KUNTHAVAI NAACHIYAR GOVT. ARTS COLLEGE FOR WOMEN(A), THANJAVUR

DEPARTMENT OF PHYSICS

M.SC. PHYSICS - STUDY MATERIAL

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SUB TITLE: NUCLEAR AND PARTICLE PHYS

UNIT II

RADIOACTIVE DECAYS

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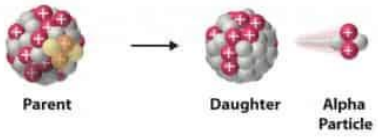
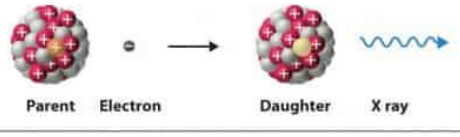
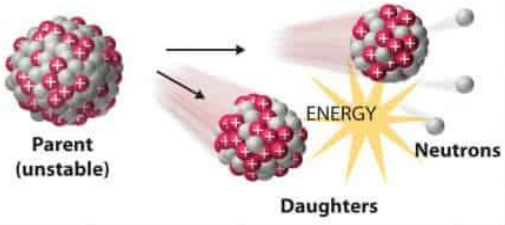
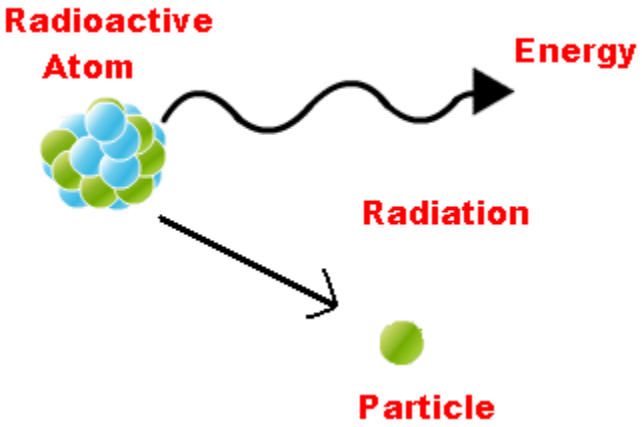
RADIOACTIVE DECAYS

Decay Type	Radiation Emitted	Generic Equation	Model
Alpha decay	$^{4}_{2}\alpha$	$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}X' + {}^{4}_{2}\alpha$	
Beta decay	$^{0}_{-1}\beta$	$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}X' + {}^{0}_{-1}\beta$	
Positron emission	$^{0}_{+1}\beta$	$^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}X' + {}^{0}_{+1}\beta$	
Electron capture	X rays	$^{A}_{Z}X + {}^{0}_{-1}e \longrightarrow {}^{A}_{Z-1}X' + X\text{ ray}$	
Gamma emission	$^{0}_{0}\gamma$	$^{A}_{Z}X^{*} \xrightarrow{\text{Relaxation}} {}^{A}_{Z}X' + {}^{0}_{0}\gamma$	
Spontaneous fission	Neutrons	$^{A+B+C}_{Z+Y}X \longrightarrow {}^{A}_{Z}X' + {}^{B}_{Y}X' + C\,{}^{1}_{0}n$	

When an atom undergoes radioactivity, it emits particles like alpha, beta and gamma rays. Radioactivity basically occurs because the **unstable** atom tries to attain stability. Hence, when they are unstable, they eventually decay by emitting a particle transforming the nucleus into another nucleus, or into a lower energy state. This chain of decays continues till the nucleus attains the stage of stability.

There are basically **three types of radiations** that are emitted by radioactive particles. These three are called the **alpha, beta and gamma** rays. All these radiations are released from the nucleus of the atom. Though all three cause some ionization and have some penetration power, but their behaviour differs from the others.



Alpha rays or alpha particles are the **positively charged** particles. A highly energetic helium nucleus which contains two protons and two neutrons is called the alpha-particle. Alpha particles have the least penetration power but the greatest ionization power.

Emission of α particle

Radioactive substances emit α particles as a means of increasing their stability by reducing their size. An important question arises why do the radioactive nuclei emit α partiicles ($_2He^4$) rather than the protons themselves. The answer lies in the fact that to escape from a nucleus, α particle must have sufficient kinetic energy. Now the mass of the α particle is suffieiently smaller than the mass of its constituent nucleons. Therefore in the formation of α particles within the nucleus, a suffieient energy is released which is converted in to kinetic energy of α particle to escape. On the other hand the proton, in order to escape, would require the kinetic energy to be supplied from outside. When an α particle is emitted from a radioactive nucleus, the following conservation laws must be obeyed.

i**)Conservation of linear momentum:** Let the parent nucleus of mass M_p at rest decay into a daughter nucleus of mass M_d with the emission of an α particle of mass $M\alpha$. Then from the principle of conservation of linear momentum.

Initial momentum= Final momentum

$$\mathbf{M_p * 0 = M_d v_d + M\alpha\, v\alpha}$$

Where v_d and $v\alpha$ are the velocities of daughter nucleus and α particle respectively. They are must have equal and opposite momenta.

ii**) Conservation of charge and nucleon- number**: In analogy with all nuclear reactions, the total charge and the total number of nucleons must be conserved in α decay. If A is the nucleon number and Z is the atomic number of the parent nucleus, then

$$_{z}P^{A} = _{z-2}D^{A-4} + _{2}He^{4}$$

where P and D refer to parent and daughter nuclei. It is obvious from equation(2) that the sum of atomic numbers and nucleon- numbers on the two sides are equal.

iii) Conservation of mass-energy: In α decay Einstein mass energy relation holds. If Eα is the kinetic energy of α particle E_d that of daughter nucleus, then according to principle of conservation of mass energy:

$$M_pC^2=M_d\,C^2 + M_\alpha\,C^2 + E_d + E_\alpha$$

As **α** particle is emitted from a radioactive nucleus, it may be supposed that this **α** particle exists inside the nucleus at least for a very short time before its emission. According to Frankel **α** particles do not pre-exist in the nucleus but their formation occurs in the process of **α** decay. This **α** particle can be held only be very strong but short range nuclear (attractive) forces. Experiments on scattering of **α** particles by heavy nuclei suggest that there exist strong repulsive force between **α** particle and heavy nucleus in accordance with Coulomb's law given by

$$F= \frac{1}{4\pi\epsilon}\frac{(z-2)e.2e}{r2}$$

Where (z-2) is the atomic number of daughter nucleus. This repulsive force is long –range force. If the **α** particle approached the nucleus from infinity(say), work has to be done on it against the Coulomb's repulsive force, which measure the potential energy of the system, when **α** particle is at a distance r from nucleus is given by

$$F= \frac{1}{4\pi\epsilon}\frac{2(z-2)e^2}{R}$$

It thus follows that as r decreases, the potential energy of the system increases so long as r > R,R being nuclear radius. This is shown by part BC. The potential energy is maximum at r=R and is given by

$$F= \frac{1}{4\pi\epsilon_0}\frac{2(z-2)e^2}{R_0A^{1/3}}$$

At distance less than R (i.e. within nucleus). Short-range strong nuclear forces come into play and the **α** particle gets confined to the nucleus. Due to attractive forces within nucleus the **α** particle has negative potential energy within the nucleus. This potential energy may be assumed to be constant equal to ($-U_0$). Hence the shape of potential energy versus separation curve. The region of negative potential from R to O is called the potential well of depth U_0 and width R; while the region of positive potential is called the potential energy barrier; the maximum potential energy U_{max} being known as barrer height.

The potential barriers, which prevents the α particles from entering the nucleus from outside, also, prevents the emission of these particles from the interior. Thus in order to escape from nucleus. The alpha particle must possess adequate energy to cross the barrier height.

Substituting numerical values for Uranium, we find that $U_{max} = 30MeV$. Thus according to classical concept an ∝ particle must have energy greater than 30MEv TO ESCAPE FROM NUCLEUS. But it has been observed that ∝ particles emitted by $_{92}U^{238}$ have energy 4.2 MeV. Classical physics which considers only the corpuscular nature of these particles can not explain how ∝ particles can cross a potential barrier which is nearly seven times as high as the energy possessed by them.

The first successful explanation of the problem or penetration of potential barrier was given by Gamow and independently by condon and Gurney in 1928, using concepts of wave mechanics. According to which there is always a finite probability that ∝ particle can leak through the barrier even when it has kinetic energy $E_{\propto}$ less than the barrier height (U_{max}).

Geiger- Nuttall Law

Different α particle emitters emit α-particles of different energies and hence of different ranges. The experimental observations show that longest lived α emitters emit least energetic particles, while shortest lived α emitters emit most energetic particles. On the basis of experimental observations Geiger –Nuttall obtained empirical relation, known as Geiger-Nuttal law, between the decay constant λ of α emitters and the range R (in air) of α particles emitted by it: given by

$$\log \lambda = a + b \log R$$

Where a and b are constants for the given radioactive series. It was further found that the constant b was the same for the three radioactive series viz. Uranium, thorium and actinium series, but the constant 'a' was different. If the relationship is represented graphically, the three straight lines for the three series are parallel to one another. More recent results show that Geiger - Nuttall law is only approximately correct: because some of radioactive elements of a particular series do not lie exactly on the straight line concerned.

Classical Physics could not give a satisfactory explanation of Geiger- Nuttall law. A proof of this law was given by Gamow on the basis of quantum physics. The theoretical study gives the law in the form

$$\log \lambda = c + d E^{-1/2}$$

where c and d are constants. The assumption made is that the changes in atomic number and nuclear radius are negligible as compared to the changes in energy E of the emitted particle.

Geiger–Nuttall law is found useful in estimating the half-lives of some elements belonging to a particular radioactive series, which can not be easily determined by direct observations.

FERMI THEORY OF BETA DECAY

In 1934 Enrico Fermi developed a successful theory of β decay, based pm Pauli's neutrino hypothesis. The assumptions of the theory are:

1. The radioactive nuclei emit electrons. These electrons do not pre-exist inside the nucleus, but are formed when a neutron transforms into a proton. Along with an electron, an antineutrino is also produced:

   $_0n^1 \rightarrow {}_1p^+ + e^- + \bar{\nu}$
   Neutron proton electron antineutrino
   On the other hand when a proton is converted into a neutron , a positive β - particle (positron) and neutrino are emitted.

   $_1p^+ \rightarrow {}_0n^1 + e^+ + \nu$
   proton neutron positron neutrino

2. β - decay obeys the law of conservation of energy. The available energy is shared among the electron and the neutrino. Due to larger mass of product nucleus, its kinetic energy is negligible.
3. The rest mass of neutrino is zero.
4. Electron- neutrino field is weak in contrast to strong nuclear force between nucleons. This means that the fundamental interaction. Hence the systems is ideal for time-dependent perturbation theory, which is an approximation of quantum mechanics. In this method, the Hamiltonian (energy) of system is splitted into two parts (i) Hamiltonian (energy) of unperturbed system (H^0) and (ii) interaction Hamiltonian due to weak interaction (H'). Here interaction is weak, therefore perturbation theory is an appropriate method for solution of problem.
5. According to time-dependent perturbation theory the rate of transition from an initial stat i to a final state f is given by

Where ρ(E) is the density of quantum mechanical states at energy E in iithe small energy ranged for final states, and H_{if} is the matrix element of interaction causing the transition.

Interaction Matrix Element

The matrix element of interaction H_{if} is defined as

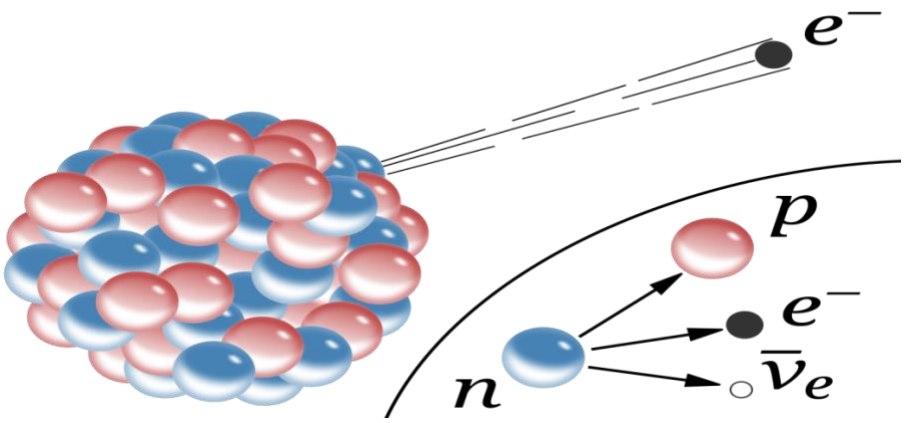
$$H_{if=} \Psi_f^* H' \Psi_i \, d\tau$$

where Ψ_{if} is wave function for final system Ψ_i is wave function for final system of initial system, H' is the Hamiltonian (energy) operator which describes the weak interaction between initial and final states and $d\tau$ is the small volume element. The physical situation of equation (1) is as follows:

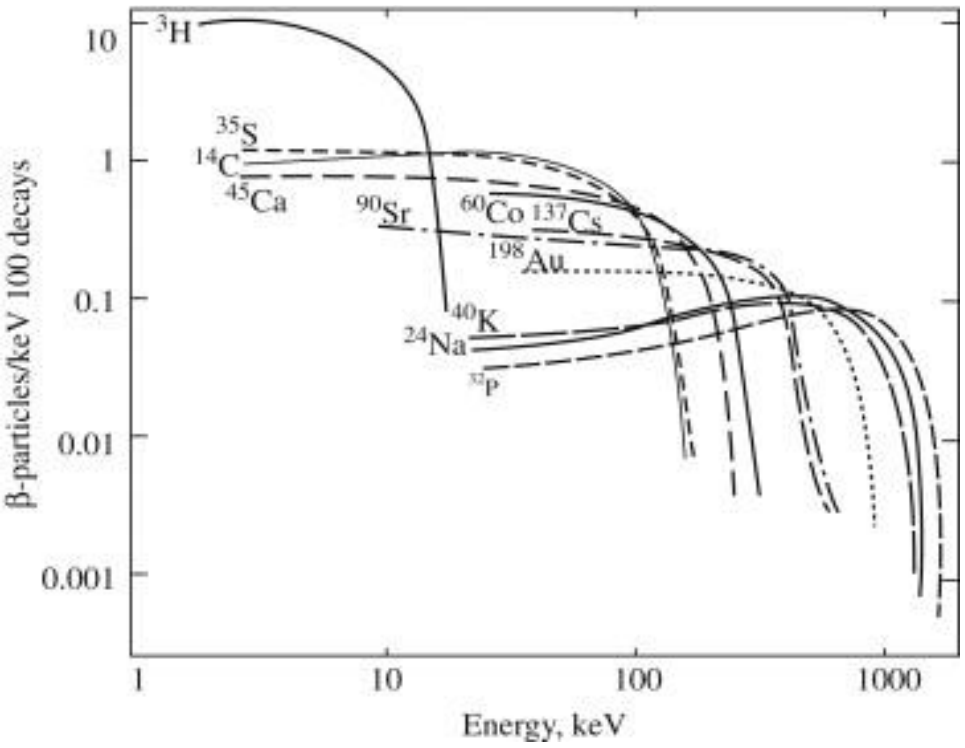
The parent nucleus in a given quantum-mechanical state can change to a daughter nucleus in a given state plus two leptons (electron and antineutrino) due to perturbing interaction H'. The two particles can travel in any direction from the nucleus and can share the available energy between them in numerous different ways. As a result there is a large number of quantum mechanical states of the final system. The energy of initial system (parent nucleus) is not perfectly sharp, so that it can overlap a certain number of states of final system.

The decay can take place to any one of the available number of final states with slightly different energies, without violating the law of conservation of energy. For simplicity we assume that the state3 of daughter nucleus is sharp, so that the uncertainty in energy i9s entirely due to the energy shared by two leptons. We may write

$\rho(E) = \frac{dN}{dEe}$, where E_e is the energy of emitted electron (β) particle



Beta Decay



Energy spectrum of β Decay

For β^--decay

${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \bar{\nu}$, therefore initial wave function $\psi_i = \psi_{iN}$ (say), where ψ_{iN} refers to normalised wave-function of parent nucleus and $\psi_f = \psi_{fN}\, \psi_e\, \psi_{\bar{\nu}}$ where ψ_{fN} is the normalised wave function of daughter nucleus, ψ_e = normalized wave-function of electron (β-particle), $\psi_{\bar{\nu}}$ = normalised wave-function of another lepton (antineutrino).

We do not know the form of interaction operator H′, but Fermi suggested that H′ should be replaced by gM, where g is a constant, which determines the strength of interaction and is called the *Fermi-coupling coefficient* and M is a dimensionless operator. Now equation (2) can be written as :

$$H'_{if} = g\int \left(\psi_{fN}^{*}\, \psi_e^{*}\, \psi_{\bar{\nu}}^{*} \right) M\, \psi_{iN}\, d\tau \qquad \ldots(3)$$

If V is the volume of system, then for weak field, the two leptons may be treated as free particles and so their normalised wave functions may be expressed as

$$\left.\begin{aligned} \psi_e &= V^{-1/2}\, e^{-i\vec{k}_e \cdot \vec{r}} \Rightarrow \psi_e^{*} = V^{-1/2}\, e^{i\vec{K}_e \cdot \vec{r}} \\ \psi_{\bar{\nu}} &= V^{-1/2}\, e^{-i\vec{k}_{\bar{\nu}} \cdot \vec{r}} \Rightarrow \psi_{\nu}^{*} = V^{-1/2}\, e^{i\vec{k}_{\bar{\nu}} \cdot \vec{r}} \end{aligned}\right] \qquad \ldots(4)$$

Substituting these values in (3), we get

$$H'_{if} = \frac{g}{V}\int \psi_{fN}^{*}\, e^{i(\vec{k}_e + \vec{k}_{\bar{\nu}}) \cdot \vec{r}}\, M\, \psi_{iN}\, d\tau \qquad \ldots(5)$$

The exponential factor can be written as

$$e^{i(\vec{k}_e + \vec{k}_{\bar{\nu}}) \cdot \vec{r}} = 1 + i(\vec{k}_e + \vec{k}_{\bar{\nu}}) \cdot \vec{r} - [(\vec{k}_e + \vec{k}_{\bar{\nu}}) \cdot \vec{r}]^2 \qquad \ldots(6)$$

Now it can be shown that the wavelengths of leptons at energies considered are substantially longer than the nuclear dimensions; so the $k_e + k_{\bar{\nu}} = 2\pi\left(\frac{1}{\lambda_e} + \frac{1}{\lambda_{\bar{\nu}}}\right) r$ has the order <<1; hence in equation (6) we need to retain only units on RHS. When we substitute (6) in (5), the matrix element H'_{if} becomes non-vanishing and independent of energies of leptons. This gives us the *allowed transitions.* Thus now equation (5) may be expressed as

$$H'_{if} = \frac{g}{V}\int \psi_{fN}^{*}\, M\, \psi_{iN}\, d\tau = \frac{g}{V} |\, M_{if}\, | \qquad \ldots(7)$$

where M_{if} is the nuclear matrix element of the final and initial wave functions of the nucleus. It is also called the overlap integral of initial and final states of nuclei. It can be computed if the structure of nuclei is well-known.

GAMOW 'S THEORY OF α PARTICLE

The assumption of wave mechanical explanation are:

1. The α particle, before emission exists as such within the nucleus
2. It moves with constant speed within the nucleus and collides again and again at the barrier surface; but is kept inside the nucleus due to high potential barrier.
3. There is a small; but finite probability that the particle may leak through the barrier each time it collides with it. This is due to de-Broglie hypothesis of wave nature of particles.
4. Once the particle leaks through the barrier. It escapes from nucleus because of its kinetic energy and Coulomb's electrostatic repulsive force.

For convenience we consider rectangular potential barrier defined by

$$V(r) = 0 \text{ for } r < 0$$
$$= V_0 \text{ for } 0 < r \leq a$$
$$= 0 \text{ for } r > a$$

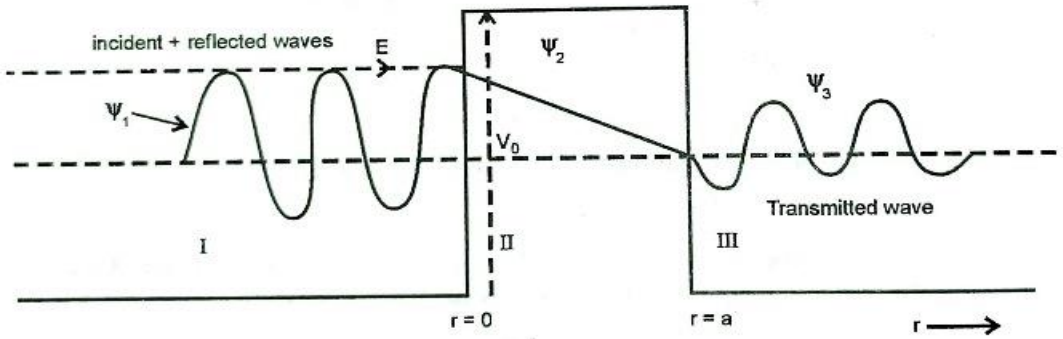


Fig. 4·9. Rectangular potential barrier

This is shown in fig. (4·9). If a particles of energy less than V_0 approaches this barrier from the left (region I), classically the particle will always be reflected and hence will not penetrate the barrier. However, the wave mechanics predicts that the particle has a finite probability of penetrating the barrier and transmitting to region III. The probability of penetration being greater if $(V_0 - E)$ and a are smaller. Moreover if $E > V_0$, classical mechanics predicts that the particle will always be transmitted; while according to wave-mechanics, the particle has a finite probability of transmission and hence it is not certain that the particle will penetrate the barrier. To solve the problem, let us write three Schrödinger's equation, one for each region. If ψ_1, ψ_2, ψ_3 are wave-functions in these regions respectively, the Schrödinger's equations for three regions are expressed as :

$$\frac{\partial^2\psi_1}{\partial r^2} + \frac{2m}{\hbar^2} E\psi_1 = 0 \qquad \text{(since } V = 0\text{)} \qquad \ldots(1)$$

$$\frac{\partial^2\psi_2}{\partial r^2} + \frac{2m}{\hbar^2} (E - V_0)\,\psi_2 = 0 \qquad \text{(since } V = V_0\text{)} \qquad \ldots(2)$$

$$\frac{\partial^2\psi_3}{\partial r^2} + \frac{2m}{\hbar^2} E\psi_3 = 0 \qquad \text{(since } V = 0\text{)} \qquad \ldots(3)$$

As first region contains both incident and reflected waves, the solutions of equation (1) may be expressed as :

$$\psi_1 = A_1 e^{ik_1 r} + B_1 e^{-ik_1 r} \qquad \ldots(4)$$

where $$k_1 = \frac{\sqrt{2mE}}{\hbar} \quad \ldots(4a)$$

As II region contains both waves, one transmitted at $r = 0$ and other reflected at $r = a$, the solution of equation (2) is

$$\psi_2 = A_2 e^{k_1 r} + B_2 e^{-k_2 r} \quad \ldots(5)$$

where $$k_2 = \frac{\sqrt{2m\,(V - E)}}{\hbar} \quad \ldots(5a)$$

As III region contains only forward moving transmitted wave, the solution of equation (3) is

$$\psi_3 = A_3 e^{ik_1 r} \quad \ldots(6)$$

In equations (4), (5), (6); A_1, B_1, A_2, B_2 and A_3 are constants to be determined by the boundary conditions that the wave functions and its first derivative exist and are continuous at the boundaries. Thus the boundary conditions are :

$$\left.\begin{aligned} \psi_1 = \psi_2 \text{ and } \frac{\partial \psi_1}{\partial r} = \frac{\partial \psi_2}{\partial r} \text{ at } r = 0 \\ \psi_2 = \psi_3 \text{ and } \frac{\partial \psi_2}{\partial r} = \frac{\partial \psi_3}{\partial r} = a \end{aligned}\right\} \quad \ldots(7)$$

The boundary condition $\psi_1 = \psi_2$ at $r = 0$ gives

$$A_1 + B_1 = A_2 + B_2 \quad \ldots(8)$$

Thus boundary condition $\frac{\partial \psi_1}{\partial r} = \frac{\partial \psi_2}{\partial r}$ at $r = 0$ gives

$$ik_1 A_1 - ik_1 B_1 = k_2 A_2 - k_2 B_2$$

$$\Rightarrow \quad A_1 - B_1 = \frac{k_2}{ik_1}(A_2 - B_2) \quad \ldots(9)$$

The boundary condition $\psi_2 = \psi_3$ at $r = a$ gives

$$A_2 e^{k_2 a} + B_2 e^{-k_2 a} = A_3 e^{ik_1 a} \quad \ldots(10)$$

The boundary condition $\left(\frac{\partial \psi_2}{\partial r} = \frac{\partial \psi_3}{\partial r} \text{ at } r = a\right)$ gives

$$A_2 k_2 e^{k_2 a} - B_2 k_2 e^{-k_2 a} = ik_1 A_3 e^{ik_1 a}$$

$$\Rightarrow \quad A_2 e^{k_2 a} - B_2 e^{-k_2 a} = i\frac{k_1}{k_2} A_3 e^{i k_1 a} \quad \ldots(11)$$

Solving equations (10) and (11) for A_2 and B_2, we have

$$A_2 = \frac{A_3}{2}\left(1 + \frac{ik_1}{k_2}\right) e^{(ik_1 - k_2)a} \quad \ldots(12)$$

and $$B_2 = \frac{A_3}{2}\left(1 - \frac{ik_1}{2}\right) e^{(ik_1 + k_2)a} \quad \ldots(13)$$

Solving equations (8) and (9) for A_1 and B_1, we have

$$A_1 = \frac{A_2}{2}\left(1 + \frac{k_2}{ik_1}\right) + \frac{B_2}{2}\left(1 - \frac{k_2}{ik_1}\right) \quad \ldots(14)$$

and $$B_1 = \frac{A_2}{2}\left(1 - \frac{k_2}{ik_1}\right) + \frac{B_2}{2}\left(1 + \frac{k_2}{ik_1}\right) \qquad \ldots(15)$$

Substituting values of A_2 and B_2 from (12) and (13) in (14), we get

$$A_1 = \frac{A_3}{4}\left(1 + \frac{ik_1}{k_2}\right)\left(1 + \frac{k_2}{ik_1}\right)e^{(ik_1 - k_2)a} + \frac{A_3}{4}\left(1 - \frac{ik_1}{k_2}\right)\left(1 - \frac{k_2}{ik_1}\right)e^{(ik_1 + k_2)a}$$

$$\Rightarrow \quad \frac{A_3}{A_1} = \frac{4e^{-ik_1a}}{\left(1 + \frac{ik_1}{k_2}\right)\left(1 + \frac{k_2}{ik_1}\right)e^{-k_2a} + \left(1 - \frac{ik_1}{k_2}\right)\left(1 - \frac{k_2}{ik_1}\right)e^{k_2a}}$$

$$= \frac{4i\,k_1k_2\,e^{-ik_1a}}{(ik_1 + k_2)^2\,e^{-k_2a} - (ik_1 - k_2)^2}$$

$$= \frac{4i\,k_1k_2\,e^{-ik_1a}}{(k_2^2 - k_1^2 + 2ik_1k_2)\,e^{-k_2a} - (k_2^2 - k_1^2 - 2ik_1k_2)\,e^{k_2a}}$$

$$= \frac{4ik_1\,k_2e^{-ik_1a}}{(k_2^2 - k_1^2)\,(e^{-k_2a} - e^{+k_2a}) + 2i\,k_1k_2\,(e^{-k_2a} + e^{k_2a})}$$

As V_0 is greater than E, k_1 and k_2 are real numbers, therefore

$$\frac{A_3}{A_1} = \frac{4i\,k_1k_2\,e^{-ik_1a}}{(k_2^2 - k_1^2).(-2\sinh k_2a) + 2ik_1k_2.2\cosh k_2a}$$

$$\Rightarrow \quad \frac{A_3}{A_1} = \frac{2i\,k_1k_2\,e^{-ik_1a}\,\operatorname{sech} k_2a}{-(k_2^2 - k_1^2)\tanh k_2a + 2i\,k_1k_2} \qquad \ldots(17)$$

Complex conjugate of ratio $\frac{A_3}{A_1}$ *i.e.*

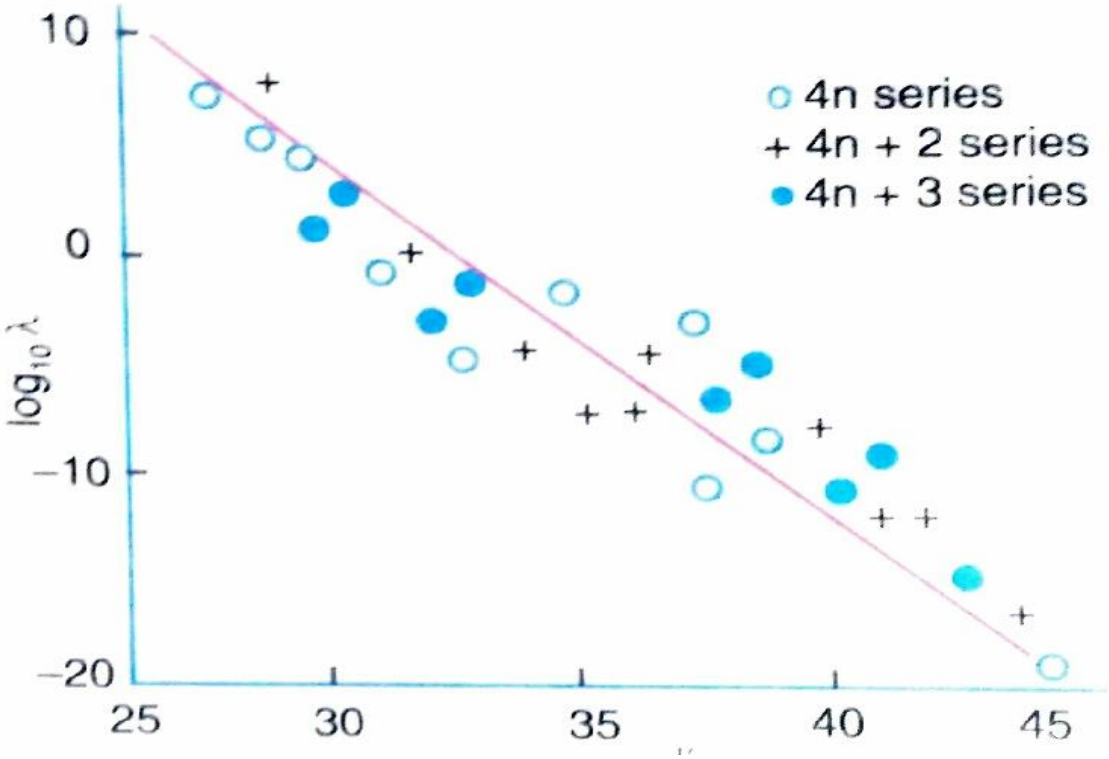
$$\left(\frac{A_3}{A_1}\right)^* = \frac{-2i\,k_1k_2\,e^{ik_1a}\,\operatorname{sech} k_2a}{-(k_2^2 - k_1^2)\tanh k_2a - 2i\,k_1k_2} \qquad \ldots(18)$$

As velocity of α-particle in first and third regions is same $v = \sqrt{\frac{2E}{m}}$, the transmission coefficient or transmission probability of incident α-particle is given by

$$T = \frac{A_3A_3^*\,v}{A_1A_1^*\,v} = \left(\frac{A_3}{A_1}\right)\left(\frac{A_3}{A_1}\right)^*$$

$$= \frac{-4\,k_1^2\,k_2^2\,\operatorname{sech}^2 k_2a}{-4\,k_1^2\,k_2^2 - (k_2^2 - k_1^2)^2\tan^2 k_2a}$$

or $$\boxed{T = \frac{4\,k_1^2\,k_2^2\,\operatorname{sech}^2 k_2a}{(k_2^2 - k_1^2)^2\tanh^2 k_2a + 4k_1^2\,k_2^2}} \qquad \ldots(19)$$



This can be used to determine the value of nuclear radius. The nuclear radius calculated in this way comes out be of the same order as obtained from scattering experiments.

Further this theory understandable the enormous variation emitting most energetic **α** particle or short lived and those emitting least energetic **α** particle are long lived. This correlation between the half life time (or disintegration constant ⅄) and the energy of the **α** particle is predicted by equation(8)

Neutrino Hypothesis

To account for the difficulties raised by continuous β Ray spectrum, Pauli, in 1931 suggested that in a β particle disintegration and additional particle called the neutrino is emitted simultaneously along with **α** β particle. The neutrino is assumed to be a fundamental particle having no charge zero mass and a spin of $\frac{1}{2}(\frac{h}{2\pi})$.

The actual disintegration energy of β disintegration is always the end point energy and the missing energy is carried away by that neutrino admitted. Fermi in 1934 developed this suggestion in to complete theory of Beta decay based on the following lines.

And atomic nucleus consists of positively charged protons and neutral neutrons. In the process of β decay the β particle is created at the time of emission by the conversion of a Neutron into Proton. When a neutron is converted into proton $_1H^1$ an electron($_{-1}\beta^0$) are emitted. Later it was observed that there are two kinds of neutrino; the neutrino(ν) itself and the antineutrino. In the presence of β decay it is actually Antineutrino that is emitted. Thus β decay process can be represented by as

$$_0\mathbf{n}^{\mathbf{1}} \rightarrow {}_1\mathbf{H}^{\mathbf{1}} + {}_{-1}\boldsymbol{\beta}^{\mathbf{0}} + \bar{\nu}$$

This equation removes the various difficulties arising in beta decay process in the following way

1) The Antineutrino carries no charge. This maintains the conservation of charge in the process of β decay.
2) The antineutrino has a linear momentum such that the net momentum of electron antineutrino and the residual nucleus is zero as that of nucleus come on that obeying the law of conservation of linear momentum.
3) Like β particle, the Antineutrino has spin $\frac{1}{2}(\frac{h}{2\pi})$. This leads to a law of conservation of angular momentum .
4) The antineutrino has a zero rest mass and hence zero rest energy. Under the assumption that and antineutrino is always a emitted along with a β particle in β decay process each β decay is accompanied by a discrete β disintegration energy E_β which is equivalent to mass difference between the parent and daughter nuclei i.e.,

Where $m_0 c^2$ is the rest is energy of electrons and K_{max} is the endpoint kinetic energy in the continuous β spectrum. The energy E_β is distributed among the electron the Antineutrino and the recoiling daughter nucleus in a continuous energy distribution extending from zero to a certain maximum and energy. Thus the neutrino hypothesis indicates that the energy is conserved in the process of β decay.

NUCLEAR ISOMERISM

Usually the life time of a nucleus in an excited state is very short less than 10^{-12}s however in rarer cases the life time of the nuclear excited stat is very much longer even days to years. Such excited states are called isomeric (or meta stable) states. The nuclear species having same atomic number (Z) and same mass(A) but having distinguishable properties such as different radioactive half-lives are called nuclear isomers. The phenomenon of the existence of nuclear isomers is called the nuclear isomerism. An isomeric state is expressed by symbol m over the chemical symbol of the element. For example $^{113}I_n$.

The phenomenon of nuclear isomerism was discovered by Otto Hahn in 1921 in $_{92}Pa^{234}$isotope belonging to (4n+2) radioactive series.

The phenomenon of nuclear isomerism was discovered by Otto Hahn in 1921 in $^{234}_{91}Pa$ isotope belonging to $(4n + 2)$ radioactive series.

Examples of Nuclear Isomers :

(*i*) **Example of UX_α (Pa^{234}).** It is found that β^{-1} active UX_1 (^{234}Th) decays into (^{234}U). β^- active UX_2 which decays to U II (^{234}U) UX_1

$$\xrightarrow[24{\cdot}1d]{\beta^{-1}} UX_2 \xrightarrow[1{\cdot}18\ \text{min}]{\beta^-} UII$$

However in 0·12% cases UX_1 undergoes a different type of transformation given below :

$$UX_1 \xrightarrow[24{\cdot}1d]{\beta^{-1}} UZ \xrightarrow[6{\cdot}7h]{\beta^-} UII$$

It is obvious that the decay products UX_2 and UZ of UX_1 have different half-lives, though they have same values of Z and A. The energy level diagram and the isomeric transitions is shown in Fig. 6.14. It may be noted that UZ is simply the ground state of UX_2.

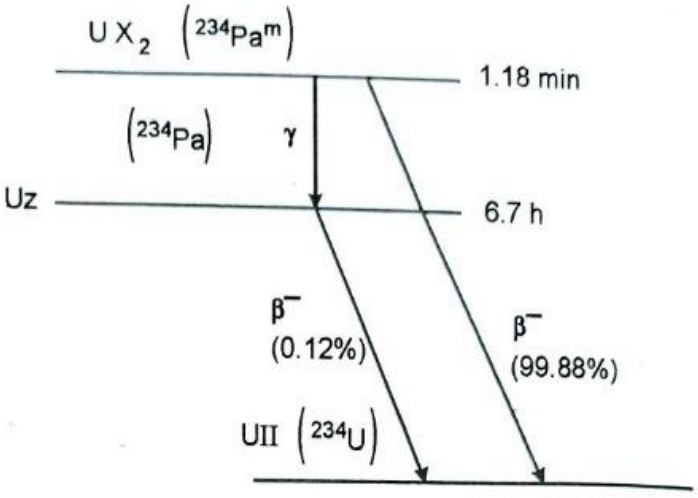
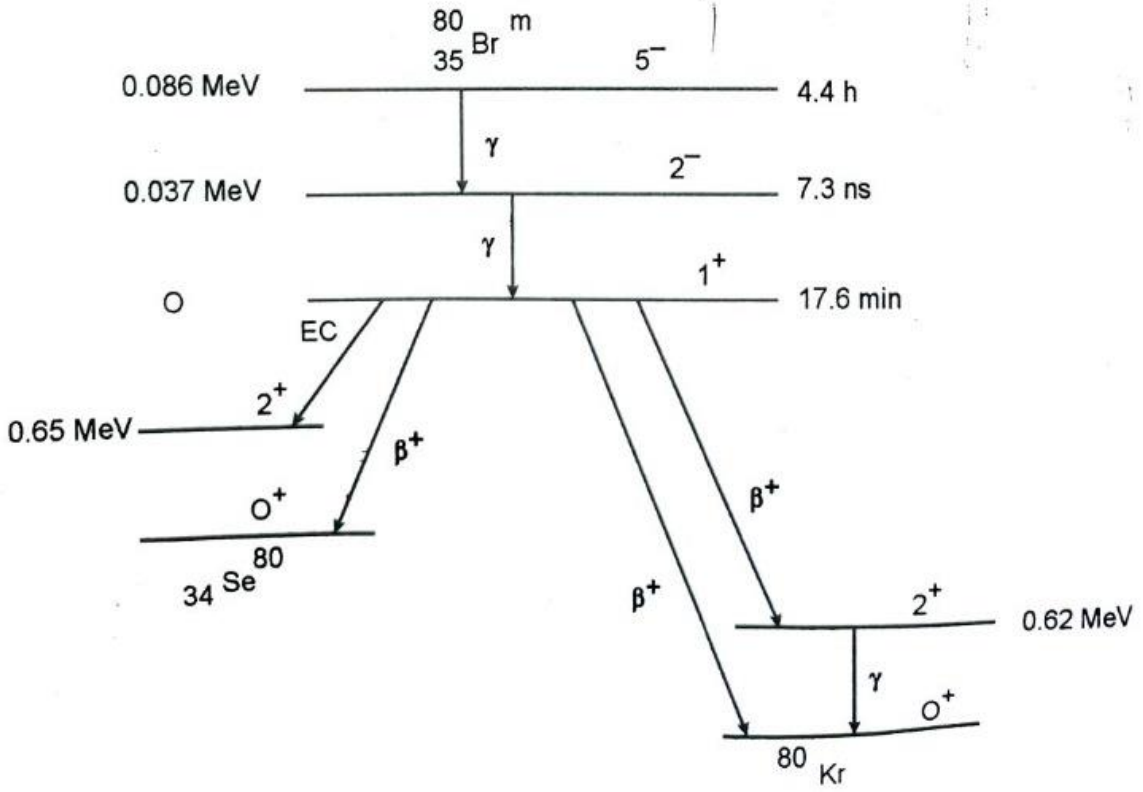


Fig. 6·14. Iosmeric transition of $^{234}_{Pa}$ m

(*ii*) **Example of Bromine $^{80}_{35}Br$.** It is formed by artificial transmutation of stable isotope ^{79}Br by bombardment with slow neutrons.

$$^{79}_{35}Br + {}^{1}_{0}n \rightarrow {}^{80}_{35}Br + \gamma$$



^{80}Br has two half lives 17·6 min and 4·4 *h*. The state of half-life 4·4 *h* is the isomeric state written as $^{80}_{35}Br^{m}$. The energy level diagram with transitions is shown in Fig. 6.15. The isomeric state $^{80}Br^{m}$ emits two γ-photons and goes to ground state of half-life 17·6 min. The intermediate state has a half life of 7·3 *ns*. The ground state of ^{80}Br goes to $^{80}_{36}Kr$ by β-decay (92%) and in some cases it goes to $^{80}_{34}Se$ by β^+ decay (3%) and by electron capture (*EC*) (5%).

(*iii*) **Example of ^{113}In.** $^{113}In^{m}$ is an isomer of $^{113}I_n$. One source of $^{113}In^{m}$ is ^{113}Sn which has a half-life 118 days. ^{113}Sn decays to $^{113}In^{m}$ with a 98·2% electron capture (*EC*) probability or decay to a higher excited state with 1·8% probability. This state promptly emits a γ-photon of energy 0·253 *MeV* to go to isomeric state $^{113}In^{m}$. The energy level diagram with transitions is shown in Fig. 6·16.

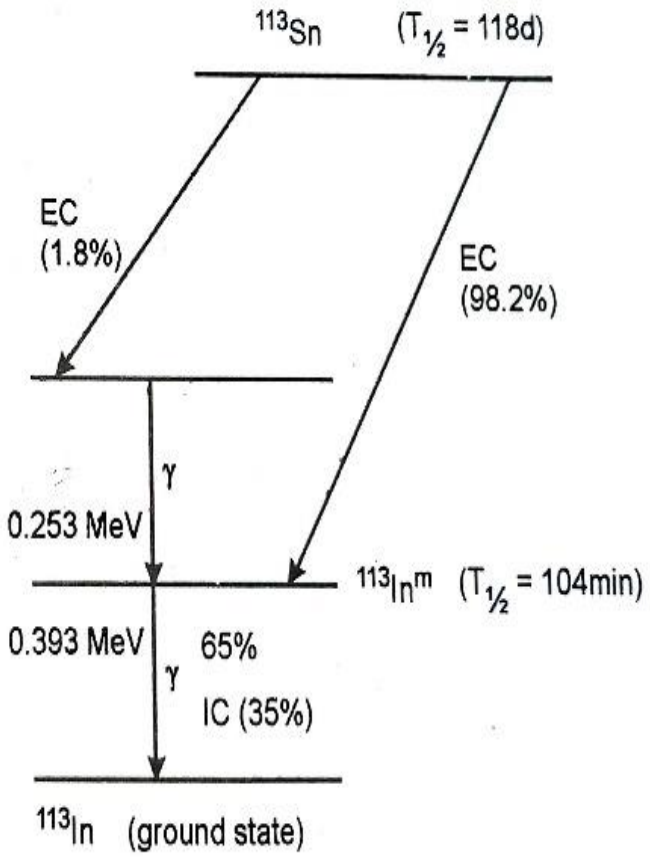


Fig. 6·16

Weizsacker in 1936 predicated on theoretical basis that isomeric transitions occur with an *observable small energy and large difference in nuclear spin.* Such transitions will result in longer and measurable life times. These characteristics help to locate isomeric islands in periodic table. The isomeric islands are combined below magic numbers 50, 82 and 126. Thus nuclei $^{86}_{37}Rb$ (N = 49 close to 50), $^{134}_{52}Te$ (N = 79 close to 82), $^{199}_{80}Hg$ (Z = 8(close to 82) possess isomeric states.

Beta spectrum selection rules

The probability of β -decay depends on the value of the matrix elements M_{if} . As M_{if} is determined by the product $f(Z,E_v)T_{1/2}$, we may say that the probability of β decay is determined by f $T_{1/2}$ values

For example, the decay ${}_8O^{14} \rightarrow {}_7N^{14} + e^+ + \nu$ is purely Fermi type because in this case the spins of O^{14} and N^{14} both are zero and their parities are identical and positive. It is forbidden by Gamow-Teller rule because of $0 \rightarrow 0$ transition.

The β-decay ${}_2He^6 \rightarrow {}_3Li^6 + e^- + \bar{\nu}$ is Gamow-Teller type because for ${}_2He^6$ the spin I = 0, while for ${}_3Li^6$, the spin I = 1, and parities are identical.

However, there are a number of transitions, which belong to *mixed type*. It takes place both by means of Fermi and Gamow-Teller transitions. In Fermi transitions the nuclear isospin suffer no change *i.e.* $\Delta T = 0$. In the Gamow-Teller transitions $\Delta T = 0, \pm 1$ (with the exception of $T_i = 0$ to $T_f = 0$)

For example,

$${}_1H^3 \rightarrow {}_2He^3 + e^- + \bar{\nu} \left(\frac{1}{2}^+ \text{ to } \frac{1}{2}^+ \right)$$

$${}_{16}S^{35} \rightarrow {}_{17}Cl^{35} + e^- + \bar{\nu} \left(\frac{3}{2}^+ \text{ to } \frac{3}{2}^+ \right)$$

follow *mixed selection rules* since here $\Delta I = 0$, no.

Superallowed and Normal Allowed Transitions

The allowed transitions are subdivided into superallowed and normal allowed transitions. Superallowed transitions are those which follow the rules of allowed transitions with no change in nuclear structure, they are characterised by the minimum $f T_{1/2}$ values. For such transition $\log f T_{1/2} \approx 2{\cdot}5 - 3{\cdot}0$ and usually take place between the neighbouring components of the same isomultiplet. The examples of superallowed transitions are

$${}_0n^1 \rightarrow {}_1p^1 + e^- + \bar{\nu}$$

$$\left(T = \frac{1}{2}, T_Z = -\frac{1}{2} \rightarrow T = \frac{1}{2}, T_Z = \frac{1}{2} \right)$$

${}_8O^{14} \rightarrow {}_7\overset{*}{N}{}^{14} + e^+ + \nu$[(Star (*) denoted excited nuclear state]

$(T = 1, T_Z = -1 \rightarrow T = 1, T_Z = 0$

Superallowed transitions are observed only in nuclei with $A \leq 40$.

The transitions obeying allowed transition rules but taking place between the components of different multiples are called normal allowed transitions. In such transitions the nuclear structure of parent and daughter nuclei suffer some change in nuclear structure and $\log (f T_{1/2}) \approx 5$. The example of normal allowed transition is

$${}_2He^6 \rightarrow {}_3Li^6 + e^- + \bar{\nu}$$

$$(T = 1, T_Z = -1) \rightarrow (T = 0, T_Z = 0)$$

The transition is normal allowed because the nuclear structures of ${}_2He^6$ and ${}_3Li^6$ are not identical but are very similar.

Forbidden Transitions. We have seen that $\Delta I = 0, \pm 1$, no; are allowed transitions. But if parity changes and $|\Delta I|$ exceeds 1, then the transitions are called forbidden transitions, their degree increases with increase of ΔI. That is if $|\Delta I|$ exceeds 1, then the transition is *once forbidden transition, i.e.*

For ***once forbidden transition***, Fermi selection rule is $\Delta I = 0, \pm 1$, yes (except $0 \rightarrow 0$) and Gamow-Teller selection rule is $\Delta I = 0, \pm 1, \pm 2$, yes $\left(\text{except } \frac{1}{2} \rightarrow \frac{1}{2}, 0 \rightleftarrows 0 \right)$

Decay type	ΔJ	ΔT	$\Delta\pi$	$\log_{10} ft_{1/2}$
Superallowed	$0^+ \rightarrow 0^+$	0	no	3.1–3.6
Allowed	0, 1	0, 1	no	2.9–10
First forbidden	0, 1, 2	0, 1	yes	5–19
Second forbidden	1, 2, 3	0, 1	no	10–18
Third forbidden	2, 3, 4	0, 1	yes	17–22
Fourth forbidden	3, 4, 5	0, 1	no	22–24

$\Delta I = \frac{1}{2} - \frac{1}{2} = 0$, no change in parity.,

This is in accordance with Fermi selection rule and Gamow-Teller selection rule, so the decay is *mixed type.*

(*iii*) In $O^{14} \rightarrow N^{14} + \beta^+ (O^+ \rightarrow O^+)$

$\Delta I = 0 - 0 = 0$, no change in parity

This is condition of Fermi-selection rule, but forbidden by Gamow-Teller selection rule due to $(O^+ \rightarrow O^+)$ transition, so the decay is *Fermi type.*

INTERNAL CONVERSION

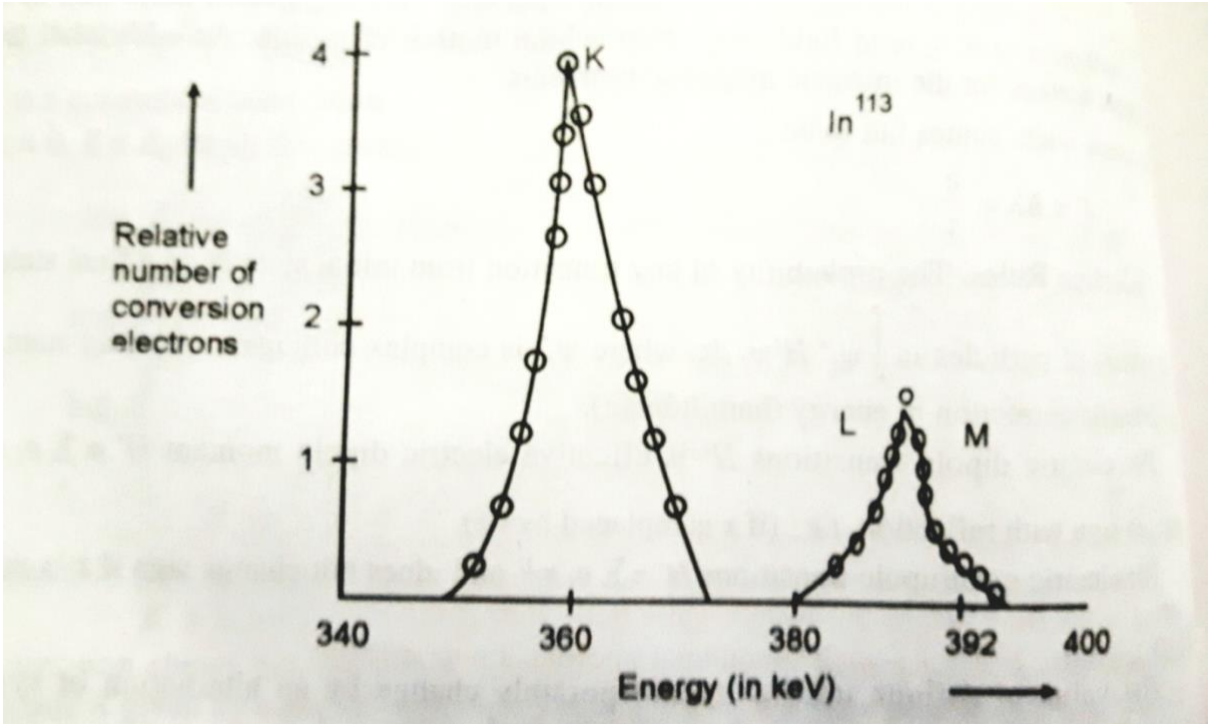
When a nucleus emits α orβ particle; it is usually left in an excited state. The transition from an excited state to some lower energy state(or ground state) results in an emission of γ- ray. That is why the emission of an α particle or β particle is accompanied by γ- ray photon. In some cases, when the nucleus goes from an excited state to a lower energy to one of the atomic electrons outside the nucleus. This electron atomic electron is called the internal conversion electron.

If E_T is the transition energy (i.e., energy difference between upper and lower energy levels of excited nucleus), then the internal conversion electron energy is

$$\mathbf{E_e = E_T - B_i}$$

B_i is the binding energy of electron in i th shell. Clearly the energy spectrum of internal conversion is discrete and it is complex too.

Fig. Shows the spectrum of conversion electrons which are ejected from K, L and M shells of Indium by internal conversion of 392 kev transition in $In^{113.}$ In addition to conversion electrons, internal conversion also produces X-ray quanta , when one of outer electrons fall to a level in the k-shell or in the L shell, vacated out electron or emission of Auger electrons.



Before the thought of internal conversion the observed discrete energy lines sitting on the top of β - ray spectrum remained a puzzling problem. The continuous β – ray spectrum was explained by Fermi by assuming simultaneous emission of antineutrino. The β particle and antineutrino share the total energy released in β decay. The discrete lines superimposed on continuous β -ray spectrum can be explained by the internal conversion, so the discrete lines were named as internal conversion lines. In 1924 C.D.Willi's and his co-workers introduced the name of electrons responsible for discrete lines as internal conversion electrons. This name gained full acceptance after H.Bethe in 1934 proposed their α, β *and* γ rays be reserved for particles coming from the nucleus and that the ejected atomic electron be called as the internal conversion electron.

It was finally accepted that the term B_i in equation (1) is not the characteristic of parent nucleus but it is of the daughter formed by the β –decay of parent nucleus. Therefore, it was concluded that the discrete lines β – ray spectrum arise from a process quite different from β – decay. Equation (1) simply represents the law of conservation of energy. In this equation no photon is involved; E_T is simply the energy difference between initial and final energy states of excited nucleus.

Internal Conversion Coefficient. When a nucleus makes a transition from one energy level to the other, the probabilities of photon- emission and internal conversion electron are the function of changes in spin and parity between the two nuclear energy levels the energy difference, atomic number (z) of nucleus and the angular momentum carried away by radiation.

The total internal conversion coefficient α for a given transition may be defined as the ratio of the probability of internal conversion electron emission from all atomic shells to the probability of photon-emission. If λ_T

If λ_γ represent the probability per unit time for the emission of a γ-proton by a radioactive nuclear multipole transition and λ_e the probability of emission of internal conversion electron per unit time in the same nuclear multiple field, then total internal conversion coefficient is given by

$$\alpha = \frac{\lambda_e}{\lambda_\gamma} \qquad \ldots(2)$$

If N_e and N_γ represent the numbers of experimentally observed conversion electrons and γ-photons respectively in the same time interval in the same sample, then

$$\alpha = \frac{N_e}{N_\gamma} \qquad \ldots(3)$$

The total transition probability between the given nuclear states is

$$\lambda = \lambda_\gamma + \lambda_e = \lambda_\gamma + \alpha \lambda_\gamma \text{ [using (2)]}$$

or $$\boxed{\lambda = \lambda_\gamma (1 + \alpha)} \qquad \ldots(4)$$

The total number of nuclei transforming is

$$N = N_\gamma + N_e = N_\gamma + N_\gamma \alpha$$

or $$\boxed{N = N_\gamma (1 + \alpha)} \qquad \ldots(5)$$

The total internal conversion coefficient α is made up of the sum of individual coefficients acting separately for each atomic sub-shell. If α_K, α_L, α_M – are the internal conversion coefficient for *K, L, M* – shells, then total internal conversion coefficient

$$\alpha = \alpha_K + \alpha_L + \alpha_M + \ldots\ldots \qquad \ldots(6)$$

The treatment to find the exact values of α_K, α_L, α_M is quite tedious. A number of approximate formulae have been developed. One of them is due to Dancoff and Morisson developed on the basis of relativistic treatment. For transition energy E_T less than m_0c^2 and neglecting the binding energy of *K*-electron, the approximate formula applicable to *electric multipols* of order $(2)^l$, $l = 1, 2, 3 \ldots$ is given by

$$(\alpha_k)_{\text{electric}} \cong \frac{l}{l+1}(Z^3)\left(\frac{e^2}{4\pi\varepsilon_0\ \hbar\ c}\right)^4\left(\frac{2m_0c^2}{E_T}\right)^{l+\frac{5}{2}} \quad \ldots(7)$$

For magnetic multipoles (parity unfavoured), the formula is

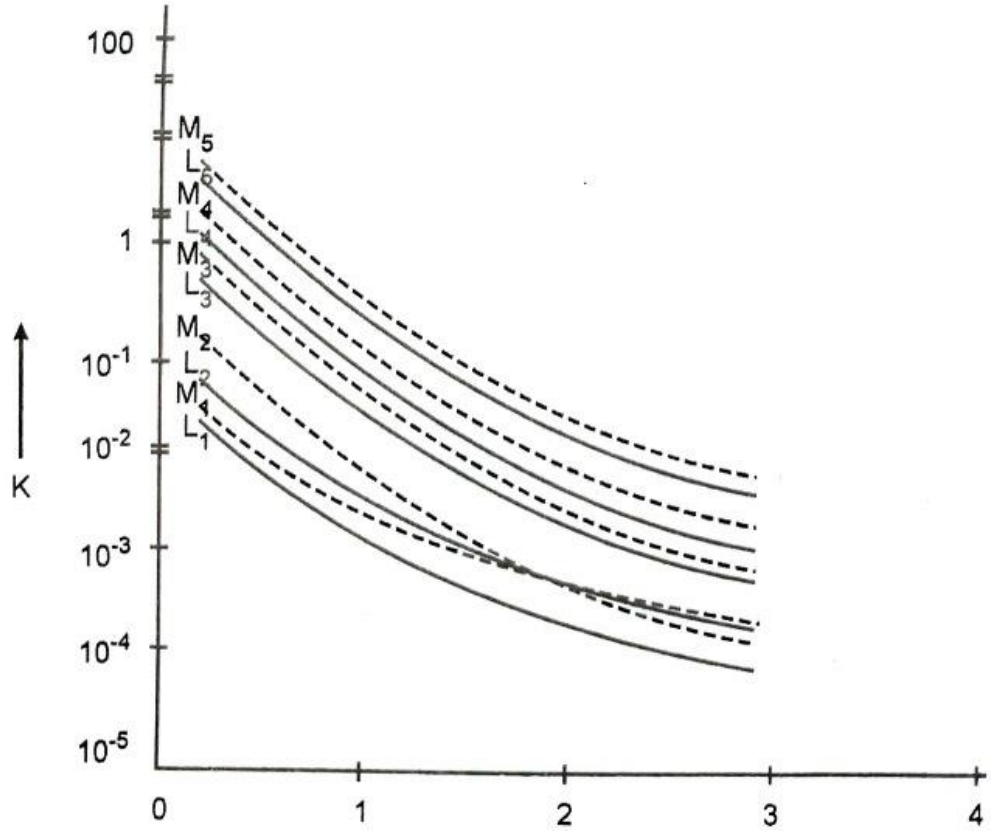
$$(\alpha_k)_{\text{magnetic}} \cong (Z^3)\left(\frac{e^2}{4\pi\varepsilon_0\ \hbar\ c}\right)^4\left(\frac{2m_0c^2}{E_T}\right)^{l+\frac{3}{2}} \quad \ldots(8)$$

From these relations the conclusions drawn are :

(*i*) The conversion coefficients are directly proportional to Z^3, this implies that the conversion dominates in heavy nuclei and γ-decay is favoured in light nuclei.

(*ii*) The conversion coefficient depends on transition energy E_T and decreases with increase of E_T. This implies that conversion internal conversion dominates for low energy and for high energy γ-decay is favoured.

(*iii*) The internal conversion process gives no information above the structure of nucleus.



UNIT III

NUCLEAR MODELS

UNIT III

NUCLEAR MODELS

CONSERVATION OF ENERGY

In analyzing nuclear reactions, we apply the **many conservation laws**. **Nuclear reactions** are subject to classical **conservation laws for charge, momentum, angular momentum, and energy** (including rest energies).

Conservation Laws in Nuclear Reactions

1. **Conservation of nucleons**. The total number of nucleons before and after a reaction are the same.

2. **Conservation of charge**. The electric charge is conserved in all reactions. This means that the sum of the atomic numbers of X and a is equal to the sum of atomic numbers of Y and b.

$$\mathbf{Z_a + Z_X = Z_Y + Z_b}$$

3. **Conservation of Mass - Energy**. In nuclear reactions neither kinetic energy nor rest mass is conserved separately. But the total mass-energy is always conserved. The kinetic energy Q –librated in any reaction is always equal to the reduction of the total rest- mass of all the constituents of the reaction, the mass energy equivalence relation is

$$E = mc^2$$

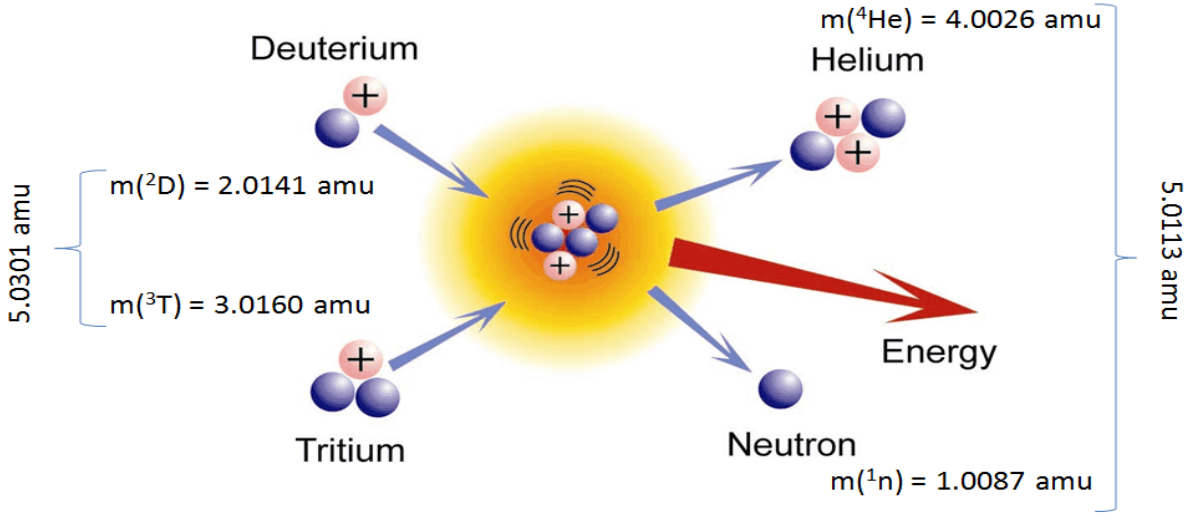
Accordingly $1\ amu = 931{\cdot}5\ MeV$

Conversely $1\ MeV = 0{\cdot}001074$ amu

For example in nuclear reaction ${}^{10}_{5}B\ (\alpha, p)\ {}^{13}_{6}C$, the Q value of reaction is

$$M_{B^{10}} + M_{\alpha} = M_{p} + M_{C^{13}} + Q$$

Where all masses are rest masses for the corresponding neutral atoms. If relativistic masses were used in above equation, then Q-value for reaction would be identically zero.



Q = 0.0188 amu x 931.481 MeV/amu = 17.5 MeV

4.Conservation of Parity. In every nuclear reactions the total parity is conserved. If π_x, π_a, π_Y and π_b are intrinsic parities of nuclei taking part in the reaction, then for the initial and final states of reaction the parities are

$$\boldsymbol{\pi_i = \pi_x \pi_a}$$

$$\boldsymbol{\pi_f = \pi_Y \pi_b}$$

5.Conservation of momentum. The total momentum of the interacting particles before and after a reaction are the same. The law of **conservation of momentum** states that in the collision of two objects such as billiard balls, the **total momentum is conserved.**

LIQUID DROP MODEL

In 1936 Bohr suggested that the nucleus might be like a droplet of dense liquid, composed of the sub droplets of the protons and neutrons, it contains. This picture of nucleus is analogous to the macroscopic picture of a small water droplet composed of molecules.

In this liquid drop model we do not attempt to describe the motion of individual nucleons, but instead concentrate on the collective behaviour due to the coherent motion of all the nucleons. It it is observed that, except for light nuclei, the binding energy per nucleon remains almost constant i.e. the total binding energy varies linearly with the number of nucleons A in the nucleus. The same is true for the volume of the nucleus, which yields the dependence of nuclear radius as

$$\mathbf{R \propto A^{1/3}}$$

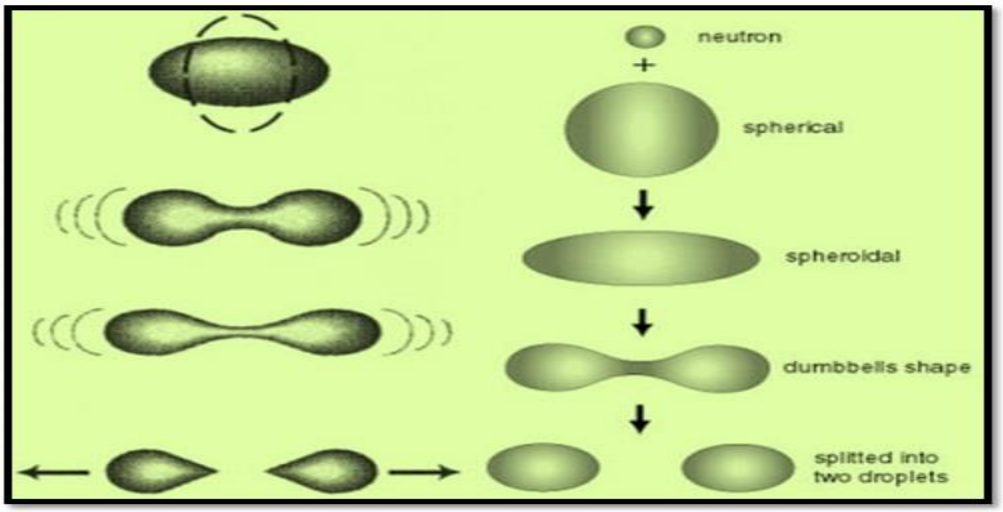
$$\mathbf{R = R_0 A^{1/3}}$$

where R_0 is a constant, equal to 1.2 fermi (1 fermi = 10^{-15} m). The following analogies between atomic nucleus and a small drop of liquid support the liquid drop model of the nucleus.

1. The ismall liquid drop is spherical because of surface tension effects and the atomic nucleus is assumed to be spherical.

2. The density of a spherical liquid drop is independent of its volume. This is also true for atomic nucleus. In this analogy there is a disparity in the sense that the density of liquid depends on the type of liquid where as the density of liquid depends on the

type of liquid where as the density of nuclear matter is independent of the type of the nucleus. This may be seen as follows:



$$V_n = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \text{ using (1)} \quad \ldots(1)$$

The mass of the nucleus is A times the mass of nucleon

i.e. $M_n = A\, m_n$, where m_n is the mass of the nucleon

$\therefore$ Density of nuclear matter, $\rho_n = \dfrac{M_n}{V_n} = \dfrac{Am_n}{\frac{4}{3}\pi R^3 A} = \dfrac{3m_n}{4\pi R_0^3}$ ...

As m_n and R_0 are constants independent of the type of nucleus, therefore the *density nuclear matter is independent of the type of nucleus.*

Substituting $m_n = 1{\cdot}67 \times 10^{-27}\ kg$ and $R_0 = 1{\cdot}2 \times 10^{-15}\ m$.

we get $\rho_n = \dfrac{3 \times 1{\cdot}67 \times 10^{-27}}{4 \times 3{\cdot}14\,(1{\cdot}2 \times 10^{-15})^3} \approx 10^{17}\ \mathbf{kg/m^3}$.

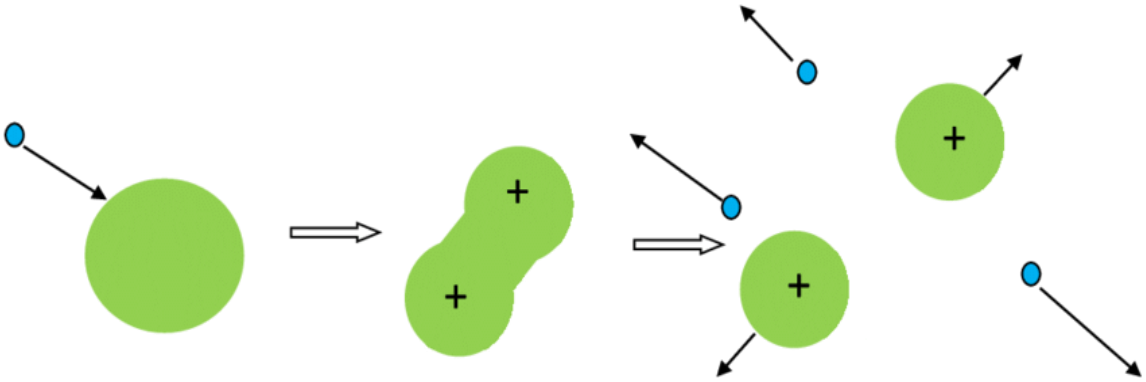
This is very much larger than the density of ordinary matter. This suggests that an atomic nucleus must be considered to be equivalent to the drop of a given liquid, say water.

3. The molecules in a liquid drop interact over short ranges as compared to the diameter of the drop. i.e. they interact only with their immediate neighbours; similarly the nucleon in a nucleus interact with their immediate neighbours.

4. The surface tension forces acting at the surface of the drop are analogous to the potential barrier effects at the surface.

5. The molecules in the drop move over short distances with thermal velocities, If the temperature is increased to increase the thermal agitation, the evaporation of molecules takes place. Similarly the nucleus possesses definite kinetic energy within the nucleus.
6. If the energy is given to the nucleus by the bombardment of light of particles, a compound nucleus is formed which emits nucleons almost immediately.
7. If a liquid drop is made to oscillate, it tends to separate into two parts of equal size. Certain heavy nucleui break into lighter fragments after capturing neutrons (nuclear fission).
8. The existence of non –zero quadropole moments shows that many nuclei are non-spherical. This phenomenon can be described well within the frame-work of liquid drop model by allowing the drop to be a non- spherical shape.

The liqiuid drop model has been utilised with a certain amount of success in the interpretation of intra-nuclear forces and nuclear fission phenomenon; but it ccould not explain why number of protons (Z) and number of neutrons (N) are equal for lighter nuclei, with N somewhat larger in heavy nuclei.



Further according to this model a nucleus is more tightly bound and more stable if it contains more neutrons and the fewer protons because this would reduce the mutual repulsion of the protons. The model could not explain why there is no helium isotope ${}_2He^3$ or an oxygen isotope ${}_8O^{15}$, which contain a greater fraction of neutrons than the actually present isotopes.

8.4. REACTION ENERGETICS : Q-VALUE

The conservation laws of momentum and energy has decisive influence upon the characteristics of all physical processes. This also holds for nuclear reactions.

The Q-value of a reaction is the change in total kinetic energy of system or the change in total rest mass energy of system. As rest mass is an invariant in relativistic mechanics, therefore Q value is same in laboratory and centre of mass reference systems.

If we consider the general nuclear reaction $X\ (a, b)\ Y$, the total initial energy in the reaction is

$$E_i = K_a + M_a c^2 + K_X + M_X c^2 \qquad \ldots(1)$$

where K_a and K_X are the kinetic energies of particles a and X respectively. The total potential energy is zero as particles 'a' and X are assumed far apart, M_a and M_X are rest masses of 'a' and X respectively.

The final energy of reaction is

$$E_f = K_b + M_b c^2 + K_Y + M_Y c^2 \qquad \ldots(2)$$

Here K_b = Kinetic energy of particle 'b', K_Y = Kinetic energy of particle Y, M_b = Rest mass of particle b, M_Y = Rest mass of particle Y.

According to law of conservation of energy

$$E_i = E_f$$

The difference between initial and final rest mass energies is called the **reaction energy** or Q-value of the reaction *i.e.*

$$Q = [M_a + M_X) - (M_b + M_Y)]\ c^2 \qquad \ldots(3)$$

Then from (1) and (2)

$$Q = (K_b + K_Y) - (K_a + K_X) \qquad \ldots(4)$$

Equation (3) may also be expressed as

$$(M_a + M_X)\ c^2 = (M_b + M_y)\ c^2 + Q \qquad \ldots(5)$$

If Q is positive (*i.e.* $Q > 0$), the reaction is exoergic and the energy liberated appears in the form of kinetic energies of reaction products; but if $Q < 0$, the reaction is endoergic *i.e.* it can occur only if required energy is supplied by the kinetic energy of the projectile.

Q-value in terms of measurable observables

In the laboratory system the $K_X = 0$ as target is stationary in this system and K_Y is usually small and therefore hard to measure. The difficulty is removed by eliminating K_Y by applying momentum conservation law

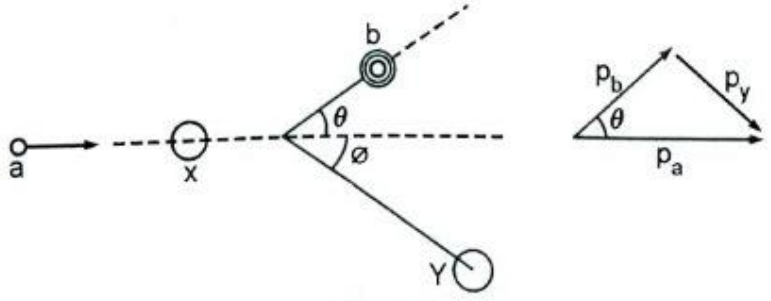


Fig. 8.1

The conservation law of momentum in vector form is expressed as

$$p_a = p_b + p_Y \qquad \ldots(6)$$

The general relativistic relation* between momentum and kinetic energy is

$$p^2 = 2M_0K + K^2/c^2 \qquad \ldots(7)$$

Then from momentum triangle we have

$$p_Y^2 = p_a^2 + p_b^2 - 2p_a p_b \cos\theta \qquad \ldots(8)$$

Using (7) for substituting value of p_a, p_b and p_Y, we get

$$2M_YK_Y + \frac{K_Y^2}{c^2} = 2M_aK_a + \frac{K_a^2}{c^2} + 2M_bK_b + \frac{K_b^2}{c^2}$$

$$- 2\cos\theta \sqrt{\left\{\left(2M_aK_a + \frac{K_a^2}{c^2}\right)\left(2M_bK_b + \frac{K_b^2}{c^2}\right)\right\}} \qquad \ldots(9)$$

Non-relativistic Q-equation. The magnitude of term K^2/c^2 is small and in non-relativistic limit it may be ignored. Thus e get

$$2M_Y K_Y = 2M_aK_a + 2M_bK_b - 2\cos\theta \sqrt{(2M_aK_a\, 2M_bK_b)}$$

$$\therefore \qquad K_Y = \frac{M_a}{M_Y}K_a + \frac{M_b}{M_Y}K_b - \frac{2\cos\theta\sqrt{2M_aM_bK_aK_b}}{M_Y} \qquad \ldots(10)$$

Now for stationary target (X) in lab-system $K_Y = 0$.

Therefore, from (4), we have, $Q = K_b + K_Y - K_a$. $\qquad \ldots(11)$

Substituting value of K_Y from (10) in (11), we get

$$Q = K_b + \frac{M_a}{M_Y}K_a + \frac{M_b}{M_Y}K_b - \frac{2\cos\theta\sqrt{(M_aM_bK_aK_b)}}{M_Y} - K_a$$

$$\text{or} \qquad Q = \left(\frac{M_a}{M_Y} - 1\right)K_a + \left(\frac{M_b}{M_Y} + 1\right)K_b - \frac{2\cos\theta\sqrt{(M_aM_bK_aK_b)}}{M_Y} \qquad (12)$$

This equation is called *non-relativistic Q-value equation.*

Relativistic Q-equation. For finding the relativistic Q-equation, equation (9) is to be solved without any approximation. This adds a correction term δ_{rel} to equation (12), so that in this case we have

---

$$*m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$E^2 = (mc^2)^2 = \frac{m_0^2c^4}{1 - \frac{v^2}{c^2}} = \frac{m_0^2c^4 - m_0^2v^2c^2 + m_0^2v^2c^2}{1 - \frac{v^2}{c^2}}$$

Also $E^2 = (K + m_0c^2)^2$

$$\therefore \quad (K + m_0c^2)^2 = \frac{(m_0^2c^2)\left(1 - \frac{v^2}{c^2}\right)}{1 - \frac{v^2}{c^2}} + p^2c^2$$

this gives $p^2 = 2m_0K + K^2/c^2$.

$$Q = \left(\frac{M_a}{M_Y} - 1\right) K_a + \left(\frac{M_b}{M_Y} + 1\right) K_b - \frac{2\cos\theta\sqrt{(M_a M_b K_a K_b)}}{M_Y} + \delta_{rel} \qquad \ldots(13)$$

where $$\delta_{rel} = \frac{1}{2M_Y c^2}\left[K_a^2 + K_b^2 - K_Y^2 - \cos\theta\sqrt{(M_a M_b K_a K_b)}\left(\frac{K_a}{M_a} + \frac{K_b}{M_b}\right)\right] \qquad \ldots(14)$$

Usually K_Y is small as compared to K_a and K_b; therefore while calculating δ_{rel}, K_Y is set equal to zero.

Q value calculation examples

- **Find Q value for the Beta decay of ^{24}Na**
  - $^{24}Na \rightarrow {}^{24}Mg^{+} + \beta^{-} + \nu + Q$
  - $Q = {}^{24}Na - {}^{24}Mg$
  - M (^{24}Na)-M(^{24}Mg)
    - → 23.990962782-23.985041699
    - → 0.005921 amu
      - * 5.5154 MeV
  - From mass excess
    - → -8.417 - -13.933
    - → 5.516 MeV
- **Q value for the EC of ^{22}Na**
  - $^{22}Na^{+} + e^{-} \rightarrow {}^{22}Ne + \nu + Q$
  - $Q = {}^{22}Na - {}^{22}Ne$
  - M (^{22}Na)-M(^{22}Ne)
  - 21.994436425-21.991385113
  - 0.003051 amu
    - → 2.842297 MeV
  - From mass excess
    - → -5.181 - -8.024
    - → 2.843 MeV

10 Ne	16	0+	24.00	9×10^{-21} s	2p
	17	1/2−	16.500	109.2 ms 6	ε, εp, εα
	18	0+	5.317	1.6670 s 17	ε
	19	1/2+	1.752	17.22 s 2	ε
	20	0+	−7.042	90.48% 3	
	21	3/2+	−5.731	0.27% 1	
	22	0+	−8.024	9.25% 3	
11 Na	18	1−	25.0	1.3×10^{-21} s 4	p
	19	(5/2+)	12.93	<40 ns	p
	20	2+	6.850	447.9 ms 23	ε, εα 20.05%
	21	3/2+	−2.184	22.49 s 4	ε
	22	3+	−5.181	2.6027 y 10	ε
	23	3/2+	−9.530	100%	
	24	4+	−8.417	14.997 h 12	β−
12 Mg	19		31.83	4.0 ps 15	2p
	20	0+	17.56	90.8 ms 24	ε, εp≈27%
	21	5/2+	10.91	122 ms 3	ε, εp 32.6%, εα<0.5%
	22	0+	−0.399	3.8755 s 12	ε
	23	3/2+	−5.473	11.317 s 11	ε
	24	0+	−13.933	78.99% 4	

COMPOUND NUCLEUS

The compound nucleus theory was proposed by N. Bohr in 1936. Before this theory that attempts where made to explain the experimentally observed variation of nuclear cross section on the assumption that incident particle in tracks that target nucleus under a square well potential. This problem is handled by quantum mechanical scattering theory. There result predicts that you

are square well potential can neither change the energy of the particle nor it can remove it from the beam, it can only deflect it.

In other words, it can explain only elastic scattering processes. Calculation also predicts that starting resonances should take place at wide energy intervals of 10 to 20 MeV. But Fermi and his co-workers discovered that slow neutron resonances in intermediate and heavy nuclei occur as close to 1ev. Does the simple square well potential theory is unable to explain the compound nuclear reactions. According to Bohr‘s compound nuclear theory, the compound nuclear reaction is two step process as explained below.

1) **Formation Mode.**

The incident particle and a target nuclear fuse together to form a compound nucleus in a highly excited state that is

$$\text{incident particle} + \text{target nucleus} \rightarrow \text{compound nucleus}$$

The kinetic energy of the incident particle together with its binding energy with the compound nucleus represents the excitation energy of the compound nucleus. After the incident particle has merged completely with the target nucleus, its energy is shared by all particle of compound nucleus. The compound nucleus exist for time sufficiently long that the incident particle (or nucleons in it) and the nucleons inside the target nucleus are mixed together completely.

2) Decay mode

After sometime (10^{-21} s long on nucleus scale). A particle or group of particles gains sufficient energy to overcome their nuclear barrier and escape from the compound nucleus. This is decay of compound nucleus. The formation and decay of compound nucleus are completely independent processes. The decay pattern is not relate to how the compound nucleus was formed .

Analysis for verification of compound nucleus theory.

We consider the low energy region where the compound nucleus hypothesis is valid. According to compound nucleus hypothesis that target nucleus observe the incident particle and forms the compound nucleus C in which all the nucleons share energy. This is the process of formation of compound nucleus. After sometime by chance a particle be get

sufficient energy and is emitted leaving the final nucleus Y. This is the process of decay of compound nucleus.

The compound nucleus cross-section is defined as

$$\sigma(a, b) = \sigma_C(a)\, G_C(b) \qquad \ldots(1b)$$

where $\sigma_C(a)$ is cross-section for the formation of compound nucleus C^* by particle 'a' and $G_C(b)$ is the relative probability of decaying of compound nucleus with the emission of particle b, having the final nucleus Y.

We now consider another case when the compound nucleus C^* decays through the emission of a particle z, having the final nucleus z, so that

$$a + X \rightarrow C \rightarrow Z + z \qquad \ldots(2a)$$

$$\sigma(a, z) = \sigma_C(a)\, G_C(z) \qquad \ldots(2b)$$

For same excitation energy, angular momentum and parity, we assume $\sigma(a)$ to be same. Consider now the reaction

$$b + Y \rightarrow C^* \rightarrow Z + z \qquad \ldots3(a)$$

we have $\sigma(b, z) = \sigma_C(b)\, G_C(z) \qquad \ldots(3b)$

For same compound nucleus $G_C(z)$ is assumed to be same. Suppose τ is the life time of compound nucleus C. Then the total level width Γ satisfies the relation

$$\Gamma \cdot \tau = \hbar \Rightarrow \tau = \frac{\hbar}{\Gamma} \qquad \ldots(4)$$

If Γ_a, Γ_b, Γ_z are the partial widths of the modes of decay a, b, z, ..., then the probability of decay through the channels 'a' and 'b' will be

$$G_C(a) = \frac{\Gamma_a}{\Gamma}\ ,\quad G_C(b) = \frac{\Gamma_b}{\Gamma} \qquad \ldots(5)$$

where $\sum_{\tau} G_C(a) = 1,\ \sum_{\tau} \Gamma_a = 1$

The mean life time of decay modes a, b ... are given by equation (4), *i.e.*

$$\tau_a = \frac{\hbar}{\Gamma_a}\ ,\quad \tau_b = \frac{\hbar}{\Gamma_b} \qquad \ldots(6)$$

Hence we can write

$$\frac{1}{\tau} = \frac{\Gamma}{\hbar} = \frac{\Gamma_a + \Gamma_b + \ldots}{\hbar} = \frac{1}{\tau_a} + \frac{1}{\gamma_b} + \ldots \qquad \ldots(7)$$

Since according to Bohr's assumption the formation and decay processes of the compound nucleus are quite independent, therefore it is possible to find a relationship between Γ_a and $\sigma_C(a)$.

From reciprocity theorem, we have

$$k_a^2\, \sigma(a, b) = k_b^2\, \sigma(b, a) \qquad \ldots(8)$$

where $k = \frac{1}{\lambda}$ where $(\lambda\text{-cross}) = \frac{\lambda}{2\pi}$.

Using equations 1(b) and (5), equation (8) gives

$$k_a^2\, \sigma_C(a)\, \frac{\Gamma_b}{\Gamma} = k_b^2\, \sigma_C(b)\, \frac{\Gamma_a}{\Gamma} \qquad \ldots(9)$$

or $$\frac{k_a^2\, \sigma_C(a)}{\Gamma_a} = k_b^2\, \frac{\sigma_C(b)}{\Gamma_b} = k_z^2\, \frac{\sigma_C(z)}{\Gamma_z} = \ldots = F \qquad \ldots(10)$$

where F is a function only of the state, *i.e.*, excitation energy, angular momentum and parity of the compound nucleus but is independent of the decay channel. From equations (5) and (10), we may write

$$G_C(a) = \frac{\Gamma_a}{\Gamma} = \frac{k_a^2\, \sigma_C(a)}{\Gamma F} = \frac{k_a^2\, \sigma_C(a)}{(\Gamma_a + \Gamma_b + \Gamma_z + \ldots)\, F}$$

$$= \frac{k_a^2\, \sigma_C(a)}{\Gamma_a F + \Gamma_b F + \Gamma_z F + \ldots}$$

$$= \frac{k_a^2\, \sigma_C(a)}{k_a^2\, \sigma_C(a) + k_b^2\, \sigma_C(b) + k_z^2\, \sigma_C(z) + \ldots} \qquad \ldots(11)$$

In fact there are a number of experimental evidences which support Bohr's hypothesis of compound nucleus formation and decay, but here we discuss the experiment of Ghoshal. In his experiment Ghoshal produced the same compound nucleus ${}^{84}_{30}Zn^*$ with the bombardment of ${}^{60}_{28}Ni$ by α-particles and by bombardment of ${}^{63}_{29}Cu$ by protons. The two reactions producing same compound nucleus ${}^{64}_{30}Zn^*$ are expressed as

$$\left.\begin{aligned} {}^{4}_{2}He + {}^{60}_{28}Ni &\rightarrow {}^{64}_{30}Zn^* \\ {}_{1}H^{1} + {}^{63}_{29}Cu &\rightarrow {}^{64}_{30}Zn^* \end{aligned}\right\} \qquad \ldots(12)$$

In order to produce the same excitation energy in ${}^{64}_{30}Zn^*$, the kinetic energy of α-particles must exceed the kinetic energy of protons by 7 *MeV*, *i.e.*,

$$E_\alpha = E_p + 7\ MeV$$

Thus $$(M_\alpha + M_{Ni})c^2 + 7\ MeV = (M_p + M_{Cu^{63}})\, c^2$$

After formation of compound nucleus ${}^{64}_{30}Zn^*$, it may decay through various channels such as

$${}^{64}Zn^* \rightarrow \begin{cases} {}^{63}Zn + n \\ {}^{62}Cu + p + n \\ {}^{62}Zn + 2n \end{cases} \qquad \ldots(13)$$

In symbols the reactions (12) and (13) may be expressed as

(*i*) ${}^{60}Ni\ (\alpha, n)\ {}^{63}Zn$

(*ii*) ${}^{60}Ni\ (\alpha, pn)\ {}^{62}Cu$

(*iii*) ${}^{60}Ni\ (\alpha, 2n)\ {}^{62}Zn$

(*iv*) ${}^{63}Cu\ (p, n)\ {}^{63}Zn$

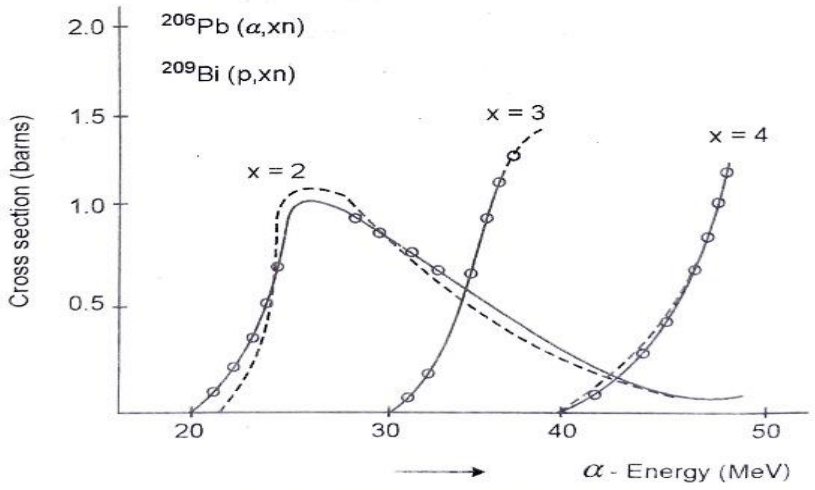
(*v*) ${}^{63}Cu\ (p, pn)\ {}^{62}Cu$

(*vi*) ${}^{63}Cu\ (p, 2n)\ {}^{62}Zn$

This relation was actually verified by Ghoshal. He plotted six cross sections as functions of kinetic energy of incident particle. The proton energy scale has been shifted by 7 Mev relative to α particle energy scale to corresponding peaks of proton and α particle curves. The results are accurate within 10%. It is worth to mention that Ghoshal's experiment was performed with high energy incident particle. This verifies the compound nucleus theory.

The verifications of compound nucleus theory in heavy elements was made by John by comparing the excitations functions of (α, xn) reaction in $^{206}_{82}Pb$ and (p, xn) reaction in $^{209}_{83}Bi$. The compound nucleus formed was $^{210}_{84}Po^*$. According to Bohr's theory we should obtain $\sigma(p, 2n) : \sigma(p, 3n) : \sigma(p, 4n) = \sigma(\alpha, 2n) : \sigma(\alpha, 3n) : \sigma(\alpha, 4n)$.

The experimental results are shown in Fig. 8·5. These confirm the validity of compound nucleus theory.



[Proton Energy = α energy – 11.9 MeV]

Fig. 8·5

Breit - Wigner Dispersion Formula

If we investigate the interaction of neutrons with nuclei at various energy, it is found that if the energy of neutron is below the least excitation energy of compound nucleus, then only elastic scattering take place because there were only channel available to the system is the entrance channel. As the energy of neutron is increased, the cross-section of interactions shows narrow peaks at definite excitation energies.

These peaks are called the resonant peaks. These peaks occur for slow neutrons in nuclei of middle and high atomic weight A while for protons and Alpha particles they occur in light nuclei A< or equal to 30. For resonance processes the first theoretical formula was given by G. Breit and E.P. wigner United States in 1936.

In the simplest form the formula gives the cross section in a neighbourhood of a single resonance level formed by the incident particle with zero angular momentum and zero charge that the spin and coulomb's effect may be ignored. The result is analogous to the theory of optical dispersion, so the formula obtained is called the dispersion formula.

where C is a constant.

To determine constant C, we consider that the reaction takes place in an enclosure of volume V. Assuming quantised state, then the number of states in the incident channel between momentum p and $p + dp$ is

$$n(p)\,dp = \frac{\text{Volume of phase space}}{\text{Volume of one cell}} = \frac{\iiint\iiint dx\,dy\,dz\,dp_x\,dp_y\,dp_z}{h^3}$$

But $\iiint dx\,dy\,dz$ = Volume of enclosure = V

$\iiint dp_x\,dp_y\,dp_z$ = Volume of spherical shell of radius p and thickness dp in momentum space.

$$= 4\pi p^2\,dp$$

$$\therefore \quad n(p)\,dp = \frac{V \cdot 4\pi p^2\,dp}{h^3} = \frac{4\pi p^2\,dp \cdot V}{(2\pi\hbar)^3} \left(\sin \hbar = \frac{h}{2\pi}\right)$$

If σ_a is the cross-section for absorption of particle a by nucleus X, then the volume swept out by the effective collision area in one second is $\sigma_a v$ where v is the velocity of incident particle; so the probability of finding the nucleus X in the enclosure will be

$$\frac{\sigma_a v}{V}$$

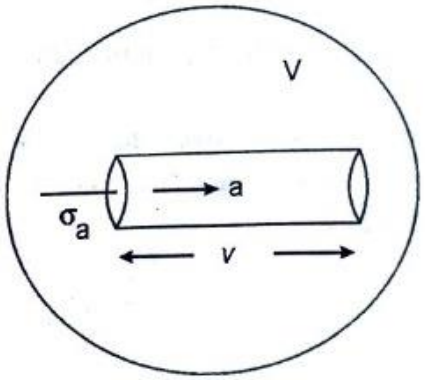


Fig. 8·6

The probability of formation of compound nucleus in the given entrance channel per second is

dP = number of possible states × probability of finding the nucleus X in enclosure

$$= \frac{4\pi p^2 dp\, V}{(2\pi\hbar)^3} \times \frac{\sigma_a v}{V}$$

$$= \frac{\sigma_a p^2 v\, dp}{2\pi^2\hbar^3}$$

As $p = \sqrt{2mE} \Rightarrow p^2 = 2mE,\ v = \frac{p}{m} = \sqrt{\frac{2E}{m}}$

and $dp = \sqrt{2m}\left(\frac{1}{2}E^{-1/2}\,dE\right) = \sqrt{\frac{m}{2E}}\,dE$

In view of above expressions, we get

$$dP = \frac{\sigma_a (2mE)\sqrt{(2E/m)}\sqrt{m/2E}\,dE}{2\pi^2\hbar^3}$$

$$= \frac{\sigma_a\, 2mE\, dE}{2\pi^2\hbar^3}$$

Substituting value of σ_a from (5), we get

$$\therefore \quad dP = \left(\frac{C}{(E - E_r)^2 + \frac{\Gamma^2}{4}}\right) \cdot \frac{p^2\,dE}{2\pi^2\hbar^3}$$

If λ is the associated wavelength, then according to de-Broglie hypothesis

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{\hbar}{\lambda\!\!\!^{-}} \text{ where } \lambda\!\!\!^{-} = \frac{\lambda}{2\pi}$$

$$\therefore \quad dp = \frac{C}{(E-E_r)^2 + \frac{\Gamma^2}{4}} \frac{(\hbar/\lambda\!\!\!^{-})^2\, dE}{2\pi^2\hbar^3}$$

$$= \frac{C}{\left((E-E_r)^2 + \frac{\Gamma^2}{4}\right)} \cdot \frac{dE}{2\pi^2\hbar\lambda\!\!\!^{-2}}$$

Integrating over whole energy spectrum, the total probability is given by

$$P = \int_{-\infty}^{+\infty} \frac{C}{2\pi^2\hbar\lambda\!\!\!^{-2}} \cdot \frac{dE}{(E-E_r)^2 + \frac{\Gamma^2}{4}}$$

As integrand has finite values only for energy within the level width (Γ) which is so narrow that we can neglect the variation of $\lambda\!\!\!^{-}$, so

$$P = \frac{C}{2\pi^2\hbar\lambda\!\!\!^{-2}} \int_{-\infty}^{+\infty} \frac{1}{(E-E_r)^2 + \frac{\Gamma^2}{4}} dE$$

$$= \frac{C}{2\pi^2\hbar\lambda\!\!\!^{-2}} \frac{2}{\Gamma}\left[\tan^{-1}\frac{2(E-Er)}{\Gamma}\right]_{-\infty}^{+\infty}$$

$$= \frac{C}{2\pi^2\hbar\lambda\!\!\!^{-2}} \frac{2}{\Gamma}\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$\therefore \quad P = \frac{C}{\pi\hbar\,\Gamma\lambda\!\!\!^{-2}} \qquad \ldots(6)$$

If Γ_a is the partial width of the compound level for the decay through the entrance channel, then the probability of decay of compound nucleus through entrance channel per second $= \frac{\Gamma_a}{\hbar}$.

\For equilibrium the probability of formation per second must be equal to the probability of decay per second of the excited state through the same channel; so

$$\frac{C}{\pi\hbar\,\Gamma\lambda\!\!\!^{-2}} = \frac{\Gamma_a}{\hbar}$$

$$\Rightarrow \quad C = \pi\lambda\!\!\!^{-2}\,\Gamma\,\Gamma_a \qquad \ldots(7)$$

$\therefore$ The cross-section for the formation of the compound nucleus is

$$\sigma_a = \frac{\pi\lambda\!\!\!^{-2}\,\Gamma\,\Gamma_a}{(E-E_\gamma)^2 + \frac{\Gamma^2}{4}} \qquad \ldots(8)$$

The relative probability of the decay of compound nucleus through the exit channel $Y + b$ can be written as $\frac{\Gamma_b}{\Gamma}$, so the cross-section for the reaction $X\ (a, b)\ Y$ is given by

$$\sigma(a, b) = \sigma_a \cdot \frac{\Gamma_b}{\Gamma} = \frac{\pi\lambda\!\!\!^{-2}\,\Gamma\,\Gamma_a}{(E-E_r)^2 + \frac{\Gamma^2}{4}} \cdot \frac{\Gamma_b}{\Gamma}$$

∴ The reaction cross-section in this case becomes

$$\sigma^{(l)}(a,b) = \pi\lambda^2 \frac{(2I_C+1)}{(2I_X+1)(2I_a+1)} \cdot \frac{\Gamma_a \Gamma_b}{(E-E_r)^2 + \frac{\Gamma^2}{4}}$$

This is Breit-Wigner formula for particles having spin.

Reciprocity theorem

Let us consider a reversible process X+X⇌Y+y in which X, x,Yand occur arbitrary numbers in a large numbers in a large box of volume V. We are interesting in the relation between the total across section σ(x→y), most generally $\sigma(\boldsymbol{\alpha}\rightarrow\beta)$ of the reaction with entrance channel ∝ and reaction channel βand the cross section $\sigma(\beta \rightarrow \boldsymbol{\alpha})$ of the inverse reaction.

For this we use the fundamental theorem of statistical mechanics (the principle of overall balance), which states that when the system is in dynamical equlibrium all energetically permissible states or occupied with equal probability.

Here we are interested in two particular, states the reaction channel **α** and β. The theorem is then equivalent to starting that in a given energy range the number of possible channel in the box is proportional to the number of possible channels in to the box. The latter is

given by

Similarly, we have $N_\beta = p_\beta^2 V dE_\beta / 2\pi^2 \hbar^3 v_\beta$. ...(86)

The energy range for the two channels must of course be the same, *i.e.* $dE_\alpha = dE_\beta$, hence

$$\frac{\text{No. of channels } \alpha \text{ in the box}}{\text{No. of channels } \beta \text{ in the box}} = \frac{N_\alpha}{N_\beta} = \frac{p_\alpha^2 v_\beta}{p_\beta^2 v_\alpha}. \quad ...(87)$$

The system is in dynamical equilibrium when the number of $\alpha \to \beta$ transitions per second is equal to the number of $\beta \to \alpha$ transitions per second. The condition usually holds and is known as the *principle of detailed balance*. Further

No. of transitions $\alpha \hookrightarrow \beta$ per sec $= N_\alpha \times T(\alpha \to \beta)$,

where $T(\alpha \to \beta)$ is the transition probability for the transition $(\alpha \to \beta)$.

Hence $$p_\alpha^2 v_\beta T(\alpha \to \beta) = p_\beta^2 v_\alpha T(\beta \to \alpha). \quad ...(88)$$

The transition probability measures the chance that one particle moving with velocity v in volume V is scattered per sec. Hence the cross-section σ which corresponds to unit incident flux is given by the relation

$$\sigma = TV/v. \quad ...(89)$$

Combining relations (88) and (89) and using $k = p/\hbar$, we have

$$k_\alpha^2 \sigma(\alpha \to \beta) = k_\beta^2 \sigma(\beta \to \alpha) \quad ...(90)$$

or $$\sigma(\alpha \to \beta)/\lambda\!\!\!^{-2}_\alpha = \sigma(\beta \to \alpha)/\lambda\!\!\!^{-2}_\beta. \quad ...(91)$$

We have assumed zero intrinsic angular momenta for the particles so far. If I is the intrinsic angular momentum of any one of the particles, the corresponding density of states then must be multiplied by $2I + 1$. Thus if there are intrinsic momenta for X, x, Y and y, we may write

$$(2I_X + 1)(2I_x + 1) k_\alpha^2 \sigma(\alpha \to \beta) = (2I_Y + 1)(2I_y + 1) k_\beta^2 \sigma(\beta \to \alpha). \quad ...(92)$$

If the initial and final states have definite angular momenta, then the above equation must be employed.

NUCLEAR SHELL MODEL

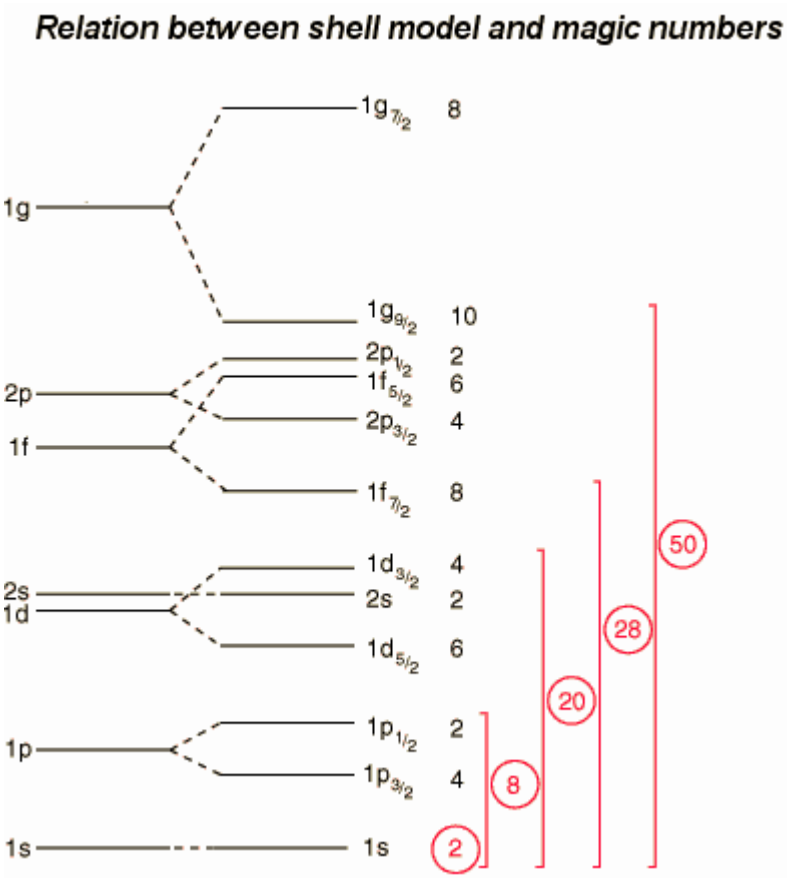
The nuclear shell model lacked a physical basis as was found in atoms. The reason is that a shell structure exists in atoms because of a relatively fixed central body (called the nucleus). All the electrons move independently around the central Coulomb force provided by the nucleus. Under this central force potential and the concept of Pauli exclusion principle, the solution of schrodinger equation result the atomic shell model structure which beautifully explain the periodic table. But there exists no Central force and physical principle for the nucleus. All the nucleus are equivalent. There exists a strong interaction among the nucleons then how can a nucleon move independently .

"Magic Numbers" in Nuclear Structure

It is found that nuclei with even numbers of protons and neutrons are more stable than those with odd numbers. In particular, there are "magic numbers" of neutrons and protons which seem to be particularly favored in terms of nuclear stability:

2, 8, 20, 28, 50, 82, 126
Magic Numbers

$^{4}_{2}He$ $^{16}_{8}O$ $^{40}_{20}Ca$ $^{48}_{20}Ca$ $^{208}_{82}Pb$



Calcium provides a good example of the exceptional stability of "doubly magic" nuclei since it has two of them. The existence of several stable isotopes of calcium may have to do with the fact that Z=20, a magic number. The two highlighted isotopes have neutron numbers 20 and 28, also magic numbers. Compared to the binding energy calculated from the Weizsaecker formula, they both have more than the expected binding energy.

Assumptions in independent particle model

To overcome the difficulty of the absence of a central Force, it was assumed that each nucleon inside the nucleus experiences as Central attractive forces which can be ascribed to the average effect of all other (A -1) nucleons in the nucleus. On this assumption each nucleon behaves as though it were moving independently in a central field, what is described as a short ranged potential well. Several potential well steps how were suggested to derive the magic numbers theoretically.

A typical form of the average potential is shown in Fig. (7.5)

The simplest potentials are

(*i*) Infinite rectangular potential well of the form

$$V(r) = \begin{cases} -V_0 & \text{for } r < R \\ \infty & \text{for } r > R \end{cases}$$

where R is nuclear radius which is a good approximation for a short-range force.

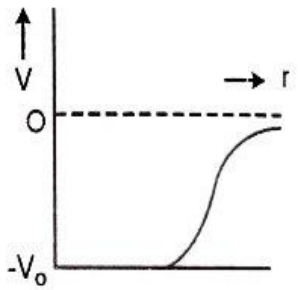


Fig. 7·5. A typical form of the average potential

(*ii*) Infinite harmonic oscillator potential (most suited for light nuclei)

$$V(r) = -V_0\left[1 - \left(\frac{r}{R}\right)^2\right]$$

With these potential wells, the solutions of Schrödinger's equation does not lead to all magic numbers. It leads to degenerate eigen states with orbital angular momentum l. For each l, there can be $(2l + 1)$ identical nucleons when spin is ignored or $2(2l + 1)$ identical nucleons when the energy is independent of spin orientation. The order of energy states for deep rectangular potential well turns out to be

Order of states	1*s*	1*p*	1*d*	2*s*	1*f*	2*p*	1*g*
Occupation number $2(2l + 1)$	2	6	10	2	14	6	18
Total number of nucleons	2	8	18	20	34	40	58

Clearly this fails to give any indication of a closed shell at 50 nucleons and fails even more clearly for still larger nucleon-numbers.

UNIT IV

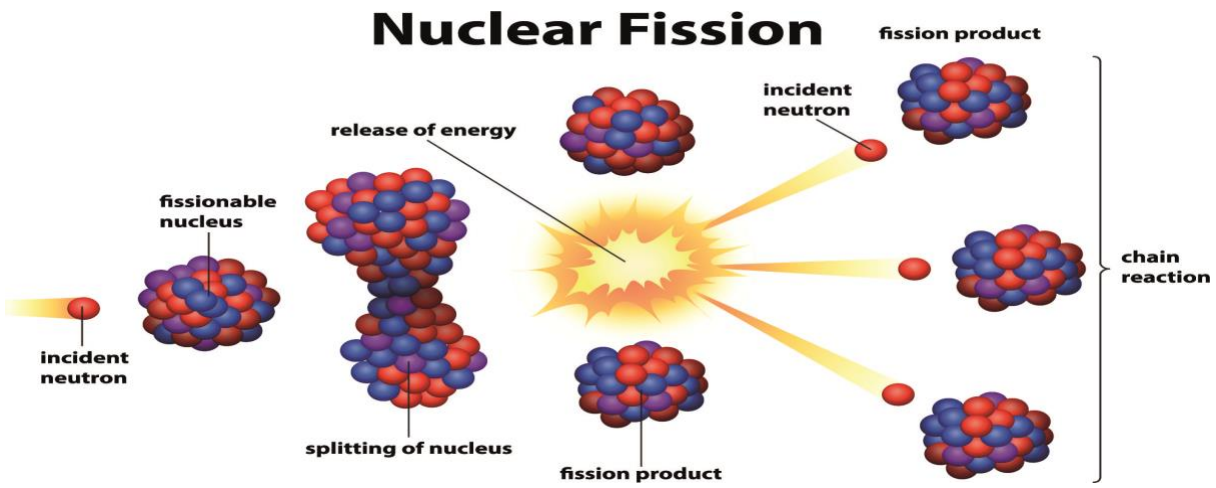
NUCLEAR REACTORS

UNIT IV

Nuclear Fission

The phenomenon of splitting of a heavy nucleus into two lighter fragments is called the nuclear fission. The nuclear fission was discovered by German scientist Otto and Fritz Strassmann in 1939 while attempting to produce transuranic elements (i.e., elements beyond uranium in the periodic table) by the bombardment of slow neutrons on ${}_{92}U^{235}$ nucleus.

In the process of nuclear fission enormous amount of energy is released because product nuclei have mass smaller than the parent nuclei. The difference of mass appears in the form of energy.



Types of fission

Thermal fission

a) since a thermal neutron adds negligible energy to the fissionable nucleus, it is clear from semi empirical mass formula that the fission of the nuclei in which the compound nuclei are of even-even structure take place even with their thermal neutrons. Fission of ^{235}U and ^{239}Pu by thermal neutrons are most important reactions.

b)Fast Fission

Other isotopes of Uranium and other elements which form compound nuclei of even-odd structure enter into (n,f) reactions with fast neutrons (>1 MeV). The example is ^{238} U.

c) charged particle Fission

Elements with Z> 90 show fission process with protons, neutrons and Alpha particles. High energy charged particles induce Fission in elements even in the middle of the periodic table.

d)The photo fission

High energy photons induce efficient in the heavier elements 5.1 Mev Gamma Rays can produce fission with ^{238} U. The photo fission of Uranium and Thorium was investigated by E.W. Tittertan and co-workers 1950 using continuous X rays from the synchrotron with a maximum energy of 24MeV.

They used the photographic plate technique and measured the range of the fission fragments and compared the kinetic energies of fission fragments from Thorium with those produced in slow neutron fission of ^{235}U. They found that the average kinetic energy released in photo fission of ^{232}Th was about 0.8 of that released in the slow neutron fission of ^{235} U.

e)Ternary fission

R. Present in 1941 using Liquid Drop model predicted the possibility of tri partition or ternary fission, that is the division of excited heavy nucleus into three nuclei of comparable masses.

L.Rosen and A.M. Hudson in 1949 found about 4.3 ternary fission per 10^6 binary fission of U^{235} bombarded with slow neutrons. K.W. Allen and J.T.Dewan in 1950 bombarded ^{233}U, ^{235}U and ^{239}Pu with slow neutron and found the third particle as a long range α particle in about 0.2 % of the fissions.

Fission cross section

Fission can be in some nuclides by certain incident particles of particular energy system. For example, U^{2235} undergoes fusion with thermal neutrons whereas ^{238}U does not. U^{238} and fission with fast neutrons in (> 1 Mev).

The nucleus of the fissionable material (^{233} U, ^{235}U , ^{238}U and ^{239} Pu) may capture the thermal neutron to form the compound nucleus or the neutron may be scattered. The compound nucleus may either undergo fission or emit Gamma Rays and decay to the ground state which may emit particles. The ratio of radioactive capture Cross section to the fission cross section is found to be about 0.18 for the nucleus ^{235}U and 0.39 for ^{239} Pu with thermal neutrons.

The fission cross section varies with energy in a complicated way. In the thermal energy range $\sigma\ is$ found to follow $1/v$ law. At higher energies (0.28 eV to 20 eV) there are many closely spaced resonance about 20 in ^{235}U. At high energies fission cross section decreases and reduces to about 1 barn for ^{235}U and 2 barn for ^{233}U with 1MeV neutrons.

Some heavy nuclides such as ^{234}U, ^{236}U , ^{238}U do not undergo fission with slow neutrons. Fission is a threshold reaction in these nuclides and the fission cross section where is with energy. The threshold energy and fission cross sections have also been measured for fission induced by charged particles. For ^{235}U the threshold for fission by deutrons is close to 8MeV, fission cross section rises to a value of 1 barn at about 20 Mev, increases slowly to a value 1.5 to 2 barns at 120-200 MeV. The threshold for the fission of ^{235}U by $\boldsymbol{\alpha}$ particles is about 21 MeV, the fission cross section rises to 1.5 barnes and above 50 MeV and remains this value up to energies of about 400 MeV.

ENERGY RELEASED IN FISSION

The energy released in nuclear fission is enormously large as compared to any conventional chemical reaction. In every fission of ^{235}U about 200 MeV energy is produced as kinetic energy. The energy of fission can be estimated in a number of ways as discussed below.

(*i*) **Estimation of energy from mass of fission fragments.** To estimate the energy we consider the fission reaction

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow X + Y + \partial n + Q \qquad ...(1)$$

The kinetic energy is given by

$$Q = (M_U + M_n)\, c^2 - (M_X + M_Y + \nu M_n)\, c^2 \qquad ...(2)$$

Where M_U is mass of uranium, M_n = mass of neutron M_X and M_Y mass of fission products and ν is number of neutrons produced.

or $$Q = (\Sigma M_{initial} - \Sigma M_{final})\, c^2 \qquad ...(3)$$

For example we take $X = {}^{98}M_0$, $Y = {}^{136}Xe$, $\nu = 2$

$M_U = 135{\cdot}124$ a.m.u., $M_n = 1{\cdot}009$ a.m.u.

$M_X = M\,(^{98}M_0) = 97{\cdot}936$ a.m.u.

$M_Y = M(^{136}Xe) = 135{\cdot}951$ a.m.u.

$\therefore$ $\Sigma M_{initial} = 235{\cdot}124 + 1{\cdot}009 = 236{\cdot}133$ a.m.u.

$\Sigma M_{final} = 97{\cdot}936 + 135{\cdot}951 + 2 \times 1{\cdot}009 = 235{\cdot}905$ a.m.u

$\therefore$ Mass defect $\Delta M = \Sigma M_{initial} - \Sigma M_{final} = 236{\cdot}133 - 235{\cdot}905 = 0{\cdot}228$ a.m.u.

As 1 a.m.u. = 931 MeV

$\therefore$ Energy released $Q = (\Delta M$ in a.m.u.$) \times 931$ MeV

$= 0{\cdot}228 \times 931$ MeV $\simeq 212$ MeV

This is the energy released per fission. Most of energy appears in the form of kinetic energy of fission fragments. On the basis of experimental data, the energy distribution is found as follows :

1.	Kinetic energy of heavy fragments	$\simeq$ 168 MeV
2.	Kinetic energy of fission neutrons	$\simeq$ 5 MeV
3.	Energy of β^- particles	$\simeq$ 8 MeV
4.	Energy of antineutrinos	$\approx$ 12 MeV
5.	Energy emitted as γ-rays	$\approx$ 12 MeV
	Total energy released per fission	$\approx$ 205 MeV

(*ii*) **Estimation from binding energy.** From binding energy per nucleon (B_n) versus mass number curve it is found that the binding energy per nucleon for ^{235}U is 7·6 MeV. The average binding energy per nucleon of two fission fragments lying in the middle of binding energy curve is about 8·5 MeV.

The difference of binding energy per nucleon is 0·9 MeV.

Hence the energy released per fission

$Q = 235 \times 0{\cdot}9$ MeV

$= \mathbf{211{\cdot}5\ MeV}$

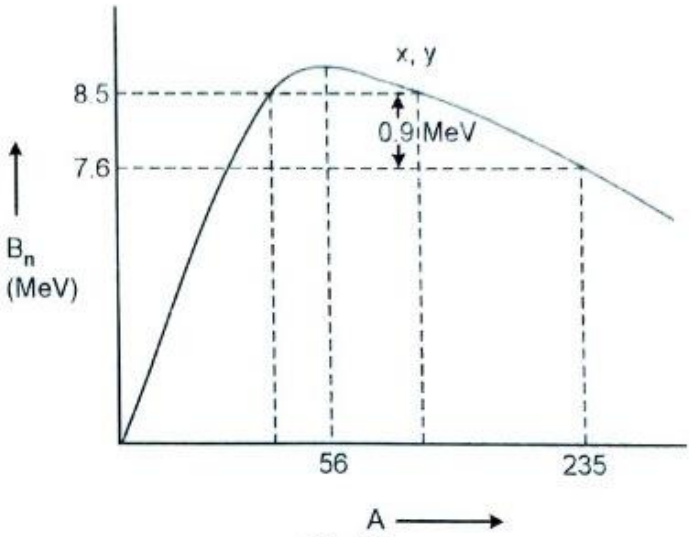


Fig. 9·2

9·5. DISTRIBUTION OF ENERGY OF FRAGMENTS

The nucleus undergoing fission can be considered at rest initially and the mass of neutrons is very small as compared with fission products X and Y. Then conservation of momentum gives

$$\vec{p_1} + \vec{p_2} = 0 \quad \Rightarrow \quad \vec{p_2} = -\vec{p_1} \qquad \ldots (1)$$

i.e., two fission fragments carry equal and opposite momentum. If M_1 and M_2 are masses of fragments and v_1, v_2 their velocity respectively, then numerically

$$M_1 v_1 = M_2 v_2 \Rightarrow \frac{v_1}{v_2} = \frac{M_2}{M_1} \qquad \ldots(2)$$

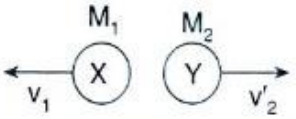


Fig. 9·3. Fission fragments after formation

If E_1 and E_2 are the kinetic energies of fission fragments, then

$$\frac{E_1}{E_2} = \frac{\frac{1}{2} M_1 v_1^2}{\frac{1}{2} M_2 v_2^2} = \frac{M_1}{M_2} \cdot \left(\frac{v_1}{v_2}\right)^2 = \frac{M_1}{M_2} \cdot \left(\frac{M_2}{M_1}\right)^2 = \frac{M_2}{M_1} \qquad \ldots(2)$$

thus the kinetic energies of fission fragments are inversely proportional to their masses.

The energy of fragments can be experimentally measured by a cloud chamber or proportional counter. The experimental curve of kinetic energy distribution of fission fragments of U^{235} is shown in Fig. 9·4. From the graphs it is clear that the energy maxima lie as 68 MeV and 100 MeV, the ratio of energy of the fragment is $\frac{E_2}{E_1} = \frac{100}{68} \approx 1{\cdot}5$. Also it is known that the mass ratio of the most abundant fragments is $\frac{M_1}{M_2} = \frac{135}{95} \approx 1{\cdot}5$.

Thus the experimental results verify the theoretical predictions. The measurement of fission-fragment energies gives clear evidence for the asymmetry of the fission process.

An approximate energy distribution of fission fragments is as follows :

Total kinetic energy of fission fragments = 168 MeV

Kinetic energy of lighter fragment = 100 MeV

Kinetic energy of heavier fragment = 68 MeV

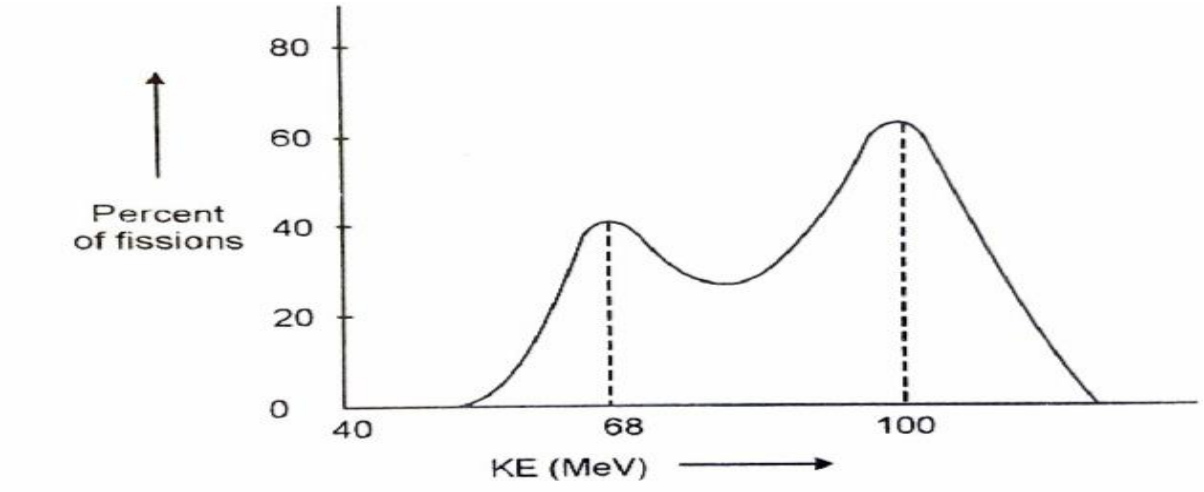


Fig. 9-4

9·10. BOHR AND WHEELER'S THEORY OF NUCLEAR FISSION

The first theoretical analysis of nuclear fission was carried out by Bohr and Wheeler in 1939 on the basis of liquid drop model. They considered the nucleus as a uniformly charged and incompressible drop (of constant density) and having a well defined surface. The requirement of incompressibility imposes the condition that the total volume of the drop remains constant. Hence the oscillations of the excited nucleus will deform the surface only. For simplicity the drop is considered to deform from a spherical shape in such a way that the axis of symmetry is retained. The symmetry axis is chosen as polar axis for spherical coordinates. The radial coordinate of a point on the distorted surface may be expressed in terms of Legendre polynomials :

$$R(\theta) = R_0\left[\, 1 + \sum_{l=0}^{\infty} \alpha_l P_l(\cos\theta)\right] \qquad \ldots(1)$$

where R_0 is the radius of undeformed spherical drop, α'_s are deformation parameters and $P_l(\cos\theta)$ are Legendre polynomials. The requirement of constant volume is specified by $\alpha_0 = 0$ and if centre of mass of the drop is assumed constant, then $\alpha_1 = 0$, so equation (1) gives

$$R(\theta) = R_0 (1 + \alpha_2 P_2 + \alpha_3 P_3 + \ldots) \qquad \ldots(2)$$

The surface energy of a spherical drop

$$E^{\circ}_{surface} = 4\pi R_0^2\ T \text{ where } T \text{ is surface tension.} \qquad \ldots(3)$$

The surface energy of a deformed drop is expressed as

$$E_{surface} = T \int dS \qquad \ldots(4)$$

According to Bohr and Wheeler the surface energy of deformed (spherical) drop in terms of deformation parameter α_l is expressed as

$$E_{surface} = 4\pi R_0^2 T\left(1 + \frac{2}{5}\alpha_2^2 + \frac{5}{7}\alpha_3^2 + \cdots\right) \qquad \ldots(5)$$

The Coulomb energy of an undeformed (spherical) drop as calculated from semi-empirical mass formula is

$$E^\circ_{coulomb} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{3}{5}\frac{(Ze)^2}{R_0} \qquad \ldots(6)$$

For deformed drop, the Coulomb energy is

$$E_{coulomb} = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{(Ze)^2}{R_0}\left[1 - \frac{1}{5}\alpha_2^2 - \frac{10}{49}\alpha_3^2 - \cdots\right] \qquad \ldots(7)$$

Therefore the total deformation energy E_T is given by

$$E_T = E_{surface} + E_{coulomb}$$

$$= 4\pi R_0^2 T\left(1 + \frac{2}{5}\alpha_2^2 + \frac{5}{7}\alpha_3^2 + \ldots\right) + \frac{1}{4\pi\varepsilon_0} \cdot \frac{3}{5}\frac{(Ze)^2}{R_0}\left(1 - \frac{1}{5}\alpha_2^2 - \frac{10}{49}\alpha_3^2 - \cdots\right)$$

Neglecting higher order terms and using equations (3) and (6), we get

$$E_T = E^\circ_{surface}\left(1 + \frac{2}{5}\alpha_2^2\right) + E^\circ_{coulomb}\left(1 - \frac{1}{5}\alpha_2^2\right) \qquad \ldots(8)$$

The change in energy due to deformation of the liquid drop is given by

$$\Delta E = (E_{surface} + E_{coulomb}) - (E^\circ_{surface} \div E^\circ_{coulomb})$$

$$= E^\circ_{surface}\left(1 + \frac{2}{5}\alpha_2^2\right) + E^\circ_{coulomb}\left(1 - \frac{1}{5}\alpha_2^2\right) - (E^\circ_{surface} + E^\circ_{coulomb})$$

$$= \frac{2}{5}E^\circ_{surface}\,\alpha_2^2 - \frac{1}{5}E^\circ_{coulomb}\,\alpha_2^2 = \frac{1}{5}\alpha_2^2\,(2\,E^\circ_{surface} - E^\circ_{coulomb}) \qquad \ldots(9)$$

The surface energy appears with positive sign, it prevents the disruption of liquid drop, while Coulomb energy appears with negative sign; it promotes the disruption of drop therefore

(*i*) If $2E^\circ_{surface} > E^\circ_{coulomb}$, ΔE is positive and so the drop will be stable against spontaneous fission.

(*ii*) If $2E^\circ_{surface} < E^\circ_{coulomb}$, ΔE is negative, so the nucleus will be unstable against spontaneous fission.

(*iii*) If $2E^\circ_{surface} = E^\circ_{coulomb}$, $\Delta E = 0$ and the case is critical. In this case

$$2(4\pi R_0^2 T) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{3}{5}\frac{(Ze)^2}{R_0} \qquad \ldots(10)$$

As $R_0 = r_0 A^{1/3}$, equation (10) gives

$$\therefore \quad 8\pi\,(r_0^2 A^{2/3})\,T = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{(Ze)^2}{r_0 A^{1/2}}$$

This gives $\frac{Z^2}{A} = 4\pi\varepsilon_0 \frac{40\pi r_0^3 T}{3e^2} = 50$ ...(11)

This defines the limiting value of $\frac{Z^2}{A}$; it is expressed as $\left(\frac{Z^2}{A}\right)_{lim}$. Clearly nuclei with $\frac{Z^2}{A} > 50$ will be unstable against spontaneous fission. The value of $\frac{Z^2}{A}$ for uranium is nearly 36; so uranium is stable against spontaneous fission. The limiting value is reached for a transuranic nucleus with $Z = 117$ and $A = 270$, for which $\frac{Z^2}{A} = 50{\cdot}7$. As a result of shell structure, the actual value may be towards smaller Z.

Now we define a quantity called fissionable parameter x by the relation

$$x = \frac{(Z^2/A)}{(Z^2/A)_{lim}} \qquad \ldots(12)$$

and discuss fission in terms of x.

If $x < 1$ nucleus will be stable against spontaneous fission.

If $x > 1$, the nucleus will be unstable against spontaneous fission.

The critical energy for fission (E_f) is defined as the energy required to deform a drop when it is at the verge of splitting into two equal drops :

$$E_f = 4\pi r_0^3 T A^{2/3} f(x) \qquad \ldots(13)$$

where x is given by equation (12)

The critical energy to the first order in $\frac{Z^2}{A}$ may be estimated by neglecting the second order change in energy due to the neck joining the two fission fragments and comparing only the sum of surface energy and electrostatic energy of the spherical nuclei of half in size of parent nucleus and in contact with each other. That is

$$E_f = 2\,(4\pi r_0^2)\,T\left(\frac{A}{2}\right)^{2/3} - 4\pi r_0^2 T A^{2/3} + 2 \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{3}{5} \cdot \frac{(Ze/2)^2}{r_0(A/2)^{1/3}} + \frac{1}{4\pi\varepsilon_0} \frac{(Ze/2)^2}{2r_0\,(A/2)^{1/3}}$$

$$- \frac{1}{4\pi\varepsilon_0}\left(\frac{3}{5}\right) \cdot \frac{(Ze)^2}{r_0 A^{1/3}} \qquad \ldots(14)$$

Dividing by $4\pi r_0^2 T A^{2/3}$, we get

$$\frac{E_f}{4\pi r_0^2 T A^{2/3}} = f(x) = 0{\cdot}260 - 0{\cdot}215x \qquad \ldots(15)$$

For an uncharged droplet $x = 0$ and so $f(x) = 0{\cdot}260$. This condition implies that there exist no electrostatic forces to aid the fission; hence the critical energy is just the work done against surface tension in separating the two drops.

For $x = 1$, a small deformation from the spherical shape causes the drop to reach the spherical shape and get separated. If we plot $f(x)$ against x for $x = 0$ and 1, the curve is shown in Fig. 9·10 by a smooth curve connecting these values. The curve $f^*(x)$ determines for comparison the energy required to deform the nucleus into two spherical nuclei in contact with each other. The threshold energy for uranium-235 is 5·7 MeV. This corresponds to value of $x = 0{\cdot}74$. From this we conclude that $\left(\frac{Z^2}{A}\right)_{lim} = 48$ in agreement with experiments.

Four Factor Formula

In natural uranium both isotopes U^{235}, and U^{238} are present. $U^{235}$are present has large probability of spontaneous fissions with the capture of slow neutrons while U^{238} may observe slow neutrons without causing fission, it mainly causes radiator capture reaction. In such an assembly of materials the first generation neutrons will have a certain life history.

Consider the absorption of n thermal neutrons U^{235} in nuclei to produce fission in a very large size of natural uranium. If provided with a moderator whose function is to slow down neutrons. If v fast neutrons are produced per fission, than that total number of fast neutrons produced as a result of fission of n-nuclei U^{235}will be nv. In actual practice the number of fast neutrons produced will be slightly greater than nv due to fast fission of U^{235}- nuclei. If the fast neutrons produced due to fission of U^{238} is increased by a factor explain greater than 1 then the total number of fast neutrons produced = **$n\nu\Sigma$.**

Fast neutrons will collide uranium nucleus and some will get slowed down without being captured in U^{238} resonance states. suppose a fraction P of neutrons escape resonance capture while slowing down this fraction. This fraction p is called the probability of resonance escape then the number of neutrons slowed down to thermal energy to produce fission of U^{235} is **$n\nu\Sigma p$,** while the numbers **$n\nu\Sigma(1-p)$** is captured to produce non fission reactions while slowing down.

The neutrons attaining thermal energies are either abserved $U^{235}$to cause for the fission will be used without fission. let f be the fraction of thermal neutrons which are observed in fuel. The factor f is called the thermal utilisation factor, so that the total number of thermal neutrons observed in U^{235} is **$n\nu\Sigma pf$**. The remaining neutrons **$n\nu\Sigma p(1-f)$** will produce non fission reaction. All the thermal neutrons observed in235 uranium do not produce fission, some produce non fission reaction. If g is the fraction of thermal neutrons in uranium which cause fission U^{235} in second generation, then the total number of thermal neutrons available for producing Fission in second generation due to n thermal neutrons fission in first generation will be

$$\mathbf{V_0 = n\nu\Sigma pfg}$$

The quantity $\eta = \nu g$ gives number of fast neutron for each thermal neutrons observed in U^{235} then $V_0 = n\Sigma pf\eta$

The neutron multiplication factor (k) which is the ratio of the second generation neutron to first generation neutrons is given by

$$\mathbf{K}=\frac{n\Sigma\eta pf}{n}=\mathbf{\Sigma pnf}$$

this relation is called four factor formula and holds for infinite sized system.

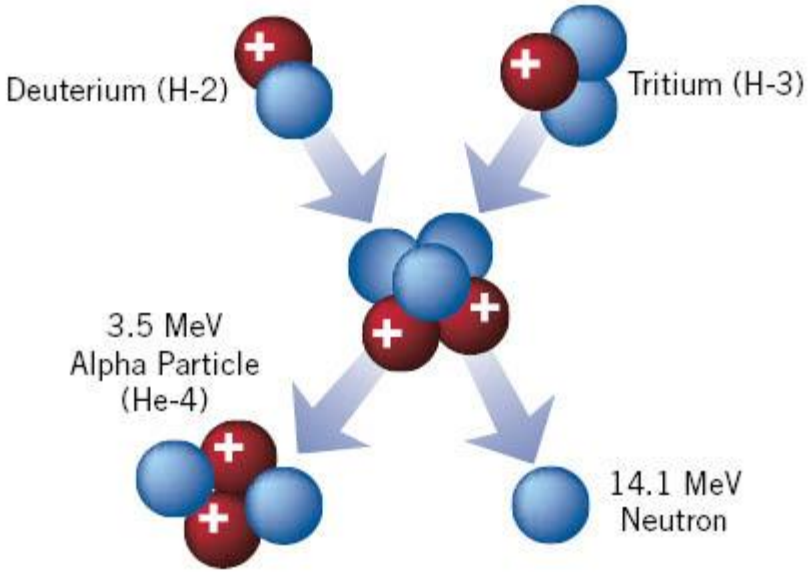
Basic Fusion Process

Nuclear fusion is a reaction in which two or more atomic nuclei are combined to form one or more different atomic nuclei and subatomic particles (neutrons or protons). The difference in mass between the reactants and products is manifested as either the release or the absorption of energy.

This difference in mass arises due to the difference in atomic binding energy between the nuclei before and after the reaction. Fusion is the process that powers active or main sequence stars and other high-magnitude stars, where large amounts of energy are released.

In cases where the interacting nuclei belong to elements with low atomic numbers e.g., hydrogen [atomic number] or its isotopes deuterium and tritium, substantial amounts of energy are released. The vast energy potential of nuclear fusion was first exploited in thermonuclear weapons, or hydrogen bombs, which were developed in the decade immediately following World War II. For a detailed history of this development, *see* nuclear weapon.

Meanwhile, the potential peaceful applications of nuclear fusion, especially in view of the essentially limitless supply of fusion fuel on Earth, have encouraged an immense effort to harness this process for the production of power.



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The release of energy with the fusion of light elements is due to the interplay of two opposing forces: the nuclear force, which combines together protons and neutrons, and the Coulomb force, which causes protons to repel each other. Protons are positively charged and repel each other by the Coulomb force, but they can nonetheless stick together, demonstrating the existence of another, short-range, force referred to as nuclear attraction. Light nuclei (or nuclei smaller than iron and nickel) are sufficiently small and proton-poor allowing the nuclear force to overcome repulsion.

This is because the nucleus is sufficiently small that all nucleons feel the short-range attractive force at least as strongly as they feel the infinite-range Coulomb repulsion. Building up nuclei from lighter nuclei by fusion releases the extra energy from the net attraction of particles. For larger nuclei, however, no energy is released, since the nuclear force is short-range and cannot continue to act across longer nuclear length scales. Thus, energy is not released with the fusion of such nuclei; instead, energy is required as input for process.

The D-T fusion reaction has a positive Q-value of 2.8×10^{-12} joule. The H-H fusion reaction is also exoergic, with a Q-value of 6.7×10^{-14} joule. To develop a sense for these figures, one might consider that one metric ton (1,000 kg, or almost 2,205 pounds) of deuterium would contain roughly 3×10^{32} atoms. If one ton of deuterium were to be consumed through the fusion reaction with tritium, the energy released would be 8.4×10^{20} joules. This can be compared with the energy content of one ton of coal—namely, 2.9×10^{10} joules. In other words, one ton of deuterium has the energy equivalent of approximately 29 billion tons of coal.

Radio Activity Decay Process

An unstable nucleus will decompose spontaneously, or decay, into a more stable configuration but will do so only in a few specific ways by emitting certain particles or certain forms of electromagnetic energy. Radioactive decay is a property of several naturally occurring elements as well as of artificially produced isotopes of the elements. The rate at which a radioactive element decays is expressed in terms of its half-life; i.e., the time required for one-half of any given quantity of the isotope to decay. Half-lives range from more than 1,000,000,000 years for some nuclei to less than 10^{-9} second. The product of a radioactive decay process—called the daughter of the parent isotope - may itself be unstable, in which case it, too, will decay. The process continues until a stable nuclide has been formed.

Radioactive Emissions

The emissions of the most common forms of spontaneous radioactive decay are the alpha (α) particle, the beta (β) particle, the gamma (γ) ray, and the neutrino. The alpha particle is actually the nucleus of a helium-4 atom, with two positive charges $^{4}/_{2}\mathrm{He}$. Such charged atoms are called ions. The neutral helium atom has two electrons outside its nucleus balancing these two charges. Beta particles may be negatively charged (beta minus, symbol e^{-}), or positively charged (beta plus, symbol e^{+}). The beta minus [β^{-}] particle is actually an electron created in the nucleus during beta decay without any relationship to the orbital electron cloud of the atom.

The beta plus particle, also called the positron, is the antiparticle of the electron; when brought together, two such particles will mutually annihilate each other. Gamma rays are electromagnetic radiations such as radio waves, light, and X-rays. Beta radioactivity also produces the neutrino and antineutrino, particles that have no charge and very little mass, symbolized by ν and ν, respectively.

Alpha Decay

In alpha decay, an energetic helium ion (alpha particle) is ejected, leaving a daughter nucleus of atomic number two less than the parent and of atomic mass number four less than the parent. An example is the decay (symbolized by an arrow) of the abundant isotope of uranium, $^{238}\mathrm{U}$, to a thorium daughter plus an alpha particle:

$$^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He} \qquad Q_\alpha = 4.268\ \mathrm{MeV}, \quad t_{1/2} = 4.51 \times 10^{9}\ \mathrm{years}$$

Beta Decay

In beta-minus decay, an energetic negative electron is emitted, producing a daughter nucleus of one higher atomic number and the same mass number. An example is the decay of the uranium daughter product thorium-234 into protactinium-234:

$$^{234}_{90}\mathrm{Th} \longrightarrow {}^{234}_{91}\mathrm{Pa} + \varepsilon^{-} + \bar{\nu} \qquad Q_{\beta^+} = .263\ \mathrm{MeV}, \quad t_{1/2} = 24.1\ \mathrm{days}$$

In the above reaction for beta decay, $\boxed{\nu}$ represents the antineutrino. Here, the number of protons is increased by one in the reaction, but the total charge remains the same, because an electron, with negative charge, is also created.

Nuclear Reactor

Nuclear reactor is a system in which year self sustaining fission chain reaction occurs in a controlled manner (K=1). The chain reaction may be controlled by absorbing a desired number of neutrons so that on the average one neutron from each fission is left to to cause further fission.Due to controlled chain reaction nuclear reactor is a source of useful energy. The design varies depending on the purpose of use. In general a nuclear reactor consist of the following parts.

1.Fuel

The fissionable material used in reactor is termed as fuel. The generally used fuels are Uranium enriched inU-235,Pu239 or U-233. These three materials have much larger fission cross section at thermal energies of neutrons than any other nuclide occurring in quantity on earth or other nuclide that can be produced on a large scale. Reacters usingU-233 or Pu-239, are called fast breeder reactors because such a reactor produces more fuel then it consumes.

2. Moderator

A moderator is a suitable material which is used to slow down. The fast neutrons produced in fission. The neutrons emitted in fission process have a spectrum of energies with most probable value nearly e 0.7 Mev. In a fast reactor, these neutrons are utilised directly to produce fission. In a common type of nuclear reactor called the thermal reactor, the fission is predominantly produced by thermal neutrons, it is necessary to slow down neutrons considerably in order to increase their chance of inducing further fissions.

Thus it is important to reduce the energies of neutrons to thermal value 0.025eV; this is achieved by using a velocity moderator in the vicinity of the fuel, the moderator must reduce the kinetic energy of neutrons, but not absorb them. Further it must be present in the sufficient quantity and distributed in such a way between the lumbs or rods of uranium fuel that a sufficiently large fraction of fast neutrons get slowed down due to collisions with the atoms of moderator instead of being captured by uranium itself.

A best moderator we must choose an element of low atomic weight A. The first six elements in order of increasing atomic weight or hydrogen, helium, lithium, beryllium, boron and carbon.

Of these there are no suitable solid or liquid compounds of helium and lithium. Hydrogen has a fairly large absorption cross section for slow neutrons. Boron has too large absorption cross section for slow neutrons that it is used as an absorber of neutrons in reactor, hence cannot be used as a moderator.

Heavy hydrogen or deuterium has such a low neutron capture cross section that its utility for slowing down neutrons is greater than light hydrogen. It is frequently used as moderator in the form of heavy hydrogen for which M=12000.

Beryllium is a difficult element to handle because of its chemical properties and high cost; so its use as a moderator is limited.

Carbon (C^{12}) has a low thermal neutron capture cross section =0.0045 barns and is often used in the allotropic form of graphite for which M=160.

Nitrogen and much more commonly oxygen in molecular substances which are liquid or solid are used as moderators. Water is the most widely used oxygen containing substance.

The consequence is that heavy water (D_2O) and carbon (graphite) or the best choices as moderators.

3.Reflector

In order to minimize the loss of neutrons from the surface, the core of a reactor is often surrounded by a reflector. The reflector is a solid element of low absorbtion and high scattering cross section. Graphite is frequently used as reflector.

4. Controller

In order to control a chain reaction it is necessary to adjust the number of neutrons: This is usually achieved by means of control rods made of an element having a large neutron capture cross-section. The control rods are inserted into the core of the reactor. Cadmium possesses fairly large absorption cross-section for neutrons, hence the control rods are usually made of cadmium. The additional characteristic of cadmium is that it does not undergo transmutation or become radioactive.

$$_0n^1 + {}_{48}Cd^{113}\ (21{,}000\ \text{barns}) \rightarrow (\ {}_{48}Cd^{114}) \rightarrow {}_{48}Cd^{114} + \gamma$$

Compound nucleus

The delayed neutrons produced in fission are also helpful for controlling the chain reaction. Due to delayed neutrons the fluctuations in neutron density are sufficiently low during the operation of a reactor. These delayed neutrons provide sufficient time to control the mechanism. When the reactor is operating the component of the effective multiplication constant (ke) due to prompt neutrons alone is slightly less than unity. The remainder of Ke upto 1 is due to delayed neutrons.

5. Coolant.

The reactor generates a considerable amount of heat energy due to fission reaction. The excess heat energy is conducted away by a suitable cooling fluid called the coolant. The coolant-fluid is circulated round fuel elements in the readtor. The coolant –fluid must have a low neutron capture cross-section and a large thermal capacity. Commonly used coolants are air, carbon-dioxide, light and heavy water and a liquid sodium/potassium alloy.

6.Biological shield

All nuclear reactors except those operating at very low power produce intense neutrons, gamma radiations (due to β particles from fission fragments). These radiations are fatal for humans. To protect the operators from these fatal radiations, the reactor core should be surrounded by a reactor

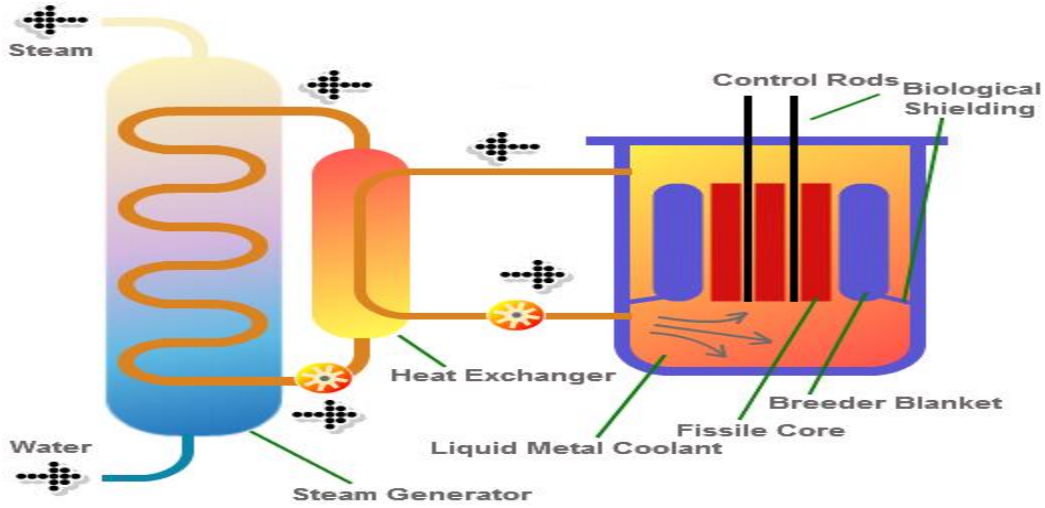
shield. Reactor shield is made of concrete with a high water content to stop neutrons and steel or concrete containing barium compounds to stop γ radiations.

Fast Nuclear Reactor

All current fast neutron reactor designs use liquid metal as the primary coolant, to transfer heat from the core to steam used to power the electricity generating turbines. FBRs have been built cooled by liquid metals other than sodium—some early FBRs used mercury, other experimental reactors have used a sodium-potassium alloy called NaK. Both have the advantage that they are liquids at room temperature, which is convenient for experimental rigs but less important for pilot or full-scale power stations. Lead and lead-bismuth alloy have also been used.

Three of the proposed generation IV reactor types are FBRs:

- **Gas-cooled fast reactor** (GFR) cooled by helium.
- **Sodium-cooled fast reactor** (SFR) based on the existing liquid-metal FBR (LMFBR) and integral fast reactor designs.
- **Lead-cooled fast reactor** (LFR) based on Soviet naval propulsion units.
    - FBRs usually use a mixed oxide fuel core of up to 20% plutonium dioxide (PuO_2) and at least 80% uranium dioxide (UO_2). Another fuel option is metal alloys, typically a blend of uranium, plutonium, and zirconium (used because it is "transparent" to neutrons). Enriched uranium can also be used on its own.



- Many designs surround the core in a blanket of tubes that contain non-fissile uranium-238, which, by capturing fast neutrons from the reaction in the core, converts to

fissile plutonium-239 (as is some of the uranium in the core), which is then reprocessed and used as nuclear fuel. Other FBR designs rely on the geometry of the fuel itself (which also contains uranium-238), arranged to attain sufficient fast neutron capture. The plutonium-239 (or the fissile uranium-235) fission cross-section is much smaller in a fast spectrum than in a thermal spectrum, as is the ratio between the $^{239}Pu/^{235}U$ fission cross-section and the ^{238}U absorption cross-section.

- This increases the concentration of $^{239}Pu/^{235}U$ needed to sustain a chain reaction, as well as the ratio of breeding to fission.[16] On the other hand, a fast reactor needs no moderator to slow down the neutrons at all, taking advantage of the fast neutrons producing a greater number of neutrons per fission than slow neutrons. For this reason ordinary liquid water, being a moderator and neutron absorber, is an undesirable primary coolant for fast reactors.
- Because large amounts of water in the core are required to cool the reactor, the yield of neutrons and therefore breeding of ^{239}Pu are strongly affected. Theoretical work has been done on reduced moderation water reactors, which may have a sufficiently fast spectrum to provide a breeding ratio slightly over 1.
- This would likely result in an unacceptable power derating and high costs in a liquid-water-cooled reactor, but the supercritical water coolant of the supercritical water reactor (SCWR) has sufficient heat capacity to allow adequate cooling with less water, making a fast-spectrum water-cooled reactor a practical possibility.[26]
- The type of coolants, temperatures and fast neutron spectrum puts the fuel cladding material (normally austenitic stainless or ferritic-martensitic steels) under extreme conditions. The understanding of the radiation damage, coolant interactions, stresses and temperatures are necessary for the safe operation of any reactor core.
- All materials used to date in sodium-cooled fast reactors have known limits, as explored in ONR-RRR-088 review.[45] Oxide Dispersion Strengthened (ODS) steel is viewed as the long-term radiation resistant fuel-cladding material that overcome the shortcomings of today's material choices. There are only two commercially operating breeder reactors as of 2017: the BN-600 reactor, at 560 MWe, and the BN-800 reactor, at 880 MWe. Both are Russian sodium-cooled reactors.

Plasma confinement

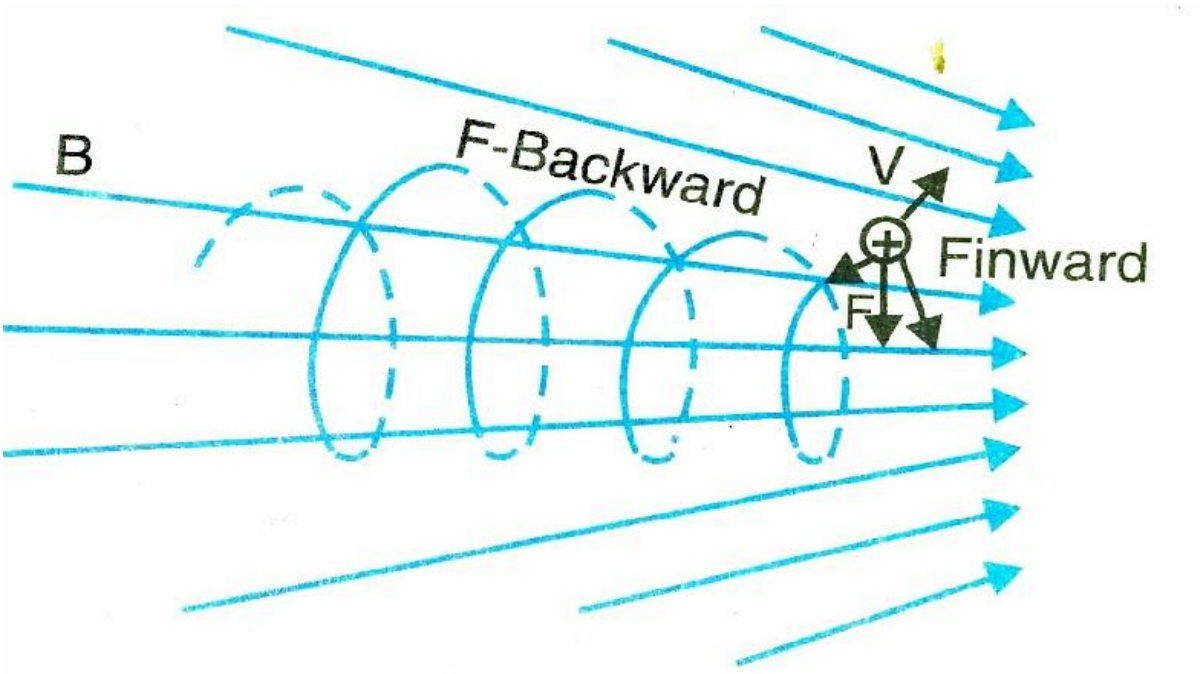
i)**Magnetic Confinement**

For the confinement of the plasma, one cannot use the walls of any vessel. Any contact with the wall will not only quickly cool the plasma. But also cause the wall to evaporate. There are at present two schemes under investigation for confining the thermo nuclear fuel: i) magnetic confinement ii)inertial confinement

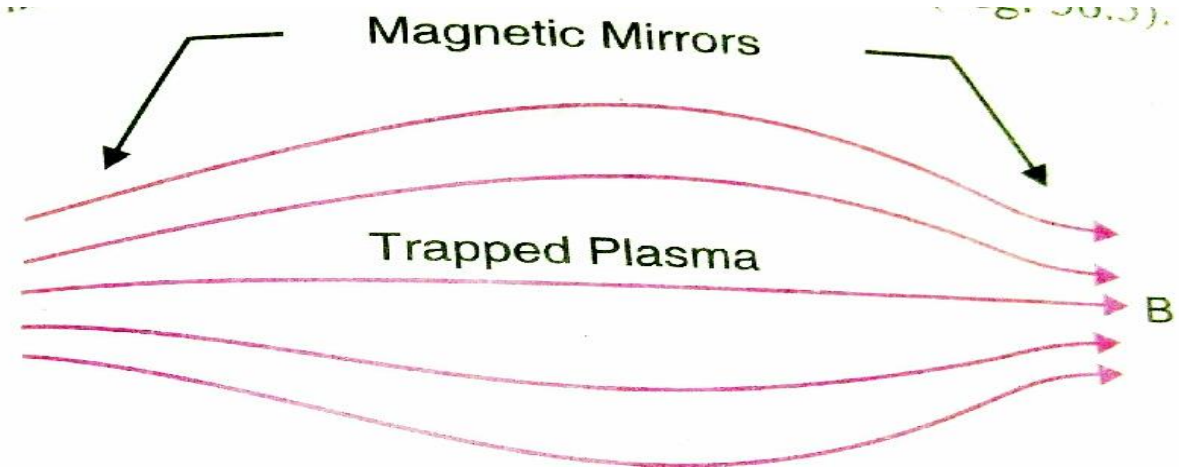
In magnetic confinement the plasma is confined by carefully designed magnetic field.Two classes of field is geometry or being studied. They are 1) magnetic bottle 2)Tokamak.

1)Magnetic bottle

It is based on the reflection of an Ion that moves in a magnetic field B whose lines of force converge. Converse the magnetic force on an ion of velocity v is perpendicular to both



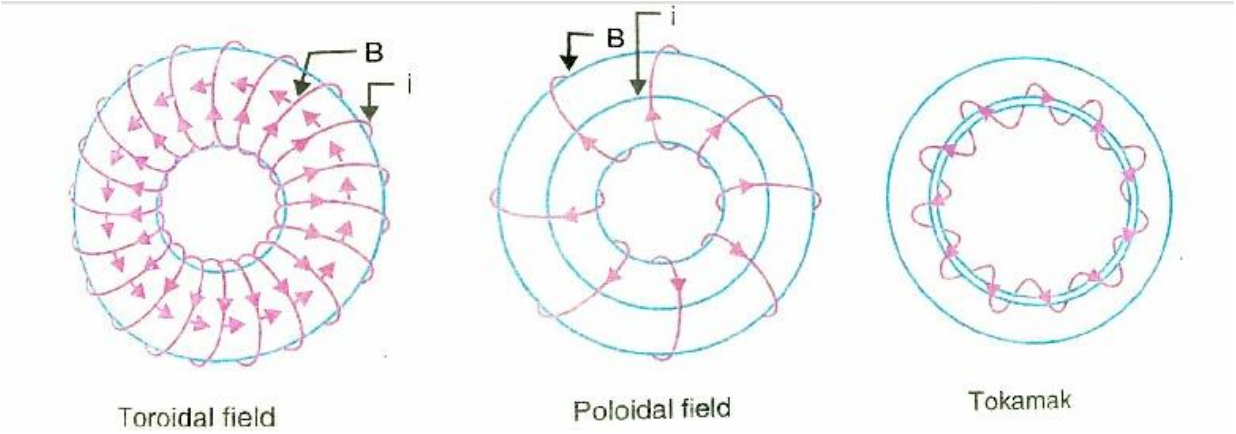
v and B. So the force has a backward component in such field as well as in board compound component that causes the ion to follow helical path around the lines of force. If the backward force is powerful enough the covering field act as a mirror to reverse the ions direction of motion.



That we can form a high density of magnetic field lines which reflects the particles back into the low field region and the scans known as a magnetic mirror. A pair of magnetic mirror constitutes a magnetic bottle.

2. Tokamak

A ‘tokamak’ is a magnetic bottle in which the confining field is created by a combination of currents flowing and external coils and currents flowing in the plasma. It is toroidal magnetic trap.

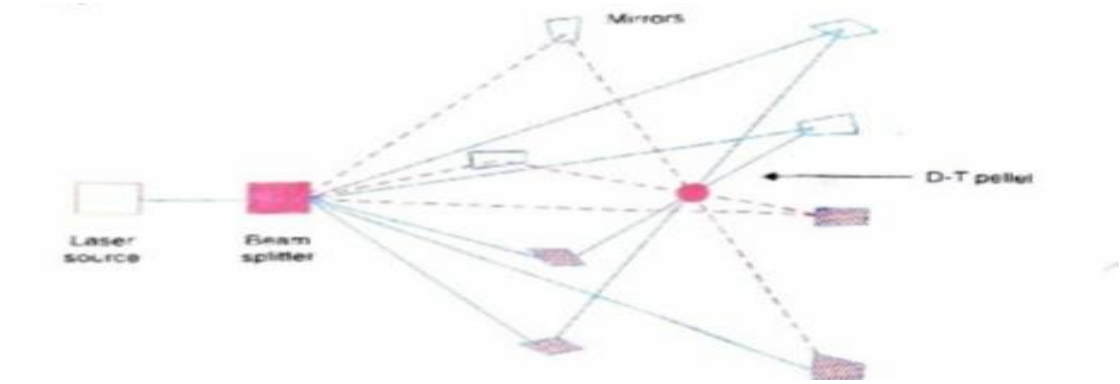


The principle of tokamak method of magnetic confinement shown in figure. A toroidal field is produced by a winding of oils and a poloidal field is produced by an axial current. These two fields are combined in the Tokamak design. The resulting field lines form a helix through which the ions can travel in close to orbits.

2) **Inertial confinement**

This scheme employs energetic beams to both heat and compress tiny deuterium- tritium Pellets. Here the fusion fuel in the form of a pellets is imploted from all sides by energy sources

such as laser beams high energy electrons or Ion beams. The intense compression pressure and the high temperatures produced in the pellet may produce conditions conducive to fusion.The difficulties in this approach or the low efficiency of laser or other sources and the need to produce stables symmetrical implosion.



Both the above schemes are in advanced experimental stage. The experiments have shown that fusion reactions can be initiated but it has not been possible to extract the useful amounts of energy .

Fusion reactors have two important advantages over fission reactors as sources of mechanical and electrical power.

1. The fuel for fusion reactors (Deuterium and tritium) is much more abundant on earth than that for fission reactors (Uranium or thorium).
2. Fusion Reactors do not produce large amounts of radioactive residues

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