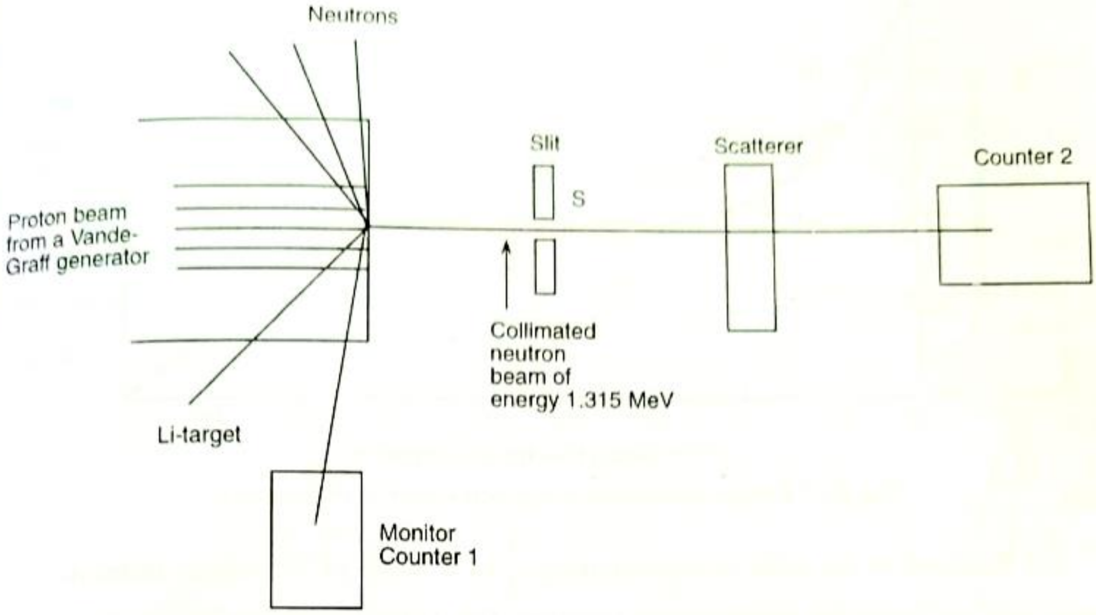


Fig. 2·12. Schematic arrangement of *n-p* scattering experiment

If N and N_0 are the number of counts registered by counter 2 in the presence and absence of scatterer respectively, then the value of scattering cross-section σ_c may be determined.

For *graphite scatterer*, the direct formula used is

$$N = N_0\, e^{-n\sigma_c x} \qquad \ldots(1)$$

where n is concentration of nuclei per unit volume in the target and x is the thickness of the target and σ_c is cross-section for carbon scatterer. For CH_2 *scatterer*, the sum $n_c\,\sigma_c + n_H\,\sigma_H$ is found from the formula.

$$N = N_0\, e^{-(n_c\,\sigma_c + n_H\,\sigma_H)\,x} \qquad \ldots(2)$$

when n_c and σ_H are concentration of carbon atoms and hydrogen atoms. By subtracting the contribution from carbon, hydrogen cross-section may be determined.

The results of experiment are

$$\left.\begin{aligned}\sigma_c &= (2\cdot192 \pm 0\cdot020)\text{ barns}\\ \sigma_H &= (3\cdot675 \pm 0\cdot020)\text{ barns}\end{aligned}\right\}(1\text{ barn} = 10^{-28}\text{ m}^2)$$

Similar experiments have been performed by different researchers for different energy ranges using different techniques. The most extensive measurements of *n-p* scattering cross-section for low energy have been performed by Melkunian and Rainwater et al. For these experiments the neutron source was a pulsed cyclotron at Columbia University. In the device the short bursts of deuterons was made incident an a beryllium target; as a result the neutrons are produced by nuclear reaction. The high energy experiments were performed by R.K. Adair. The results of experiments for low and high energy scattering are plotted in fig. Graph shows increased *n-p* scattering cross-section below 1eV.

The reason is that (*i*) upto 1eV to protons can not be regarded as completely free. For this part of energy range the scatterer was H_2 gas which has dissociation energy about

4·5 eV/ molecule. Therefore below this energy the scatterer is the hydrogen-molecule instead of a free proton.

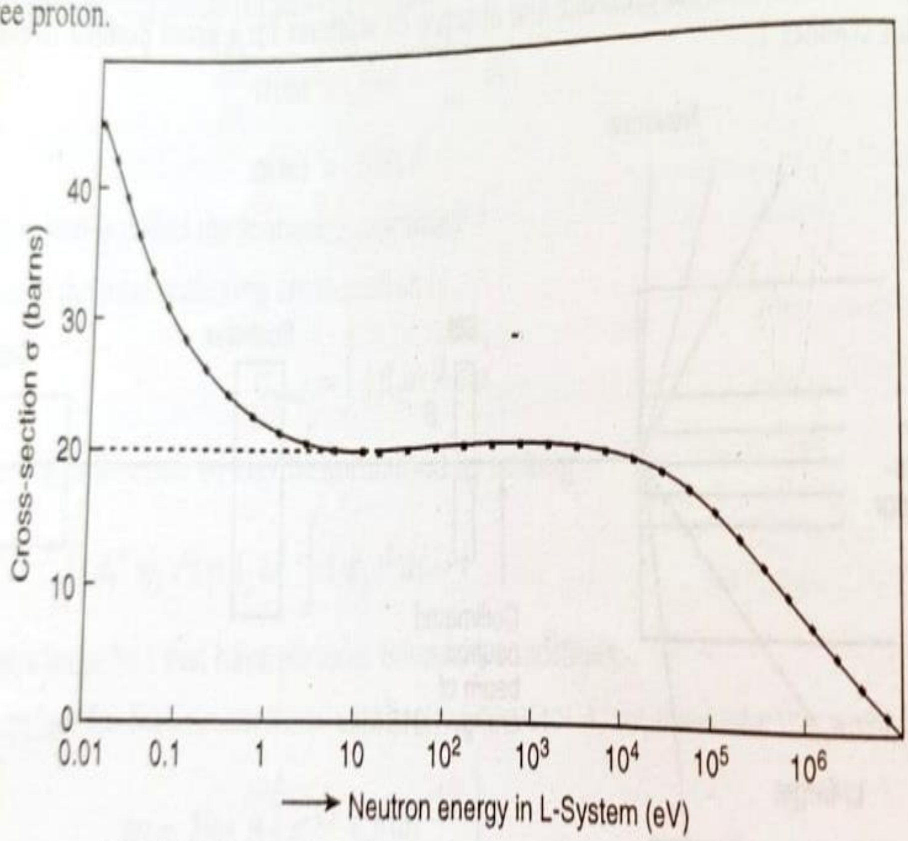


Fig. 2·13. Energy dependence of *n-p* total scattering cross-section

(*ii*) Neutrons of the order of thermal energy $\frac{3}{2}$ kT = 0.025 eV effectively increases the target area and hence the scattering cross-section. The dotted line in fig (2·13) indicates the result derived from theory assuming the protons to be completely free and at rest.

DEUTRON

Let us first consider deuteron in order to exhibit some of the concepts involved in discussing nuclear potentials and the quantum states of nuclei. The deuteron does possess measurable properties which might serve as a guide in the search for the correct nuclear interaction. These properties are :

1. The extraordinary stability of the alpha particle shows that the most stable nuclei are those in which number of neutrons and protons are equal. The deuteron consists of two particles of roughly equal masses M, so that the *reduced mass of the system is* $\frac{1}{2}M$.

2. The binding energy of the deuteron is very small. Its experimental value is 2.22457312 ± 22 MeV. Since the energy needed to pull a nucleon out of a medium mass nucleus is about 8 MeV, we must regard the *deuteron as loosely bound.*

3. The angular momentum quantum number, often called the *nuclear spin*, of the ground state of the deuteron determined by a number of optical, radiofrequency and micro-wave methods is one. It suggests that the spins are parallel (*triplet state*) and the orbital angular momentum of the deuteron about their common center of mass is zero. Thus the *ground state is* ^3S *state.*

4. The parity of deuteron as measured, indirectly, by studies of nuclear disintegrations and reactions for which certain rules of parity changes exist, *is even.*

5. The sum of the magnetic dipole moments of the proton ($2.792847386 \pm 63\mu_N$) and neutron ($-1.91304275 \pm 45\mu_N$), does not exactly equal to magnetic moment of the deuteron (0.857406 ± 1 μ_N) measured by magnetic resonance absorption method.

6. A radiofrequency molecular beam method has been employed to determine the quadrupole moment of the deuteron as $Q = 0.0028590 \pm 30 \times 10^{-28}\text{cm}^2$. This shows the departure from spherical symmetry of a charge distribution. The +ve sign indicates that this *distribution is prolate* rather than oblate.

   The electric quadrupole moment and the magnetic moment discrepancy can be explained if the ground state is a mixture of the triplet states $^3S_1$ and $^3D_1$ having even parity. The percentage probability of finding the deuteron in D-state is 4 ± 2%. As deuteron spends most of the time in the spherically symmetrical state (S-state), we will for the moment ignore the D-state contribution to the deuteron wavefunction.

7. Since the neutron has no charge, the force between the neutron and proton can not be electrical. This force can not be magnetic as magnetic moments are very small. It can not be gravitational force, as the masses are very small. So we must accept the *nuclear force as a new type of force*. This force is short range, attractive and along the line joining the two particles (*central force*). Since a central force can not account for the quadrupole moment of the deuteron. As a quadrupole moment is small, the assumption will be approximately correct.

8. The *force depends only on the separation* of the nucleons not on the relative velocity or orientation of the nucleon spins with respect to the line. This force can be derived from a potential. Since the force is attractive, $V(r)$ is negative and decreases with decreasing r. Since it is short range, $V(r)$ vanishes for $r > b$, where $b \sim 3$ fermi.

MAGNETIC DIPOLE MOMENT OF DEUTRON

The value of P_D depends

Magnetic moment of deuteron. We know that the small discrepancy between the sum of the magnetic moment of proton and neutron and the measured value for the deuteron can be interpreted as a contribution of the orbital motion of the proton in the *D*-state in the deuteron ground state. If nuclear forces are supposed to be central forces, the difference will be zero. This contribution can appear only with the non-central forces. The operator describing the magnetic moment of deuteron is

$$\mu = \mu_p \sigma_p + \mu_n \sigma_n + l_p, \qquad \ldots(153)$$

where μ_n and μ_p are the magnetic moments of neutron and proton measured in nuclear magnetons, σ_n and σ_p their unitary spin operators and l_p is the orbital angular momentum of the proton about the center of mass of the bound system. The uncharged neutron cannot contribute any magnetic moment by orbital motion alone. In the centre of mass system the orbital angular momentum of the proton is half of the combined orbital angular momentum *L*.

Since $$\mu_n \sigma_n + \mu_p \sigma_p = \tfrac{1}{2}(\mu_n + \mu_p)(\sigma_n + \sigma_p) + \tfrac{1}{2}(\mu_n - \mu_p)(\sigma_n - \sigma_p)$$

Therefore equation (153) can be written as

$$\mu = \tfrac{1}{2}(\mu_n + \mu_p)\tfrac{1}{2}(\sigma_n + \sigma_p) + \tfrac{1}{2}(\mu_n - \mu_p)(\sigma_n - \sigma_p) + \tfrac{1}{2}\mathbf{L}$$

Since the operator $(\sigma_n - \sigma_p)$ in the second term vanishes for a triplet state (as spins are parallel) and $\mathbf{I} = \mathbf{L} + \mathbf{S} = \mathbf{L} + \frac{1}{2}(\sigma_n + \sigma_p)$, hence the magnetic moment operator becomes

$$\mu = (\mu_n + \mu_p)\mathbf{I} - \left(\mu_n + \mu_p - \tfrac{1}{2}\right)\mathbf{L}. \qquad \ldots(154)$$

The observed value of μ is the expectation value of this expression in the state with $I_z = I$. We therefore, can replace **L** by L_z. With the usual theory of quantum operators, we may write

$$\therefore \qquad \mathbf{L} \to L_z = \frac{\mathbf{L}.\mathbf{I}}{I^2} I_z = \frac{I(I+1) + L(L+1) - S(S+1)}{2I(I+1)} I_z. \qquad \ldots(155)$$

For the deuteron, $I(I + 1) = S(S + 1) = 2$ and in a mixture of *S* and *D* states, $L(L + 1)$ is 0 for the *S*-state and is 6 for the *D*-state. Therefore $[L(L + 1)]_{av} = 0 \times p_S + 6 \times p_D = 6p_D$. Here p_D and p_S represent the *D*-and *S*-state probabilities respectively. For the deuteron $I = 1$, the value of I_z in the state with $I_z = I$ is unity. Thus the expectation value of $\mathbf{L}_z$ is obtained by using eqn. (155).

$$\langle \psi | \mathbf{L} | \psi \rangle = 0 \text{ for the } S\text{-state}$$
$$= 3/2 \text{ for the } D\text{-state}. \qquad \ldots(156)$$

The deuteron's magnetic moment in units of the nuclear magneton is given by

$$\mu_d = P_S \langle \psi_S | \mu | \psi_S \rangle + P_D \langle \psi_D | \mu | \psi_D \rangle \qquad \ldots(157)$$

Substituting the values from equations (154) and (156) in the above equation, we get

$$\mu_d = P_S(\mu_n + \mu_p) + P_D\left[(\mu_n + \mu_p) - \tfrac{3}{2}\left(\mu_n + \mu_p - \tfrac{1}{2}\right)\right].$$

Since $P_S + P_D = 1$, therefore

$$\mu_d = (\mu_n + \mu_p) - \tfrac{3}{2}P_D\left(\mu_n + \mu_p - \tfrac{1}{2}\right). \qquad \ldots(158)$$

The last term gives a direct measure of the *D*-state probability P_D. The experimental results imply a *D*-state probability of about 3.93%. The above relation does not give accurate results. There are various other causes which can give corrections, especially of relativistic effects. Hence the measured magnetic moment gives only a rough estimate of the *D*-state probability. One may expect that the *D*-state probability P_D lies somewhere between 2 and 8%. Due to this reason, the *n-p* system is considered to be made up of 96% *S*-state and 4% *D*-state in the deuteron.

2-3. SIMPLE THEORY OF GROUND STATE OF DEUTERON

Deuteron is the simplest of two nucleon-bound system. It consists of one proton and one neutron. Therefore deuteron is a two body problem. To solve the problem we assume.

(*i*) Deuteron is a system of two particles–neutron and proton, of nearly equal masses (each of mass *m* say). *i.e.*, $m_1 = m_2 = M$

The reduced mass of system

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{M \times M}{M + M} = \frac{M}{2}$$

(*ii*) *The force between two particles is central and spherically symmetric i.e.*, it depends on distance between two particles and acts along their line-joining. It is independent of orientation. However this assumption appears to be incorrect because it cannot account for the quadrupole moment of deuteron; but it is taken for the sake of simplicity. (We shall correct later). Under central force-field the potential energy is expressed as $V = V(r)$.

(*iii*) The potential function $V(r)$ is independent of time and is square well potential.

The time-independent Schrödinger-equation in centre of mass system is written as

$$\nabla^2\psi + \frac{2\mu}{\hbar^2}\{E - V(r)\}\,\psi = 0 \qquad ...($$

where μ is the reduced mass $\mu = \frac{M}{2}$. E is total energy of system.

In spherical polar coordinates*

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

$\therefore$ Schrödinger equation (1) may be expressed as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{2M}{\hbar^2}\{\mu - V(r)\}\,\psi = 0 \qquad ...$$

Under the central force assumption, the wave-function φ may be expressed as

$$\psi = \left(\frac{u_l}{r}\right) Y_{lm}(\theta, \phi) \qquad ..$$

where $\frac{\mu_l}{r}$ is the radial function and $Y_{lm}(\theta, \phi)$ is spherical harmonic function

from (3)

$$\frac{\partial\phi}{\partial r} = \left[\frac{1}{r}\frac{\partial u_l}{\partial r} - \frac{1}{r^2}\,u_l\right] Y_{lm}(\theta, \phi)$$

Substituting values of ψ, $\frac{\partial \psi}{\partial r}$, $\frac{\partial \psi}{\partial \theta}$ and $\frac{\partial^2 \psi}{\partial \phi^2}$ in equation (2), we get

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left\{\left(r\frac{\partial u_l}{\partial r} - u_l Y_{lm}(\theta, \phi)\right)\right\} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left\{\sin\theta\left(\frac{u_l}{r}\right)\frac{\partial Y_{lm}(\theta, \phi)}{\partial \theta}\right\}$$

$$+\frac{1}{r^2 \sin^2\theta}\cdot\left\{\frac{u_l}{r}\frac{\partial^2 Y_{lm}(\theta, \phi)}{\partial \phi^2}\right\} + \frac{2\mu}{\hbar^2}\left\{E - V(r)\left(\frac{u_l}{r}\right)Y_{lm}(\theta, \phi) = 0\right.$$

Dividing by $\frac{u_l Y_{lm}(\theta, \phi)}{r^3}$, we get

$$\frac{r}{u_l}\frac{\partial}{\partial r}\left(r\frac{\partial u_l}{\partial r} - u_l\right) + \frac{1}{Y_{lm}(\theta, \varphi)}\frac{\partial}{\partial \theta}\left\{\sin\theta\frac{\partial Y_{lm}}{\partial \theta}(\theta, \phi)\right\}$$

$$+\frac{1}{Y_{lm}(\theta, \phi).\sin^2\theta}\frac{\partial^2 Y_{lm}}{\partial \phi^2} + r^2\frac{2\mu}{\hbar^2}\left\{E - V(r)\right\} = 0$$

$$\Rightarrow \quad \frac{r^2}{u_l}\frac{\partial^2 u_l}{\partial r^2} + \frac{2\mu r^2}{\hbar^2}\{E - V(r)\}$$

$$= -\left[\frac{1}{Y_{lm}(\theta, \phi)}\frac{\partial}{\partial \theta}\left\{\sin\theta\frac{\partial Y_{lm}(\theta, \varphi)}{\partial \theta}\right\} + \frac{1}{Y_{lm}(\theta, \phi)\sin^2\theta}\frac{\partial^2 Y_{lm}}{\partial \phi^2}\right] \quad \ldots(3)$$

In this equation L.H.S. is a function of r only; while R.H.S. is a function of θ and ϕ only, therefore this equation is satisfied only if each side is equal to same constant $l(l+1)$ (say).

$$\frac{r^2}{u_l}\frac{\partial^2 u_l}{\partial r^2} + \frac{2\mu r^2}{\hbar^2}\{E - V(r)\} = l(l+1) \quad \ldots(4)$$

$$\text{and } -\frac{1}{Y_{lm}(\theta, \phi)}\left[\frac{\partial}{\partial \theta}\left\{\sin\theta\frac{\partial Y_{lm}}{\partial \theta}(\theta, \phi)\right\} + \frac{1}{\sin^2\theta}\cdot\frac{\partial^2 Y_{lm}}{\partial \phi^2}\right] = l(l+1) \quad \ldots(5)$$

Equations (4) and (5) may be expressed as

$$\frac{\partial^2 u_l}{\partial r^2} + \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{l(l+1)\hbar^2}{r^2}\right]u_l = 0 \quad \ldots(6)$$

$$\text{and} \quad \frac{\partial}{\partial \theta}\left\{\sin\theta\frac{\partial Y_{lm}}{\partial \theta}(\theta, \phi)\right\} + \frac{1}{\sin^2\theta}\cdot\frac{\partial^2 Y_{lm}}{\partial \phi^2} - l(l+1)Y_{lm}(\theta, \phi) = 0 \quad \ldots(7)$$

In our solution we have assumed spherically symmetric potential function; hence the solution of equation (6) is significant. The last term in bracket *i.e.* $\frac{l(l+1)}{r^2} = \hbar^2$ appears to a straight addition in actual potential $V(r)$ and is known as the *centrifugal potential,* because its derivative with respect to r is equal to the classical centrifugal force when angular momentum is $\sqrt{l(l+1)}\ \hbar$. The centrifugal potential versus r graphs for $l = 0, 1, 2$ are plotted in figure (2·1). Clearly the effect of centripetal potential is to tear the particles apart. For binding the proton and neutrons together, the potential $V(r)$ must be attractive at least over a limited range of r and should have the value at least to compensate the repulsive centrifugal potential. This can be most easily achieved in $l = 0$ state. Therefore the lowest quantum mechanical state for a two body system like deuteron is always an $l = 0$ state (or an s-state).

The potential function providing the simplest solution of Schrödinger's equation is square-well (shown in Fig. (2·2)) and expressed as $V = \begin{cases} -V_0 \text{ for } r \le r_0 \\ 0 \text{ for } r > r_0 \end{cases}$

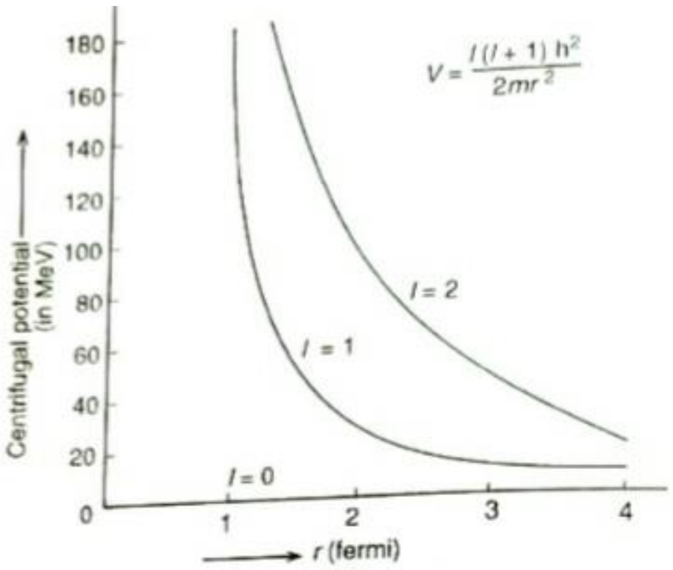


Fig. 2.1. Centripetal potential for *n-p* system.

The Schrödingers equation for *I* and *II* regions in $l = 0$ state may be expressed as

$$\frac{d^2 u(r)}{dr^2} + \frac{2\mu}{\hbar^2}(V_0 - E_B)\, u(r) = 0,\ r \le r_0 \qquad \ldots(8a)$$

and
$$\frac{d^2 u(r)}{dr^2} - \frac{2\mu}{\hbar^2} E_B\, u(r) = 0,\ r > r_0 \qquad \ldots(9a)$$

where $u_l = u(r)$; dropping the suffix '*l*'

and $E = -E_B = -2{\cdot}225\text{MeV}$

where E_B is the binding energy of deuteron.

Substituting $\sqrt{2\mu(V_0 - E_B)} = k_1$ (8*b*)

and $\sqrt{\frac{2\mu E_B}{\hbar}} = k_2$ (9*b*)

equations (8*a*) and (9*a*) take the form

$$\frac{d^2 u(r)}{dr^2} + k_1^2 u(r) = 0 \quad ; r \le r_0 \qquad \ldots(10)$$

$$\frac{d^2 u(r)}{dr^2} - k_2^2 u(r) = 0 \quad ; r > r_0 \qquad \ldots(11)$$

The general solutions of equations (10) and (11) may be expressed as

$$u_I(r) = A \sin k_1 r + B \cos k_1 r \quad \ldots(12)$$

and
$$u_{II}(r) = Ce^{k_2 r} + D\, e^{-k_2 r} \quad \ldots(13)$$

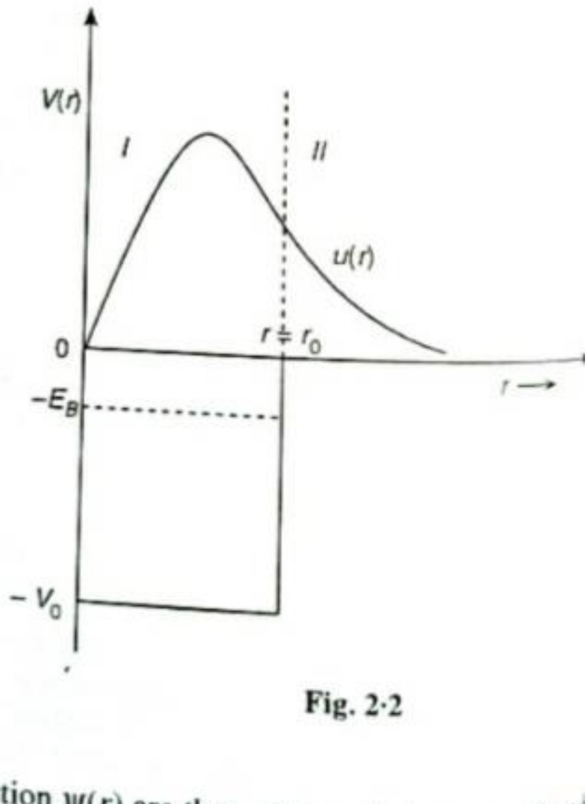


Fig. 2·2

The usual conditions satisfied by wave function $\psi(r)$ are that $\psi(r) =$ finite at $r = 0$ and zero at $r = \infty$. The condition $\psi(r) = 0$ requires that $u(r) = r.\ \psi$ vanishes at $r = 0$ and is square-integrable. In view of this equation $u_I(r) = 0$ at $r = 0$ implies that $B = 0$; Therefore equation (12) takes the form

$$u_I(r) = A \sin k_1 r$$

The condition $\psi(r) = 0$ at $r = \infty$ requires that $C' = 0$; so that $u(r)$ remains finite; therefore equation (13) takes the form

$$u_{II}(r) = D\,e^{-k_2 r} \quad \ldots(15)$$

The boundary conditions that u_I and u_{II} and their first order derivatives with respect to r must exist and be continuous, demands that

$$\psi_I = \psi_{II} \Rightarrow u_I = u_{II} \quad \text{at } r = r_0 \quad (i)$$

$$\frac{d\psi_I}{dr} = \frac{d\psi_{II}}{dr} \Rightarrow \frac{du_I}{dr} = \frac{du_{II}}{dr} \quad \text{at } r = r_0 \quad (ii)$$

Condition (*i*) gives

$$A \sin k_1 r_0 = D\,e^{-k_2 r_0} \quad \ldots(16)$$

$$A\,k_1 \cos k_1 r_0 = -k_2\,D\,e^{-k_2 r_0} \quad \ldots(17)$$

Dividing (17) by (16), we get

$$k_1 \cot k_1 r_0 = -k_2 \quad \ldots(18)$$

Equation (18) gives implicit relation between binding energy E_B of two nucleon system to range of potential r_0 and the depth V_0. The binding energy E_B is known experimentally $E_B = 2{\cdot}225$ MeV; still there remain two unknown parameters r_0 and V_0 in single equation (18) and in order to solve the puzzle we must have one more equation relating r_0 and V_0. In other words the transcendental equation (18) can not be solved analytically ; but may be solved graphically; however we may choose any arbitrary value of r_0 and find V_0 and viceversa. A few values are given in the table.

Range r_0 fermi ($= 10^{-15}$ m)	*Depth of potential V_0 MeV*
1·0	120
1·5	59
2·0	36
2·5	25
∞	2·83

A number of experiments suggest that the range of nuclear force is 1 fermi = 10^{-15} *m*. Taking appropriate value of range $r_0 = 2 \times 10^{-15}$ *m*, we get the value of potential depth $V_0 = 36$ MeV.

Substituting values of k_1 and k_2 in equation (18), we get

$$\cot k_1 r_0 = -\frac{k_2}{k_1} = -\sqrt{\frac{E_B}{V_0 - E_B}} \quad \ldots(19)$$

Approximate Solution.

In case of deuteron $E_B \ll V_0$, therefore equation (19) takes approximate form

$$\cot k_1 r_0 \approx -\sqrt{\frac{E_B}{V_0}} \quad \ldots(20)$$

Clearly $\cot k_1 r_0$ is negative and has a small fractional value. Putting $E_B \to 0$, we get $\cot k_1 r_0 \to 0$

or $$k_1 r_0 \longrightarrow \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots\ldots\ldots$$

$u(r)$ can not have radial node(s) inside the well because this would indicate that u (r) and hence $\psi(r)$ will not correspond to lowest energy level. As we are considering ground state, two ways.

1. If the width of nuclear potential well $r_0 = 2 \times 10^{-15} m$ then $k_1 = \frac{\pi}{2r_0}$

$$\Rightarrow \qquad \frac{\sqrt{2\mu (V_0 - E_B)}}{\hbar} = \frac{\pi}{2r_0}$$

Substituting reduced mass $\mu = \frac{M}{2}$ (M being mass of each nucleon) and $E_B \to 0$, we get

$$\frac{\sqrt{M V_0}}{\hbar} = \frac{\pi}{2r_0}$$

$$\Rightarrow \qquad V_0 r_0^2 \approx \frac{\pi^2 \hbar^2}{4M} \qquad \ldots(21)$$

For usual range $r_0 = 2 \times 10^{-15} m$, we get $V_0 \approx 25$ MeV

More Exact Solution

The exact solution of equation (19) may be derived as follows :

Let $k_1 r_0 = x$, so that equation (19) becomes

$$\cot x = -\frac{k_2}{k_1}$$

or $\qquad k_1 r_0 \cot x = -k_2 r_0$ (multiplying both sides by x)

$$\Rightarrow \qquad x \cot x = -k_2 r_0 \qquad \ldots(22$$

$$\Rightarrow \qquad \cot x = -\frac{k_2 r_0}{x} = y(\text{say})$$

As x lies between $\frac{\pi}{2}$ and π, let us put

$$x = \left(\frac{\pi}{2} + \epsilon\right)$$

S_0 that equation (22) can be rewritten as

$$\left(\frac{\pi}{2} + \epsilon\right) \cot \left(\frac{\pi}{2} + \epsilon\right) = -k_2 r_0$$

$$\Rightarrow \qquad \left(\frac{\pi}{2} + \epsilon\right) \tan \epsilon = k_2 r_0$$

As ϵ is small quantity $\tan \epsilon = \epsilon$, so
above equation becomes

$$\left(\frac{\pi}{2} + \epsilon\right) \epsilon = k_2 r_0$$

As ϵ^2 is very small quantity, we may write

$$\frac{\pi}{2} \epsilon = k_2 r_0$$

$$\mathbf{K_1\ r_0} = \frac{\pi}{2} + \frac{2k2\ \ r0}{\pi}$$

From this equation $V_0 r_0^2$ be calculated more exactly. $V_0 r_0^2$ is an important parameter in nuclear calculation, because this parameters enables to determine the shape and depth of nuclear for different kinds of interactions.

MESON THEORY OF NUCLEAR FORCES

In 1935, a Japanese Physicist H. Yukawa proposed a theory of nuclear forces, which is usually referred as meson-theory of nuclear forces. Yukawa proposed the existence of a new kind of field, now known as meson field, and pointed out that the nuclear forces arise due to continuous exchange of mesons (Yukawa particles) between the nucleons.

Yukawa took the analogy from the quantum field theories of electromagnetic field in which the exchange of photon takes place and gravitational field in which the exchange of graviton is assumed. Both these field particles (photon and graviton) have zero rest mass; but the nuclear field particle has a finite rest mass. The reason for this is that the nuclear force is short-range force while the other two fields are not. The rest mass Mit of the field particle may be computed as follows

When one nucleon exerts a force on the other, a meson is created. The creation of meson violates the conservation of energy by an amount AE, corresponding to meson rest mass

$$\Delta E = m_\pi \cdot C^2 \qquad \ldots(1)$$

The time for which meson can exist is determined by the uncertainty relation of wave mechanics given by

$$\Delta E = m_\pi \,.\, c^2 \qquad \ldots(1)$$

The time for which meson can exist is determined by the uncertainty relation of wave-mechanics given by

$$\Delta E\, \Delta t \sim \hbar \quad \Rightarrow \quad \Delta t \sim \frac{\hbar}{\Delta E} \qquad \ldots(2)$$

The maximum distance traversed by meson in this time at maximum possible speed *viz* the speed of light c is

$r_0 = c\Delta t$, Substituting value of Δt from (2)

$$r_0 = c\,\frac{\hbar}{\Delta E} = c\,\frac{\hbar}{m_\pi c^2} \text{ [using (1)]}$$

$$= \frac{\hbar}{m_\pi \,.\, c}$$

$$\Rightarrow \qquad m_\pi = \frac{\hbar}{r_0 c} \qquad \ldots(3)$$

Assuming range of nuclear force $r_0 = 1{\cdot}4\text{F} = 1{\cdot}4 \times 10^{-15}$ m, we get

$$m_\pi = \frac{(1{\cdot}05 \times 10^{-34} \text{ joule-second})}{(1{\cdot}4 \times 10^{-15} \text{ metre}) \times (3 \times 10^{8} \text{ metre/second})} \text{ metre}$$

$$= 0{\cdot}25 \times 10^{-27} \text{ kg.}$$

In terms of mass of electron $m_e = 9{\cdot}1 \times 10^{-31}$ kg.

$$m_\pi = \frac{0{\cdot}25 \times 10^{-27}}{9{\cdot}1 \times 10^{-31}}\, m_e = 270\, m_e$$

After Yukawa's hypothesis a search for π-mesons was started. In 1945 Powell and his co workers discovered π-mesons in cosmic radiation, having mass 273 m_e. This is in excellent agreement with theoretical mass of π-mesons (= 270 m_e). The π -mesons are of three kinds positive (π +), negative (π^-) and neutral (π^0); all have intrinsic spin I= 0. Their rest masses are 139•6, 139•6 and 135•0 MeV respectively.

The force field between two protons or two neutrons is carried by a neutral π -meson, while that between a neutron and proton by a charged with the conversion of one nucleon into the other. The process is given below

Exchange of Neutral Pion

$$\mathbf{n^0 \rightarrow n^0 + \pi^\circ}$$

$$\mathbf{p + \pi^0 \rightarrow p+}$$

Exchange Charged Pion

$$n_1 \rightleftharpoons p_1^+ + \pi^- ; \qquad p2 + \pi \rightleftarrows n_2$$

$$P_1^+ \rightleftarrows n_1 + \pi^+ \qquad n_2 + \pi^+ \rightleftarrows p_2$$

In the first case the first nucleon (neutron n_1) emits a negative pion that is absorbed by second nucleon (proton P_2). In the exchange process n_1 is converted into proton p_1 while another proton p_2 is converted into a neutron n_2. The process then goes in opposite direction .Similarly in second case the exchange is through positive pion (π^+).

Before the discovery of pion (π-meson); the muon (u-meson) was discovered and taken for Yukawa particle. The spin of muon is 1/2 ; the exchange of half-spin particles violates the law of conservation of angular momentum; hence after discovery of pion, the pion was assumed to be a good Yukawa particle.

According to Yukawa the pion exchange among nucleons gives rise to strong attractive force, now called the exchange force. The π-mesons are regarded as the quanta for the nuclear field. A nucleon is regarded as a source of field quanta and hence of meson-field.

A nucleon is supposed to be surrounded by virtual photons. The nature of potential function of meson-field

$$E^2 = p^2c^2 + m^2c^4 \quad \ldots(4)$$

In Quantum Mechanics the energy operator is $\hat{E} = i\hbar \frac{\partial}{\partial t}$ and momentum operator $\hat{p} = \frac{\hbar}{i}\nabla$.

Substituting values of E and p in operator form, we get

$$\left(i\hbar \frac{\partial}{\partial t}\right) \cdot \left(i\hbar \frac{\partial}{\partial t}\right) = \left(\frac{\hbar}{i}\nabla\right) \cdot \left(\frac{\hbar}{i}\nabla\right) c^2 + m^2 c^4$$

$$\Rightarrow \quad -\hbar^2 \frac{\partial^2}{\partial t^2} = -\hbar^2 c^2 \nabla^2 + m^2c^4$$

$$\Rightarrow \quad \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2c^2}{\hbar^2} = 0 \quad \ldots(5)$$

If $\phi\,(r, t)$ is the pion-wave-function, then the wave equation for pion takes the form

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\right)\phi\,(r, t) = 0 \quad \ldots(6)$$

This is relativistic Schrödinger equation (also called Klein-Gordan equation) for a free particle of spin 0. If we set $m \to 0$, then equation (6) takes the form

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) \cdot \phi(r, t) = 0 \qquad \ldots(7)$$

This is Maxwell's equation from which electromagnetic field is derived. This equation may be thought to be derived from the energy equation for particles of rest mass zero (*e.g.* photons)

$$E^2 - p^2c^2 = 0$$

The simplest type of electromagnetic field is the electrostatic field for which $\frac{\partial \phi}{\partial t} = 0$, so it obeys the equation

$$\nabla^2 \phi = 0 \quad (\textit{Laplace's equation}) \qquad \ldots(8)$$

Its solution is

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{e}{r} \qquad \ldots(9)$$

For π-mesons, equation (6) may be expressed as

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \frac{m_\pi^2 c^2}{\hbar^2}\right)\phi = 0 \qquad \ldots(10)$$

The time independent part of equation is

$$\left(\nabla^2 - \frac{m_\pi^2 c^2}{\hbar^2}\right)\phi = 0 \qquad \ldots(11)$$

Setting $\beta = \frac{m_\pi c}{\hbar}$, we get

$$\left(\nabla^2 - \beta^2\right)\phi = 0 \qquad \ldots(12)$$

From equation (3), $\beta = \frac{m_\pi c}{\hbar} = \frac{1}{r_0}$ ...(13)

Equation (l l) is valid for a meson field in the absence of source term is analogous to Laplace's equation in electromagnetic field in the absence of charge density function. In the presence of charge density function p, the Poisson's equation $V^2\phi = -\frac{\rho}{\epsilon}$ holds. If g

represents the source strength analogous to charge density p in electromagnetic field, then equation (12) modifies to

$$(\nabla^2 - \beta^2)\,\phi = \frac{1}{\epsilon_0}\, g\, \delta(r)$$

where g is measure of nuclear field and is called the *mesonic charge.*

$\delta(r)$ is the Dirac-delta function

$$\left.\begin{aligned}\delta(r) &= 1 \text{ at } r = 0\\ &= 0 \text{ at } r \neq 0\end{aligned}\right\}$$

The solution of equation (14) may be expressed as :

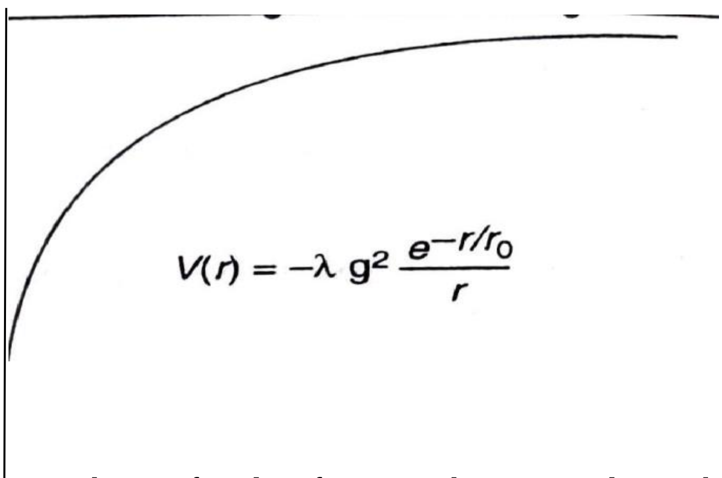
$$\phi(r) = -\lambda g\,\frac{e^{-\beta r}}{r}$$

The meson potential will be

$$\mathbf{V} = \mathbf{g}\boldsymbol{\varphi}(\boldsymbol{r}) = -\boldsymbol{\lambda g}\ \frac{\boldsymbol{e2(-\beta r)}}{\boldsymbol{r}}$$

This is the required Yukawa potential and shown graphically.

r(fermi)



V(r)

In this simple meson theory of nuclear-forces, we have ignored a number of important factors. For example pion has negative intrinsic parity, therefore it cannot be transferred from a neutron to a proton on S-state (l = 0) and still conserve parity. To conserve angular momentum and parity both, the only possible state for the pion is P-state (l = 1).

UNIT V

ELEMENTARY PARTICLES

INTRODUCTION

The problems concerning the elementary particles are today the focus of interest and of research for the experimental as well as the theoretical physicists. Experimental investigations of elementary particles all involve some source of particles to study and some way of detecting those particles and measuring their behaviour.

Many of the practical problems of such investigations are caused by the fact that particles are unstable. The investigation of electromagnetic phenomena suggested that the atom had an internal structure. At that time the typical photo-type of the elementary particle was the electron. The problem of the dualistic nature of matter was resolved by the quantum theory of fields, the elementary particles are nothing but the quanta of a corresponding field. The study of elementary particles is the basis to the understanding of radiation phenomena, or one may regard any kind of radiation as a flux of elementary particles.

In 1932, when Chadwick identified the neutron and Heisenberg suggested that atomic nuclei consisted of neutrons and protons, it seemed if. p, n and e were sufficient to account for the structure of matter. Besides these, there was the Photon, the intermediary or field particle for electromagnetic forces.

Which exists between the nucleus and electrons in thé atom. If anti-matter exists it would then be made up of anti-electrons, i.e. positrons, anti-protons an anti neutrons Thus we see that seven particles could explain both matter and anti-matter. In 1935, Yukawa postulated the existence of another particle with a mass $m \sim 200\ m_e$ the field particle for the strong nuclear forces.

Recently the extensive studies made partly on high energy cosmic ray particles and even more with the help of high energy accelerators have revealed the existence of numerous new nuclear particles. Apart from a dozen or so the particles have very short lifetimes, very much less than 10^{-6} sec. They cannot therefore be regarded as normal constituents of matter. They are characterised by the parameters, mass, spin, electric charge and magnetic moment.

They have been described by such adjectives, as fundamental, strange and elementary, but none of these is quite appropriate. The word fundamental implies that the particles are the basic building blocks of matter, but un stability of most of the particles indicates that the great majority are certainly not. It is true that their behaviour was strange in the early 1950, but it is much less now. For the want of better one the term elementary particles is now commonly used. These particles are elementary in much the same sense as are chemical elements.

According to the ancient Indian literature, the physical' world is composed of five bhutas basic elements); (i) Kshiti (earth), (ii) Ap (water), (iii) Taijas (fire), (iv) Vayu (air) and (v) Akasha (ether) five elements theory of matter is based upon the fact that our senses are being affected by the bhutas in five different ways. These elements were not considered as ordinary earth, water, fire, air and ether, but as basic principles. These may represent the solid, liquid, plasma, gaseous and field states of matter respectively.

According to the recent studies, the fundamental constituents of matter are three generations of leptons and three generations of quarks. According to the quantum field theory, the forces between the fundmental constituents of matter arise due to the exchange of gauge bosons. The mediators of fundamental forces are graviton, photon, bosons (W^+,W^-, and Z°) and 8 coloured gluons (G). The gluons and quarks are not seen as free particles.

CLASSIFICATION OF ELEMENTARY PARTICLES

The elementary particles are separated into two general groups called bosons and fermions. These two groups have different types of spin and their behaviour is controlled respectively by a different kind of statistic (i.e. the Bose statistic or the Fermi statistic, hence the names). Bosons are particles with intrinsic angular momentum equal to an integral multiple of h .

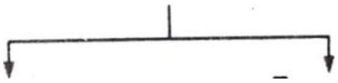
Fermions are all those particles in which the spin is half integral. The most important difference between the two classes of particles is that there is no conservation law controlling the total number of bosons in the Universe, whereas the total number of fermions strictly conserved.

Boson is a term, which not only includes material particles but also includes those quanta and photons which arise from interactions. Thus in the case of the simple electromagnetic field the bosons are merely the light photons or the X-ray photons. The photon has a mass of zero and a spin of unity and is consequently described as a massless boson. A massless boson, called a graviton with a probable spin of two units has been postulated as a field particle for gravity.

The weak nuclear interactions are mediated by heavy vector bosons (W^+,W^-, and Z) also called weak bosons. Each carries a spin of unity and mass of the order of 90 GeV. The strong interactions, are mediated by massless neutral gluons. Each carries a spin of unity. All these bosons are the quanta of gauge fields and are thus called the gauge bosons.

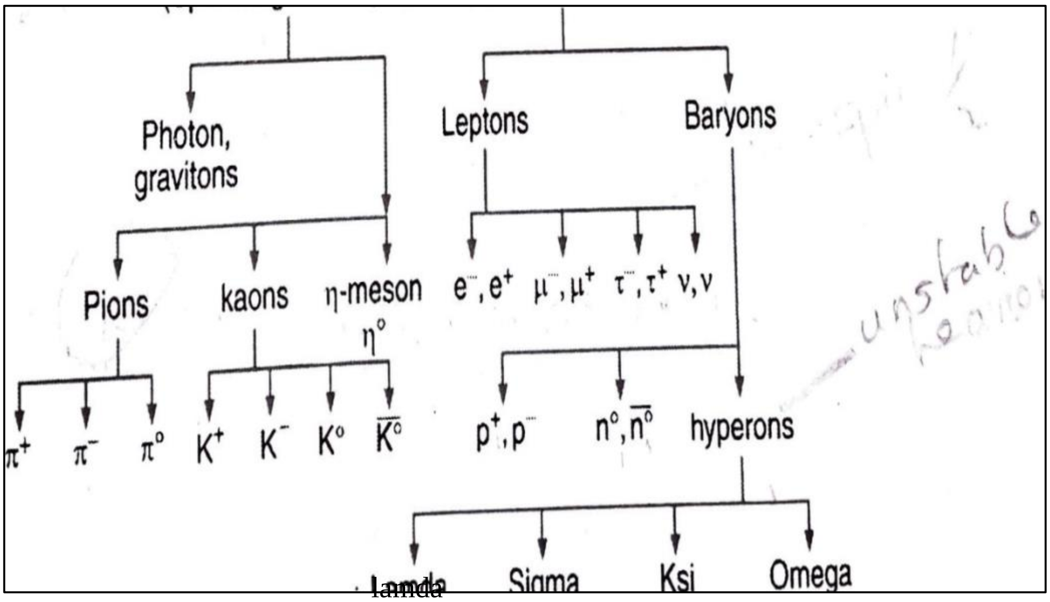
These bosons, created by the field interactions are essentially of one kinds, while the bosons formed in the strong interaction are of two distinct kinds. First there are those which are known as pions or π-mesons (π , π and π). The second group of bosons are much heavier than that of pions, and are known as kaons or K-mesons (K^+, K^-, K^O) and η^0-meson.

Classification of Elementary Particles
Elementary Particles



Bosons (Spin integral multiple)

Fermions (Spin half integral)



hyperons Hyperon hyperon hyperons

The fermions fall in two main classes, according .to whether they are light or heavy. Those in the lighter group are often called leptons (after the Greek word meaning light in weight), while those in the heavier group are called baryons (after the Greek word for heavy). The leptons are the electron, muon, tau meson and respective neutrinos and their anti-particles. These are all with spin half and not made of quarks. Leptons with other particles.

The total number of leptons minus the total number of anti-leptons remains unchanged fn all reactions and decay processes involving leptons and anti-leptons. The baryons consist of the two nucleons with their anti-particles (n°, $\bar{n}^\circ$; p+, p-) and the hyperons. Hyperons are the extremely unstable somewhat heavier particles and can be divided into four sub-groups, Λ°-particle (a neutral particle of mass about 1116MeV/c^2), the Σ-particles (Σ^+, Σ° and Σ^-) with masses in the range 1189 to 1197 MeV/c^2), the Ξ-particles (Ξ^- and Ξ° with masses 1321 and 1315 MeV/c^2) and the Ω^- particle (of mass about 1672 MeV/c^2). There is no reason to doubt the existence of the anti-particles of these fermions.

The total number of baryons minus the total number of anti-baryons is absolutely conserved, in all interactions. The kaons and pions together with the baryons are placed into a group of strongly interacting particles, called hadrons, The quarks are the basic building blocks of hadrons.

There are certain quantum numbers which are necessary to describe the behaviour of elementary particles.

(*a*) **The nucleon number.** The nuclear interactions are charge independent. This suggests us to think that neutron and proton as the states of only one particle, the ***nucleon***, which is supposed to exist in one of the two states the proton and neutron characterised by charge +1 and 0.

The *nucleon number N* is defined as

$$N = \text{(number of nucleons)} - \text{(number of antinucleons)}$$

The nucleon number N is a quantum number and remains conserved in decay processes. When neutron converts into proton and vice–versa, the number of nucleons remains the same, neutron is unstable particle while proton is stable with a life time $\approx 10^{21}$ years.

(*b*) **The lepton number :** ***The quantum number for electron*** similar to necleon number is lepton number L defined as

$$L = \text{number of leptons} - \text{number of antileptons.}$$

This quantum number remains constant in decay processes. Accordingly the decay scheme for pions and muons is

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$

(*c*) **The baryon number.** The behaviour of nucleons, leptons and hyperons is collectively expressed by a new quantum number called the *baryon number B* which takes the values as follows:

$$B = +1 \text{ for nucleons and hyperons (baryons)}$$
$$B = -1 \text{ for antibaryons}$$
$$B = 0, \text{ for all other elementary particles.}$$

The baryon number remains constant in all interactions.

(*d*) **Spin quantum number.** The intrinsic spin of the particle is expressed by a quantum number known as *spin quantum number* denoted by s. In terms of spin quantum number, the *spin angular momentum*, according to wave mechanics, is expressed as

$$L_S = \sqrt{[s(s+1)]}\,\hbar.$$

The spin quantum number s takes half integral values for Fermions *e.g.* electron, proton, neutron etc. and integral values for Bosons *e.g.* photon, common isotopes of He^4. The intrinsic spin is a property that arises due to invariance of the particle wave functions, under rotation. The invariance gives two kinds of functions, symmetric or antisymmetric, which have respectively an even or odd parity.

The Fermi particles (Fermions) are described by antisymmetric wave functions and obey Fermi Dirac Statistics, while Bose particles (Bosons) are described by symmetric wave functions and obey Bose–Einstein Statistics.

(*e*) **Isospin and Iso–spin quantum number.** We know that neutrons and protons have almost the same mass, same intrinsic spin and moreover the nuclear forces are charge independent. These results led Heisenberg to think that these two particles are different charge manifestation of the same entity–the nucleon. Thus the particles are grouped like isotopic grouping of the elements. The charge of the nucleon is treated as variable for different states of the nucleon and this variable is described by a new quantum number known as *isospin quantum number.* This variable plays similar role as spin quantum number. The isospin quantum number is $I = 1/2$ for nucleon and $I = 1$ for pion. The different states of particle are differentiated in terms of the Z – component of isospin vector *viz.*

For nucleon $$I_S = \begin{cases} \frac{1}{2} \text{ for proton state} \\ -\frac{1}{2} \text{ for neutron state} \end{cases}$$

For pion $$I_S = \begin{cases} +1 \text{ for } \pi^+ \text{ state} \\ 0 \text{ for } \pi^0 \text{ state} \\ -1 \text{ for } \pi^- \text{ state} \end{cases}$$

(*f*) **Strangeness quantum number S.** Every elementary particle is assigned a strangeness number in such a way that the *strangeness* is conserved in the reaction. This quantum number was introduced by Gellmann and Nishima by observing the strange behaviour of κ-mesons and hyperons. These particles have the peculiar property that they are produced readily in high energy nucleon-nucleon collisions but decay only very reluctantly by way of weak interactions. Another strange property of these particles is that they are produced in pairs. The strangeness number (S) is assigned zero value for particles which are not strange, but finite non-zero value for strange particles.

For Σ^+ hyperons, $S = -1$

For κ^+ meson, $S = +1$

For protons and π –mesons $S = 0$

Consider the reaction $\pi^+ + p \rightarrow \Sigma^+ + \kappa^+$

is allowed because strangeness is conserved, because the sum of the strangeness number is zero for the product particles as well as for the reactants.

Conservation of Hypercharge

Like the electric charge, the additive quantum number the strangeness is conserved in the production process. The strange particles are produced in pairs but may decay weakly. Thus it is convenient to introduce a quantum number hypercharge denoted by Y, which is the sum of the baryon number B and strangeness S.

It is convenient, since it treats baryons and mesons on an equal basis. Since the baryon number is always conserved, Y is a good quantum number conserved in strong interactions. Unlike the electric charge, the hypercharge does not conserve in weak interactions.

Thus the charge-is-related with isospin, baryon number and strangeness as

$$\mathbf{Q = T_3 + \frac{1}{2}\,Y = \ T_3 + \frac{1}{2}\ B + \frac{1}{2}}\ S$$

This is called Gell-Mann-Nishijima relation. This tells us that in deciding whether strong interaction takes place we need to check only three out of four quantities Q, T_3 B and S.

For strange mesons, B = 0 and the above relation reduces to

$$\mathbf{Q = \ T_3 + \frac{1}{2}\ S}$$

Similarly for non-strange particles, S = 0 and we have

$$\mathbf{Q = \ T_3 + \frac{1}{2}\ B}$$

To cover a whole range of strongly interacting particles, the formalism has also been extended to mesons. Since all the hyperons have negative strangeness, they can be produced only in association with K-mesons of positive strangeness. On the assumption that strangeness is a conserved quantity in a reaction such as

$$\mathbf{p^+ + p^+ \rightarrow p^+ + \Lambda^\circ + K^+}$$
$$\mathbf{0 + 0 \longrightarrow 0 + (-1\ ) + s_k}$$

It gives that the strangeness S_k = Similar argument gives strangeness S = I for neutral kaon. From the associated production reaction

$$\pi + p \longrightarrow \Lambda^\circ + K^\circ$$
$$S\pi + 0 = -1 + 1$$

It follows that S = 0. It also applies to $\pi, \pi, and\ \eta,$ mesons. Since for mesons the baryon number B = 0, hence strangeness S = hypercharge Y.

Thus we see that strangeness is not an independent new quantity, but is related to a combination of Q, T_3 and B, each of which is regulated by conservation laws.

Let us now see what kinds of particle type can be formed by various choices of T, B and S.

If $S = 0$, *there are three possibilities,*

(a) $B = 0$, $T = 0$ yields $Q = 0$ (*neutral meson*), η° *meson.*

(b) $B = 0$, $T = 1$ yields $Q = +1, 0, -1$ (*pions*).

(c) $B = 1$, $T = \frac{1}{2}$ yields $Q = +1, 0$ (*nucleons*).

If $S = 1$, *the multiple charge can be avoided only when*

(a) $B = 0$, $T = \frac{1}{2}$ and $Q = +1, 0$ (K^+, K°).

(b) $B = 1$, $T = 0$ and $Q = +1$ (*Baryon singlet*).

If $S = -1$, *the multiple charges can be avoided only when*

(a) $B = 0$, $T = \frac{1}{2}$ and $Q = 0, -1$ ($\bar{K}^\circ$, K^-).

(b) $B = 1$, $T = 0$ and $Q = 0$, (*singlet baryon* Λ°).

(c) $B = 1$, $T = 1$ and $Q = +1, 0 - 1$ (hyperons Σ^+, Σ° and Σ^-).

If $S = -2$, *the multiple charges can be avoided only when*

(a) $B = 1$, $T = \frac{1}{2}$ and $Q = 0, -1$ (hyperons Ξ°, Ξ^-).

If $S = -3$, *the multiple charges can be avoided only when*

(1) $B = 1$, $T = 0$ and $Q = -1$ (Ω^--hyperon).

All these results are summarised in the table 18.3.

Table 18.3. *Gell-Mann-Nishijima Scheme*

S	B	T	T_3	Q	*Particle*	S	B	T	T_3	Q	*Particle*
0	0	0	0	0	η^0	−1	0	1/2	+1/2	0	$\bar{K}^\circ$
	0	1	+1	+1	π^+				−1/2	−1	K^-
			0	0	π^0		1	0	0	0	Λ°
			−1	−1	π^-		1	1	+1	+1	Σ^+
	1	1/2	1/2	+1	p				0	0	Σ°
			−1/2	0	n				−1	−1	Σ^-
1	0	1/2	+1/2	1	K^+	−2	1	1/2	1/2	0	Ξ°
			−1/2	0	K°				−1/2	−1	Ξ^-
	1	0	0	1	Baryon	−3	1	0	0	−1	Ω^-

In the case of hadrons, the strangeness must be conserved ($\Delta S = 0$) for fast reactions. The decays of kaons and hyperons are very slow because they involve a breakdown of strangeness conservation. The decays $\Xi^- \to n + \pi^-$ and $\Xi^\circ \to p + \pi^-$, which involve $\Delta S = 2$, would be expected to be exceptionally slow and the decays $\Xi^- \to \Lambda^\circ + \pi^-$ and $\Xi^\circ \to \Lambda^\circ + \pi^\circ$ with $\Delta S = 1$ to be slow. Since charge and baryon number are always conserved, hence for weak decay processes, equation (5) gives

$$|\Delta S| = 2\,|\Delta T_3|$$

or

$$|\Delta T_3| = \tfrac{1}{2}.$$

As T_3 is the component of total isospin along a particular direction, hence the general form of above equation for weak decays is

$$|\Delta T| = \tfrac{1}{2}.$$

Let us apply conservation laws to high energy pion-nucleon collisions which often give large quantities of kaons. Examples of possible equations are

$$\pi^+ + n^\circ \to \Lambda^\circ + K^+$$
$$\to K^\circ + K^+.$$

Using the conservation of baryons and strangeness, we see that the second reaction violates baryon conservation and therefore cannot occur. Similarly equation $\pi^- + p^+ \rightarrow \Lambda^o + K^o$ is possible, whereas $\pi^- + p^+ \rightarrow \Lambda^o + \pi^o$ is not possible due to strangeness violation.

Let us consider equation $p^- + p^+ \rightarrow 2\pi^+ + 2\pi^- + \pi^o$. Applying various conservation laws and remembering that the pairs of pions are ejected with opposite isospins, we have

$$Q = -1 + 1 \rightarrow 2 - 2 + 0, \quad \therefore \quad \delta Q = 0$$

$$B = -1 + 1 \rightarrow 0 + 0 + 0, \quad \therefore \quad \delta B = 0$$

$$T = 1/2 + 1/2 \rightarrow 0 + 0 + 1, \quad \therefore \quad \delta T = 0$$

$$Y = -1 + 1 \rightarrow 0 + 0 + 0, \quad \therefore \quad \delta Y = 0$$

$$S = 0 + 0 \rightarrow 0 + 0 + 0, \quad \therefore \quad \delta S = 0.$$

(d) Charge Conjugation. Charge conjugation is the operation which changes the sign of the charge of a particle without affecting any of the properties unrelated to charge. It may be defined as the transformation between a particle and its antiparticle. It has been found experimentally that the operation of both the strong and electromagnetic interactions is invariant to charge conjugation. For instance, such invariance is found experimentally in the strong interaction annihilation of a proton and an antiproton into the particle antiparticle pair K^+K^- and is also found in measurements of the electromagnetic decay of the η^o meson. We thus believe that the nucleus of the antideuterium atom (strong interaction behaviour) and also the positron (electromagnetic interaction behaviour) would act in the same way. *The charge conjugation is not conserved in the weak interaction, i.e.*, the weak interaction does distinguish between a system and its charge conjugate. The charge conjugation does not simply mean a change over the opposite electric charge or magnetic moment, the sign of other charge quantum numbers [hypercharge Y, baryon number B, lepton numbers (l_e, l_μ)] is also reversed without changing mass M and spin s. Thus a unitary operator, also known as charge conjugation operator C, satisfies the following relations :

$$CQC^{-1} = -Q, \; CYC^{-1} = -Y, \; CBC^{-1} = -B, \; Cl_eC^{-1} = -l_e \text{ and } Cl_\mu C^{-1} = -l_\mu. \qquad \ldots(8)$$

Some elementary particles *e.g.*, γ, π^o and η^o – mesons and the positronium atom $(e^- + e^+)$ are transformed into themselves by charge conjugation. They are their own anti-particles. These are known as *self-conjugate or true neutral particles.* The neutron $(B = 1, Y = 1)$ and K^o-mesons $(Y = 1, B = 0)$ are not invariant under C

A system is said to possess charge conjugation symmetry or to be invariant under charge conjugation if the system (or the process) is such that it is impossile to know that it has undergone charge conjugation. For example, the operation C converts the negative pion decay $(\pi^- \rightarrow \mu^- + \bar{\nu}_\mu)$ into the positive pion decay $(\pi^+ \rightarrow \mu^+ + \nu_\mu)$, since the π^+ is the anti-particle of π^-. Until about four decades ago it was believed that the entire universe is invariant under C. In the end of 1956, experiments revealed that weak interactions violated it. It turned out that the μ^+ and μ^- decay electrons have angular distributions of opposite asymmetry : that the π^+ and π^- decay muons have opposite polarizations and that while a free neutrino is left handed an anti-neutrino is right handed. Charge conjugation applied to a free moving neutrino then results in a process which does not exist in nature.

Positronium is a hydrogen like bound state of an electron and a positron. According to the Dirac's heory the charge conjugate, which corresponds to the wave functions of positrons, has the form

$$C = ie^{i\phi}\begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix} = e^{i\phi} i\,\alpha_y. \qquad \ldots(9)$$

The phase ϕ is arbitrary. For zero phase $(\phi = 0)$

$$C = i\alpha_y, \qquad \ldots(10)$$

here α_y is a square matrix and σ_y is the Pauli spin matrix having value

$$\sigma_y = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} \quad ...(11)$$

For a single photon state

$$C|\gamma> = -|\gamma>. \quad ...(12)$$

For a system of n-photons

$$C|n\gamma> = (-1)^n |n\gamma>. \quad ...(13)$$

Thus the n-photon state is also in an eigenstate of C with the eigen value $(-1)^n$. Since π° mesons decay through electromagnetic interaction into two photons $\pi^\circ \rightarrow 2\gamma$, it follows that the π°-meson is self conjugate and that $|\pi^\circ>$ is an eigenstate of C with eigenvalue + 1, *i.e.*,

$$C|\pi^\circ> = |\pi^\circ>$$

On the other hand $|\pi^+>$ and $|\pi^->$ are not eigenstates of C as

$$C|\pi^+> = -|\pi^-> \text{ and } C|\pi^-> = -|\pi^+>$$

As the triplet spin state is symmetrical and the singlet spin state anti-symmetrical, hence to exchange an electron with a positron we must introduce a factor $(-1)^{s+1}$ as well as a factor $(-1)^l$. Thus we have

$$(-1)^{l+s+1} C = -1 \text{ or } C = (-1)^{l+s}. \quad ...(14)$$

and the positronium atom is in an eigenstate of C with eigenvalue $C = (-1)^{l+s}$.

Above relation gives that the singlet ground state ($l = 0$) decays into two photons and the triplet ground state decays into three photons. Similarly a system composed of two identical bosons of opposite charges, such as π^+, π^-, is in an eigenstate of C. Since the wave function of two bosons remains unaltered under the exchange of the two particles including charge, the eigenvalue of C is given by $C = -1$.

(e) Space-inversion Invariance (parity). The parity principle says that there is a symmetry between the world and its mirror image. This may be defined as reflection of every point in space through the origin of a co-ordinate system $x \rightarrow -x$, $y \rightarrow -y$ and $z \rightarrow -z$ or $\boldsymbol{r} \rightarrow -\boldsymbol{r}$. If a system or process is such that its mirror image is impossible to obtain in nature, the system or process is said to *violate the law of parity conservation.*

Human body is a good example of mirror symmetry. The body of a car is symmetric except for the position of the steering wheel. If we were looking at the mirror image of a normal car, it seems to violate the symmetry but it is not the case as it is also possible to design a car with the steering wheel on the other side. The mirror view of a printed page looks wrong. But there is nothing impossible about it. A printer could design inverted type and produced a page. One can read the page from right to left. The type of printing is not unnatural but is unconventional and unfamiliar. The values of the physical quantities of classical physics either remain unaltered or change sign under space inversion. Thus the quantities may have either even or odd intrinsic parity. The ordinary scalars such as temperature, electric charge, energy, have even parity. The ordinary vectors such as momentum, force, electric field have odd parity. Pseudovectors such as angular momentum, magnetic field, behave as vectors under rotations but have the even parity. Similarly pseudo scalars (the product of any ordinary polar vector with a pseudovector) have the odd parity.

All phenomena involving strong and electromagnetic interactions alone do conserve parity. In these cases the systems can be classified by the eigen values of the parity operator $\mathcal{P}$. For a single particle Schrodinger wave function ψ, the result of the parity operation is

$$\mathcal{P}|\psi(x)> = e^{i\alpha}|\psi(-x)>. \quad ...(15)$$

As α is an arbitrary real phase, hence can be set equal to zero.

$$\therefore \quad \mathcal{P}|\psi(x)> = |\psi(-x)> \quad ...(16)$$

and

$$P^2|\psi(x)> = |\psi(x)>. \quad ...(17)$$

It shows that eignvalues of $\mathcal{P}$ may be + 1 *or* – 1.

The parity of the photon depends upon the mode of transition, it is due to the change of the sign of electromagnetic current j under the parity operation. The nucleons and electrons are assigned positive (or even) intrinsic parity. The pions have negative (or odd) parity as they involve in strong interactions with nucleons. K-mesons and η° meson have negative parity.

⋋ ,Ξ, Σ, Ω hyperons have positive intrinsic parity. All anti-particles of spin ½ (the fermions) are of opposite parity to the corresponding particle, while the bosons and their anti-particles have the same parity.

The conservation of parity requires that the Hamiltonian of a free system commute with the parity operator.

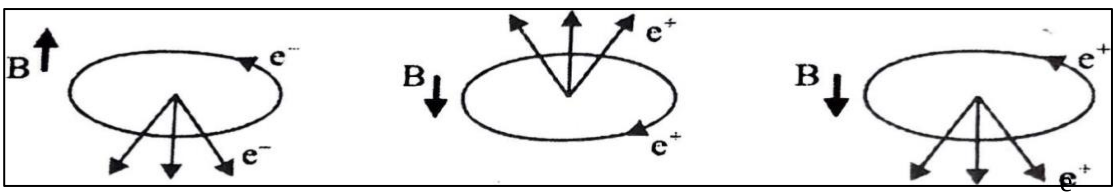
(PH-HP)=0

The transition probability must be scalar, it may contain pesudoscalar operator(I.p). The conservation of parity prevents the mixing of even and odd operators in the amplitude. Thus for the non-conservation of parity in β decay, the transition probability must contain both scalar and pesudoscalar terms or the number of electrons emitted parallel and anti-parallel to the spin of the source should be different.

The weak decay of the k mesons, which was difficult to reconcile with parity conservation and known as the τ-θ puzzle was explained by Lee andYang. They suggested that the weak interactions was not invariant to space reflection. In 1956, Wu and others, using polarized $^{60}C_0$ nuclei, found that the direction of emission of electron in the transformation to ^{60}Ni was preferentially opposite to the spin direction. The value of the pesudoscalar I.p, where I is the nuclear spin and p the electron momentum, was measured and found to be different from 0.

INVARIANCE UNDER TIME REVEALS

Time Reversal: The time reversal operator is defined as that operator which reverses the direction of time, or the direction of all motions. Under this operation displacement, acceleration and electric fields remain invariant but momenta, angular momenta and magnetic fields invert their signs. If the-time reversed process is impossible to occur in nature we can say that the process violates time, reversal symmetry. Time reversal changes the direction of flow of time, like running the movie of a phenomenon backwards. The result is usually strange.



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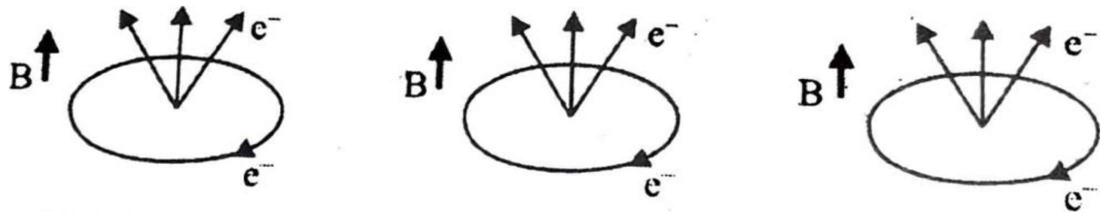

Time reversal invariance finds its simplest application in the world of particles, where it appears to govern the strong and electromagnetic interactions and possibly also the weak. It also shows that a particle possessing time reversal symmetry cannot have electric and magnetic dipole moments simultaneously. The time reversal process is the creation of an electron-positron pair by the collision of two photons.

If time reversal operation is applied to decaying $\pi^+ \rightarrow \mu^+ + \nu_\mu$. Under time reversal the momentum vectors are reversed and spins go around in the opposite directions. We have ν_μ and μ^+ with the proper handedness interacting to form a pion.

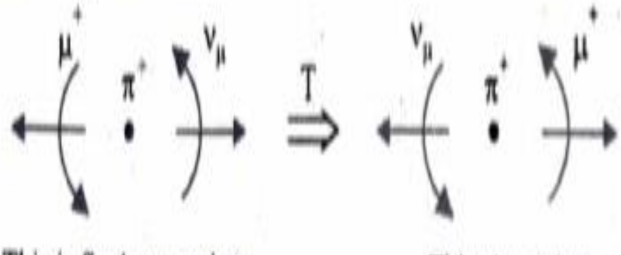

Fig. 18.5. Time reversal.

Time reversal invariance is satisfied in quantum mechanics if the Hamiltonian *H* is time independent and real. In this case $\psi^*(x, -t)$ is the time reversal wave function of $\psi(x, t)$. Thus time reversal operation T changes ψ as

Time reversal invariance is satisfied in quantum mechanics if the Hamiltonian H is time independent and real. In this case $\Psi^*(x, -t)$ is the time reversal wave function of v(x, t). Thus time reversal operation

$$T\Psi(x, t) = \Psi^*(x, -t)$$

where the asterisk indicates the complex conjugate.

The motion of a particle in an external fixed magnetic field is not invariant under inversion of time The relativistic treatment of time reversal shows that the inversion of time axis inverts the sign of the electrostatic potential. The π° - mesonic field, like the magnetostatic potential, is odd under time reversal in order to ensure that the interaction is time reversible.

Combined Inversion of CPT

The reflection symmetries are correlatd by the CPT theorem of G. Lüders and W. Pauli, sometimes known as Luders-Pauli theorem. This theorem states that the Lorentz invariant field theory is necessarily invariant to the combined operations of particle, anti-particle interchange(charge conjugation C), reflection of the coordinate system through the origin (parity, P) and reversal of time T. In other words CPT operator commutes with the Hamiltonian. The strong and the electromagnetic interactions are invariant under the separate operations of C, P and T. The weak interaction does not conserve parity and also is not invariant under charge conjugation.

All the interactions are invariant under the combined strong reflection CPT, irrespective of the order of the operations an so forth (TCP,TPC). No example of a violation of the CPT theorem is known. If follows that if T invariance is satisfied for all interactions, then these interactions will also be invariant under the combined operation of CP.

The existence of a violating interactions means that the analysis is not quite accurate. The other well known consequencs of CPT invariance are :

i)The equality of particle-antiparticle masses
(ii) equality of lifetimes of unstable particle and its antiparticle,
(iii) the magnetic moments of particle and antiparticle are equal and opposite.

The discovery of CP violations in 1964 in the system of neutral K-mesons reopened the question of CPT variance. Now observations show that at least 90% of the observed CP violation is compensated by a violation of time-reversal invariance. The less than 10% of the observed CP violation may be due to the violation of CPT This system suggests that further experimental work is required to test the validity of the CPT theorem.

The quantities conserved have the quantum numbers, which behave in two different ways, when one considers a system formed by the combination of two other systems. The quantum numbers, such as angular momentum, isospin, strangeness, baryon number, lepton number, electric charge are called additive. On the other hand, quantum numbers, such as parity, invariance under charge conjugation, invariance under time reversal are called multiplicative.

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