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Department of Physics

ELECTRICITY & MAGNETISM

18K5P07

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Electricity & Magnetism 18K5P07
Unit-I Electrostatics

Fundamental Electrostatic Phenomena

5·1 Introduction

It was discovered as early as 600 *B.C.*, that when two bodies are rubbed with one another, they acquire the property of attracting light objects, like paper, pith etc. The two bodies are said to be electrified. For example, if a rod of ebonite is rubbed with fur, both get electrified. Similarly, glass and silk also get electrified when rubbed. Further it is found that two rods of ebonite rubbed with fur, repel one another and a rod of ebonite rubbed with fur attracts a glass rod rubbed with silk. This means the electrification of the ebonite rod is different from the electrification of the glass rod. Glass is said to acquire positive electricity and ebonite negative electricity. The $-ve$ charge acquired by ebonite is equal to the $+ve$ charge acquired by fur. Thus, when two bodies are rubbed they acquire equal amounts of positive and negative charges. The Greek work for **amber** is **electron** and the branch of electricity that deals with the production of charges by friction is called **electrostatics.**

5·2 Electric Field and Electric Intensity

Electric field. The space surrounding a charged conductor within which its influence can be felt is called the electric field.

Force between two charged conductors. Let A and B be two charged bodies and $+q_1$ and $+q_2$ the charges on them respectively, separated by a distance of r.

Fig. 5·1

Then the force of repulsion F between the charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between the charges.

$$F \propto q_1q_2 \text{ and } F \propto \frac{1}{r^2}$$

$$F \propto \frac{q_1q_2}{r^2}$$

$$\therefore \quad F = A\frac{q_1q_2}{r^2}$$

Here A is constant.

In *rationalised MKS units or SI units*

$$A = \frac{1}{4\pi\,\varepsilon_r\varepsilon_0}$$

Here ε_r is the relative permittivity of the medium and ε_0 is permittivity of free space.

Substituting this value of A in equation (i)

$$F = \frac{q_1q_2}{4\pi\varepsilon_r\varepsilon_0 r^2}$$

In *rationalised MKS units or SI units*, F is in newtons, r in metres and

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{\text{newton-metre}^2}{\text{coulomb}^2}$$

and $\quad \varepsilon_0 = 8{\cdot}85\times10^{-12}\ C^2/N\text{-}m^2$

When the two charges are in free space, relative permittivity of free space $\quad \varepsilon_r = 1$, and

$$F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$$

Electric intensity at a point in an electric field. Electric intensity at a point in an electric field is defined as the force experienced by a unit $+ve$ charge placed at that point.

Unit Charge :

In *rationalised MKS units or SI units*

$$F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} \text{ in free space}$$

If $\quad q_1 = q_2 = 1$ coulomb

$r = 1$ metre

and $\quad \dfrac{1}{4\pi\varepsilon_0} = 9\times10^9$

Then, $\quad F = \dfrac{1\times1\times9\times10^9}{1^2} = 9\times10^9$ newtons

Hence one *rationalised MKS unit or SI unit* of charge is defined

Gauss's Theorem

6·1 Gauss's Theorem

Gauss's theorem states that the total normal electrical induction over a closed surface is equal to Σq the total charge present inside the surface.

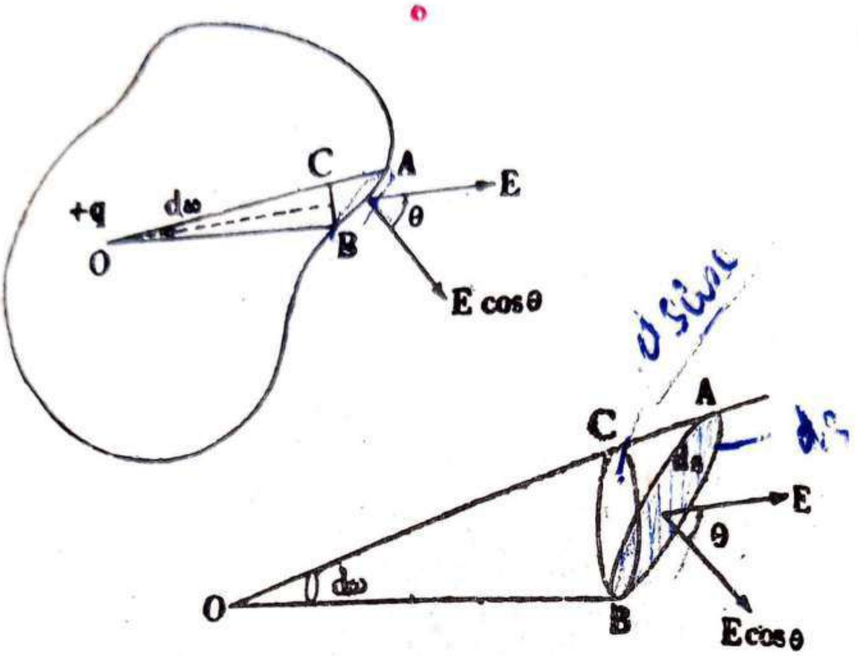


Fig. 6.1

Proof. Consider a closed surface with a charge q at the point O and a small element of the surface AB of area ds (Fig. 6·1).

Electric intensity at a point on the surface $AB = E = \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}$

Component of the intensity perpendicular to the surface

$$= E \cos\theta = \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}\cos\theta$$

T N E I over this elementary surface

= Dielectric constant × component of the intensity perpendicular to the surface × area of the surface

$$= \varepsilon_0\varepsilon_r \times \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}\cos\theta \times AB = \left(\frac{q}{4\pi}\right)\frac{AB\cos\theta}{r^2} = \frac{q\,d\omega}{4\pi}$$

$\left[\frac{AB\cos\theta}{r^2} = d\omega,\right.$ the solid angle subtended by the surface AB at O.$\left.\right]$

$T\,N\,E\,I$ over the whole surface $= \int \frac{q\,d\omega}{4\pi} = \frac{q}{4\pi}\int d\omega = \frac{q4\pi}{4\pi} = q$

(The solid angle subtended at a point inside a closed surface $= 4\pi$)

Case (*i*). If there are a number of charges q_1, q_2 etc. present inside the surface, the $TNEI = q_1 + q_2 + q_3 + \ldots = \Sigma q$.

Case (*ii*). If the charge q is outside the surface, the total normal electrical induction over the closed surface is zero. (*i*) At A, $TNEI$ inwards $= -\frac{1}{4\pi}q\,d\omega$. (*ii*) At B, $T\,N\,E\,I$ outwards $= +\frac{1}{4\pi}q\,d\omega$.

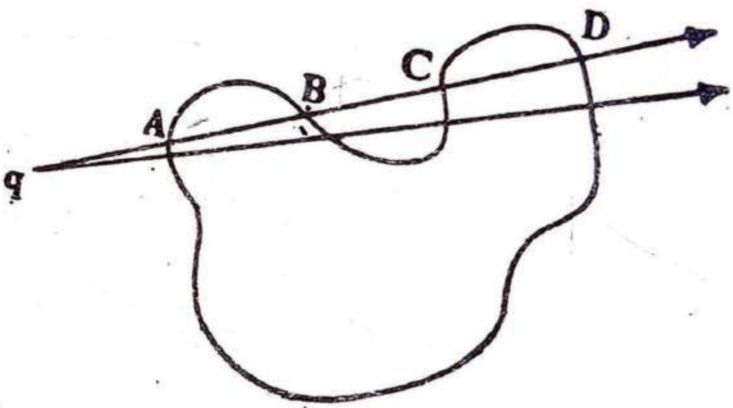


Fig. 6·2

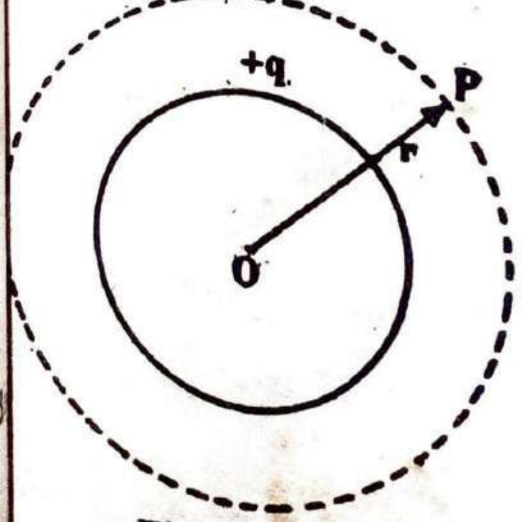
(*iii*) At C. $TNEI$ inwards $= -\frac{1}{4\pi}q\,d\omega$. (*iv*) At D, $TNEI$ outwards $= +\frac{1}{4\pi}q\,d\omega$.

$$\therefore \qquad \frac{1}{4\pi}\Sigma\, q\,d\omega = 0$$

6.2 Applications of Gauss's

(1) **Electric intensity at a point due to a charged sphere.** Consider a point P at a distance r from the centre of a sphere (Fig. 6.3).

Let the electric intensity at P be E. Charge on the sphere $= q$. $T\,N\,E\,I$ according to Gauss's theorem



$= q \qquad \ldots(i)$

$T\,N\,E\,I$ is also $= \varepsilon_0\varepsilon_r\, EA$

$= \varepsilon_0\varepsilon_r \times E \times 4\pi r^2 \ldots(ii)$

(Surface area of the sphere of radius r $= A = 4\pi r^2$)

From (*i*) and (*ii*)

$$q = \varepsilon_0\varepsilon_r\, E\, 4\pi r^2$$

$$= \frac{\sigma}{2\varepsilon_0\varepsilon_r} - \frac{\sigma}{2\varepsilon_0\varepsilon_r} = 0$$

(5) **Electric intensity at a point due to a uniform[ly] charged cylinder**. Let AB be the charged cylinder having a charge q per metre. P is a point at a distance of r from the axis of cylinder and R is the radius of the cylinder (Fig. 6.8). Construct imaginary cylinder of radius r and length l round the cylinder. Let E be the electric intensity at P. Then

T.N.E.I. over the curved surface of the cylinder of radius $r = \varepsilon_0\varepsilon_r E. 2\pi r. l$

But *T.N.E.I.* for the curved surface area of this cylinder according to Gauss's theorem $= q\,l$

[ql is the charge inside the imaginary cylinder]

(There are no electric lines of force perpendicular to the surface area of the imaginary cylinder).

From (*i*) to (*ii*), $ql = \varepsilon_0\varepsilon_r E. 2\pi\ r. l$

In *rationalised MKS units or SI units*

$$E = \frac{q}{2\pi\varepsilon_r\varepsilon_0\ R} \qquad ...(ii)$$

(where R is the radius of the charged cylinder)

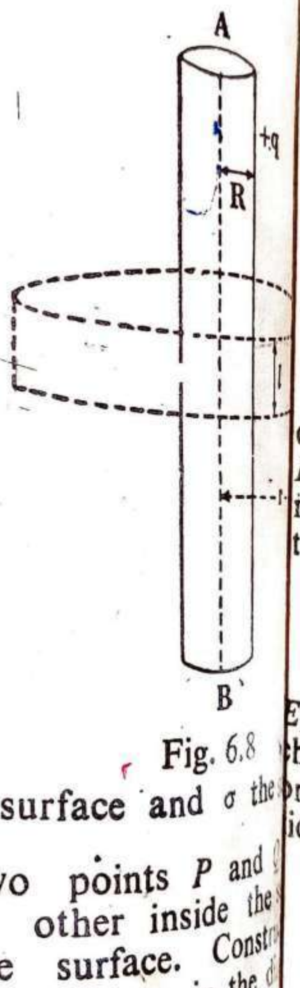


Fig. 6.8

6·3 Coulomb's Law.

It states that the electric intensity at any point near a charged surface of any shape is equal to $\frac{\sigma}{\varepsilon_0\varepsilon_r}$ where σ is the surface density of charge and $\varepsilon_0\varepsilon_r$ is the dielectric constant of the medium.

Proof. Let ABC be the charged surface and σ the density of charge.

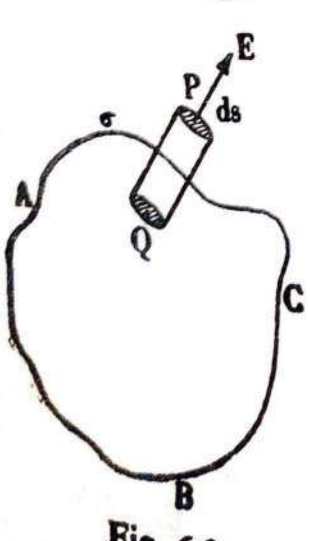


Fig. 6·9

Consider two points P and Q outside and the other inside the surface and close to the surface. Construct imaginary cylinder, as shown in the diagram of cross sectional area ds (Fig. 6·9).

T.N.E.I. over the plane surface at $P = \varepsilon_0\varepsilon_r E.\ ds$ where E is the electric intensity at P.

T.N.E.I. over the surface at Q because the electric intensity inside the charged surface is zero.

$$\therefore T.N.E.I. = \varepsilon_0\varepsilon_r E.\ ds + 0$$
$$= \varepsilon_0\varepsilon_r E.\ ds$$

But according to Gauss's theorem

$$T.N.E.I. = \sigma ds \quad ...(ii)$$

(ds is the area of the charged surface enclosed inside the cylinder)

From (i) and (ii), $\varepsilon_0 \varepsilon_r E. ds = \sigma. ds$.

$$E = \frac{\sigma}{\varepsilon_r \varepsilon_0}. \quad ...(iii)$$

6·4 Mechanical Force Experienced by Unit Area of a Charged Surface

Let σ be the surface density of charge. AB is a surface of unit area on the charged surface. Consider two points P and Q, one outside and the other inside the charged surface (Fig. 6·10).

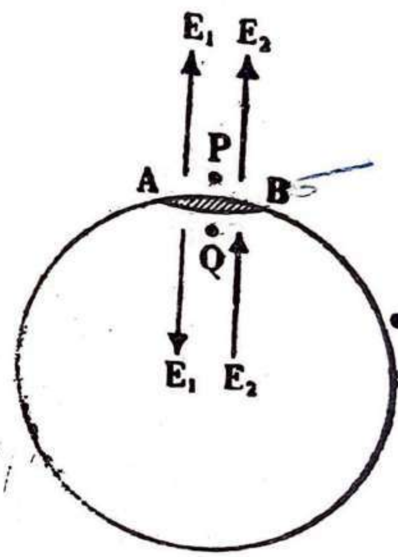


Fig. 6·10

Electric intensity at P,

$$E = E_1 + E_2 = \frac{\sigma}{\varepsilon_0 \varepsilon_r} \quad ...(i)$$

(according to Coulomb's law)

E_1 is the force experienced by a unit $+ve$ charge at P due to the $+ve$ charge on AB and E_2 is the force due to the $+ve$ charge on the rest of the surface These two forces are in the same direction.

Electric intensity at $Q = E_1 - E_2 = 0$ ...(ii)
[because electric intensity inside a closed charged surface = 0]

E_1 is the force experienced by a unit $+ve$ charge at Q due to the $+ve$ charge on the surface AB and E_2 is the force due to the $+ve$ charge on the rest of the surface. These two forces are in opposite directions.

From (ii), $\quad E_1 = E_2$

$\therefore$ From (i), $E_1 + E_2 = 2E_1 = 2E_2 = \dfrac{\sigma}{\varepsilon_0 \varepsilon_r}$

or

$$E_2 = \frac{\sigma}{2\varepsilon_0 \varepsilon_r}$$

That is, a unit $+ve$ charge on AB experiences an upward force $\dfrac{\sigma}{2\varepsilon_0 \varepsilon_r}$ due to the charge present on the rest of the surface.

Charge present on the surface AB of area 1 sq metre $= \sigma$

$\therefore$ Force experienced by unit area of the charged surface,

$$F = \frac{\sigma^2}{2\varepsilon_r\varepsilon_0} \text{ newtons/m}^2$$

But
$$E = \frac{\sigma}{\varepsilon_0\varepsilon_r} \quad \therefore \sigma = \varepsilon_0\varepsilon_r E$$

Substituting the value of σ in (*iii*), mechanical force experienced by unit area of the charged surface.

$$F = \frac{\varepsilon_r\varepsilon_0 E^2}{2} \text{ newtons/m}^2 \quad \ldots(i$$

6·5 Energy stored per Unit Volume in an Electric Field

Mechanical force experienced by unit area of a charged surface

$$= \frac{\sigma^2}{2\varepsilon_0\varepsilon_r} = \frac{\varepsilon_0\varepsilon_r E^2}{2}$$

If this charge σ is moved through a distance of 1 metre against the electric force, work done per m $= \frac{\varepsilon_0\varepsilon_r E^2}{2} \times 1 \text{ J/m}^3$

This work done per unit volume$= \frac{\varepsilon_0\varepsilon_r E^2}{2}$ J/m^3 is stored as energy in the medium.

Hence energy stored per unit volume,

$$= \frac{\varepsilon_r\varepsilon_0 E^2}{2} \text{ joules/m}^3$$

Capacity and Condensers

7·1 (*i*) **Capacity of a Conductor**

If the charge on a conductor is gradually increased, its potential also increases and at any instant the charge given to a conductor is directly proportional to its potential.

$q \propto V$, or $q = CV$, or $C = \frac{q}{V}$ where C is called the capacity of the conductor.

The capacity of a conductor is defined as the amount of charge that has to be given to it to raise its potential by unity.

Units. A conductor has a capacity of one farad if a charge of one coulomb raises its potential by one volt. The practical (rationalised *MKS* unit or *SI*) unit of capacity is *Farad.*

(*ii*) **Capacity of a spherical conductor.** A sphere of radius r is given a charge q. Let the potential of the sphere be V.

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

$$C = \frac{q}{V} = 4\pi\varepsilon_0 r \text{ farad} \quad ...(i)$$

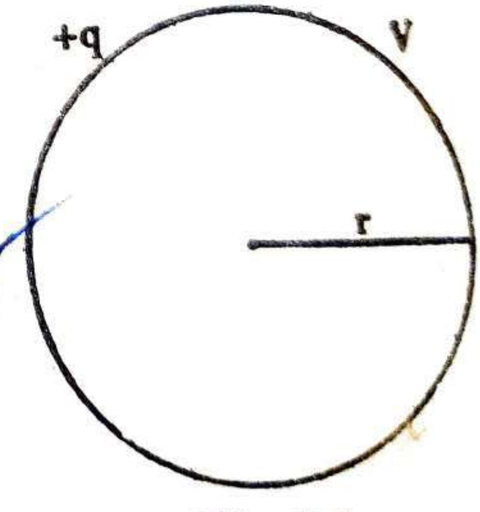


Fig. 7·1

(*iii*) **Energy of a charged conductor.** Let a conductor of capacity C be given a charge Q. The potential of the conductor $= V$. During the process of charging, let the charge on th[e co]nductor be q and the corresponding pot[en]t[ial] be V.

Fro[m] ... tion work done to give a unit $+ve$ charge to the conducto[r] ... [fo]r a charge dq, the work done $= v.\ dq$

and pro' ... nine th $v = \frac{q}{C}$ ∴ Work done $= \frac{q}{C}dq$

But,

Work done to give a charge Q to the conductor $= \int_0^Q \frac{q}{C}$

$= \left[\frac{q^2}{2C}\right]_0^Q = \frac{Q^2}{2C}$ But, $Q = VC$

∴ Work done $= \frac{C^2V^2}{2C} = \frac{1}{2} CV^2$

In *rationalised MKS units or SI units*

Work done $= \frac{1}{2} CV^2$ joules

∴ $E = \frac{1}{2} CV^2$ joules

Here C is in farads and V is in volts.

Work done in charging a conductor is stored as *potential en*... in the conductor.

(*iv*) **Sharing of charge between two charged conductors**

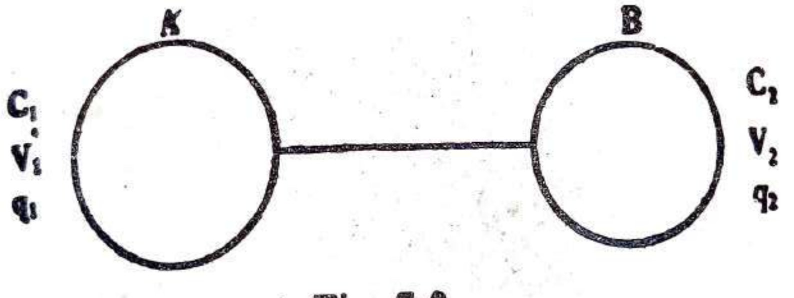


Fig. 7·2

Let A and B be the conductors of capacities C_1 and C_2, charge... potentials V_1 and V_2 respectively (Fig. 7·2).

Energy of the conductor A before contact $= \frac{1}{2} C_1V_1^2$

Energy of the conductor B before contact $= \frac{1}{2} C_2V_2^2$

Total energy before contact, $E_1 = \frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2V_2^2$

When the two conductors are joined by a wire;

the common potential $V = \frac{\text{Total charge}}{\text{Total capacity}}$

$= \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

Total energy of the conductors after contact

$E_2 = \frac{1}{2}(C_1 + C$...

$E_2 = \frac{1}{2}(C_1 + C_2) \frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)^2} = \frac{(C}{}$...

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Loss of energy due to contact,

$$E_1-E_2=\tfrac{1}{2}\,C_1V_1^2+\tfrac{1}{2}\,C_2V_2^2-\frac{(C_1V_1+C_2V_2)^2}{2(C_1+C_2)}$$

$$=\frac{1}{2(C_1+C_2)}\,[(C_1+C_2)\,(C_1V_1^2+C_2V_2^2)-(C_1V_1+C_2V_2)^2]$$

$$=\frac{1}{2(C_1+C_1)}\left[\begin{array}{c}C_1^2V_1^2+C_1C_2\,V_2^2+C_1C_2V_1^2+C_2^2V_2^2\\ -C_1^2V_1^2-C_2^2V_2^2-2C_1C_2V_1V_2\end{array}\right]$$

$$=\frac{C_1C_2}{2(C_1+C_2)}\,[V_1^2+V_2^2-2V_1V_2]$$

$$=\frac{C_1C_2}{2(C_1+C_2)}\,(V_1-V_2)^2$$

This is a $+ve$ quantity irrespective of the values of V_1 and V_2. Thus, whenever two charged conductors are connected by a wi spark is produced.

Example 7:1 A condenser of capacity 8 microfarad is ch

$\therefore$ Total capacity $C = 4\pi\varepsilon_0\varepsilon_r \left[b + \frac{ab}{b-a}\right]$

$$C = 4\pi\varepsilon_0\varepsilon_r \left[\frac{b^2 - ab + ab}{b-a}\right] = 4\pi\varepsilon_0\varepsilon_r \left[\frac{b^2}{b-a}\right]$$

for air medium $C = \frac{4\pi\varepsilon_0 b^2}{(b-a)}$

7·4 Capacity of a Cylindrical Condenser

A and *B* are two cylinders of radii *a* and *b* respectively. The inner cylinder has a charge *q* per unit length and the outer cylinder is earth connected (Fig. 7·6).

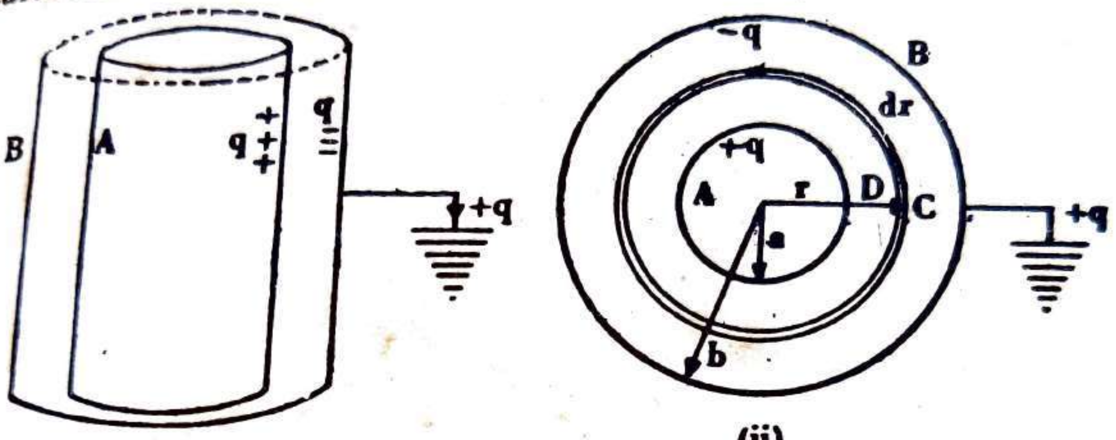


Fig. 7·6

Consider a concentric cylindrical shell of radius *r* and radial thickness *dr*.

Electric intensity at $C = \frac{q}{2\pi\varepsilon_0\varepsilon_r r}$

Work done in taking a unit +*ve* charge from *C* to *D*

$$= \frac{q}{2\pi\varepsilon_0\varepsilon_r r} dr$$

Hence the potential difference between the points *C* and *D*

$$dV = -\frac{q}{2\pi\varepsilon_0\varepsilon_r r} dr$$

Potential difference between the cylinders *A* and *B*

$$V = \int_b^a -\frac{q}{2\pi\varepsilon_0\varepsilon_r r} dr$$

$$V = \frac{-q}{2\pi\varepsilon_0\varepsilon_r} \int_b^a \frac{dr}{r} = \frac{-q}{2\pi\varepsilon_0\varepsilon_r} \left[\log_e r\right]_b^a$$

$$= \frac{-q}{2\pi\varepsilon_0\varepsilon_r} \left[\log_e a - \log_e b\right]$$

$$= \frac{q}{2\pi\varepsilon_0\varepsilon_r} \left[\log_e b - \log_e a\right]$$

$$\therefore \quad = \frac{q}{2\pi\varepsilon_0\varepsilon_r} \log_e \left(\frac{b}{a}\right)$$

The capacity per unit length of the condenser,

$$C = \frac{q}{V} = \frac{q}{\frac{q}{2\pi\varepsilon_0\varepsilon_r} \cdot \log_e\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0\varepsilon_r}{\log_e\left(\frac{b}{a}\right)}$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{2.3026 \times \log_{10}\left(\frac{b}{a}\right)}$$

This is the capacity of the cylindrical condenser for a le[ngth of one m]etre.

Therefore, the capacity of the condenser for a length l metr[e]

$$= \frac{2\pi\varepsilon_0\varepsilon_r l}{2.3026 \times \log_{10}\left(\frac{b}{a}\right)}$$

If the medium consists of a compound dielectric of relat[ive] permittivity ε_{r1} and ε_{r2},...., then the capacity of such a condenser p[er] unit length,

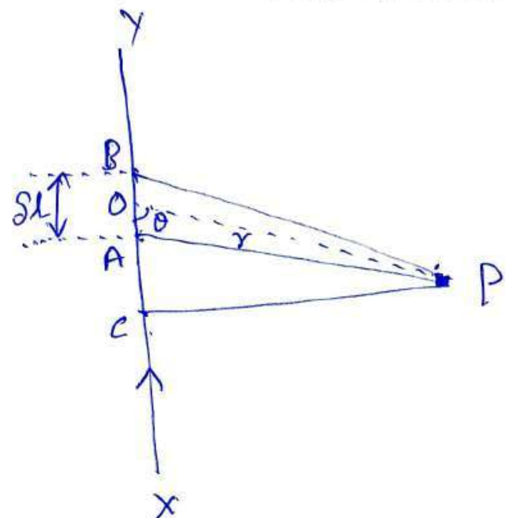
$$C = \frac{1}{\left(\frac{\log_e \frac{r_1}{a}}{2\pi\varepsilon_{r1}\varepsilon_0} + \frac{\log_e \frac{r_2}{r_1}}{2\pi\varepsilon_{r2}\varepsilon_0} + \ldots\right)}$$

$$C = \frac{2\pi\varepsilon_0}{\left(\frac{\log_e\left(\frac{r_1}{a}\right)}{\varepsilon_{r1}} + \frac{\log_e\left(\frac{r_2}{r_1}\right)}{\varepsilon_{r2}} + \ldots\right)}$$

Example 7·2. *A cable of wire* 3×10^{-3} m *in diameter and insulated with* 3×10^{-3} m *of gutta-percha (relative ...*

Unit-II Current Electricity

Biot-Savarts Law



Let XY be a conductor carrying current from X to Y. Consider an element AB of length δl of the conductor and let the current passing through the conductor be equal to I

The magnetic intensity at a point P distant r from the mid-point O of AB is given by the expression.

$$\delta F \propto \frac{I \cdot \delta l \cdot \sin\theta}{r^2}$$

Thus, the magnetic intensity at P

$\propto I$, the strength of the current

$\propto \delta l$, the length of the element

$\propto \sin\theta$,

where θ is the angle between the direction of the current and the line joining the mid-point O of the element to the point P and

$\propto \frac{1}{r^2}$, where r is the distance OP

$\therefore \delta F = \frac{k \cdot I \, \delta l \sin\theta}{r^2}$ where k is a constant.

SI units the constant $k = \frac{1}{4\pi}$

Laplace's law is

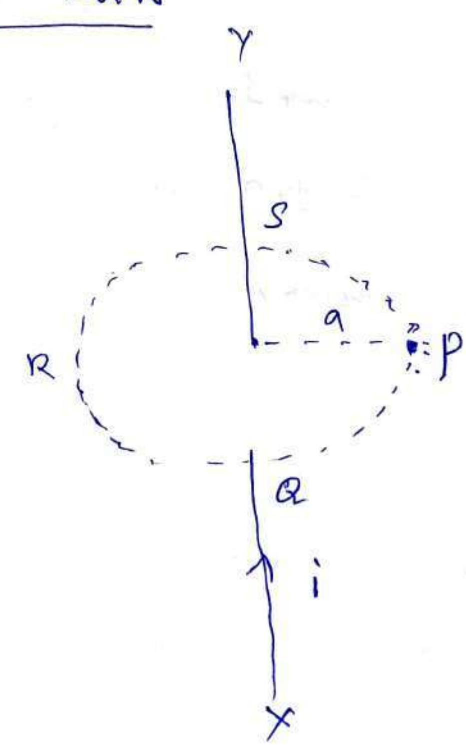
$$\delta F = \frac{1}{4\pi} \frac{I \, \delta l \sin\theta}{r^2}$$

The flux density δB depends on the nature of the medium. For free space

$$\therefore \boxed{\delta B = \frac{\mu_0}{4\pi} \frac{I \, \delta l \sin\theta}{r^2}}$$

this is Biot-Savart law

Ampere's Law



The work done in moving a unit north pole round a linear conductor carrying current can be calculated as follows. Let x y be the conductor carrying a current I

I SI units

The magnetic intensity at a point P

$$= \frac{1}{4\pi} \cdot \frac{2I}{a}$$

The flux density,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}$$

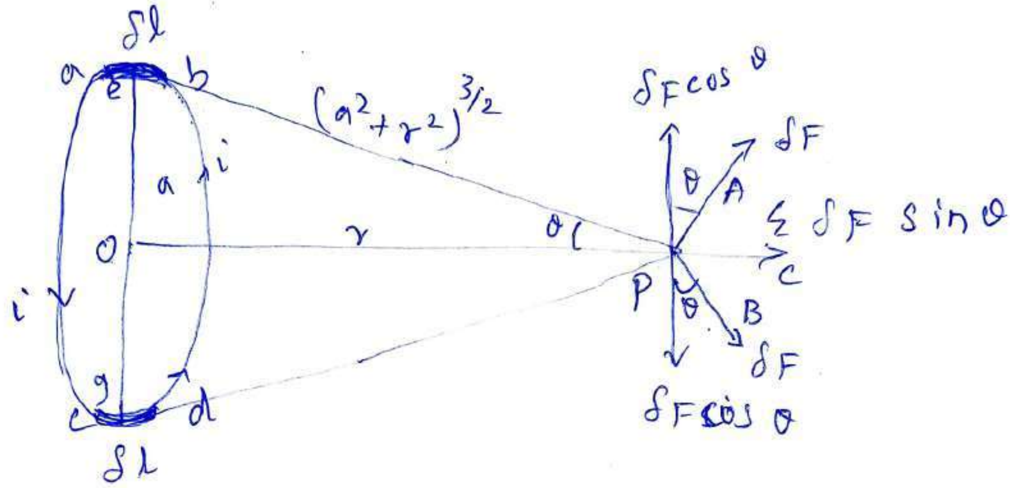
Work done

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} \cdot 2\pi a$$

$$= \mu_0 \cdot I \text{ Joules.}$$

This work done is called the magneto-motive force of the current carrying conductor.

Magnetic Intensity at a point on the Axis of a circular coil carrying current :



O is the centre of a circular coil of n number of turns and radius a. P is a point at a distance r from the centre and along the axis of the coil. The plane of the coil is perpendicular to the plane of the paper. Consider two elements ab and cd each of length δl which are diametrically opposite. Then the magnetic intensities at P due to these two elements will be δF and δF in the directions PA and PB respectively.

These directions are perpendicular to the lines joining the mid-points of the elements with the point. Resolve these intensities into two components, parallel ($\delta F \sin\theta$) and perpendicular ($\delta F \cos\theta$) to the axis of the coil.

The $\delta F \cos\theta$ components cancel one another and $\delta F \sin\theta$ components are in the same direction.

But, $\delta F = \dfrac{I\,\delta l}{4\pi(a^2+r^2)}$

because the angle between the direction of current in ab and the line ep is 90°

$\therefore$ Magnetic intensity at p due to the element ab

$$= \delta F \sin\theta = \frac{I\,\delta l \sin\theta}{4\pi(a^2+r^2)} \text{ along pc}$$

[in the Δ epo, $\theta = \angle epo$ and $\sin\theta = \dfrac{a}{(a^2+r^2)^{1/2}}$

$$\therefore \text{ Intensity} = \frac{I\,\delta l\, a}{4\pi(a^2+r^2)(a^2+r^2)^{1/2}}$$

$$= \frac{I\,\delta l\, a}{4\pi(a^2+r^2)^{3/2}}$$

Intensity at p due to one comp... turn

$$= \frac{1}{4\pi} \sum \frac{I\, \delta l\, a}{(a^2+r^2)^{3/2}} = \frac{I a}{4\pi(a^2+r^2)^{3/2}} \sum \delta l$$

$$= \frac{I a (2\pi a)}{4\pi (a^2+r^2)^{3/2}} = \frac{1}{4\pi} \left(\frac{2\pi a^2 I}{(a^2+r^2)^{3/2}} \right)$$

For n turns the magnetic intensity at p,

$$F = \frac{1}{4\pi} \cdot \frac{2\pi n a^2 I}{(a^2+r^2)^{3/2}} \text{ along PC}$$

If the direction of current in the coil is reversed, the magnitude of the intensity will be the same but the direction of the intensity will be along PC

SI units, the magnetic intensity at p.

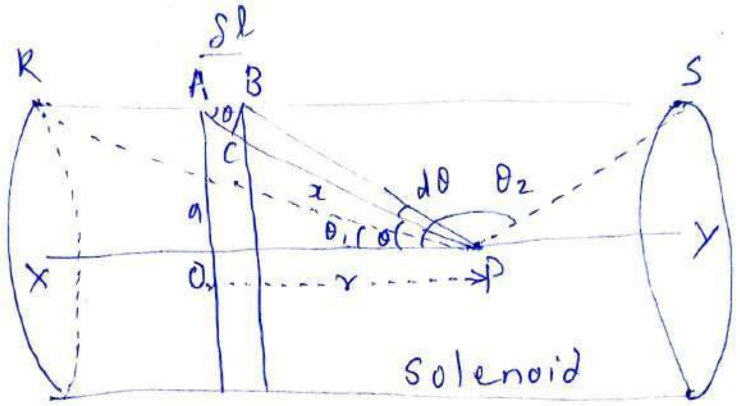
$$F = \frac{1}{4\pi} \cdot \frac{2\pi n a^2 I}{(a^2+r^2)^{3/2}}$$

Here F is in amp-turns/m, I → ampere, a, r in metres. The flux density.

$$\boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n a^2 I}{(a^2+r^2)^{3/2}}}$$

The unit of flux density is webers/m^2

Magnetic Intensity at a point on the Axis of a Solenoid



XY is the axis of a solenoid of radius a and number of turns per unit length equal to n. P is a point on the axis at a distance r from the centre O of an elementary coil of length δl and number of turns $n\delta l$

According to the expression derived in circular coil, the magnetic intensity at P due to this elementary coil.

$$\delta F = \left(\frac{1}{4\pi}\right)\frac{2\pi \cdot n\delta l a^2 I}{(a^2+r^2)^{3/2}}$$

$$= \left(\frac{1}{4\pi}\right)\frac{2\pi n \cdot \delta l I a^2}{x^3}$$

$$= \left(\frac{1}{4\pi}\right)\frac{2\pi \cdot n I a^2}{x^3} \cdot \frac{x\, d\theta}{\sin\theta}$$

[In ΔABC, $\angle BAC = \theta$

$x^2 = a^2 + r^2$, $\angle APO = \theta$

$x^3 = (a^2+r^2)^{3/2}$

$\therefore \sin\theta = \frac{BC}{AB}$

$BC = AB\sin\theta$

$x\, d\theta = \delta l \sin\theta$

$\delta l = \frac{x\, d\theta}{\sin\theta}$

$$= \left(\frac{1}{4\pi}\right) 2\pi n I \frac{a^2}{x^2} \cdot \frac{d\theta}{\sin\theta} \qquad \sin\theta = \frac{a}{x}$$

$$= \left(\frac{1}{4\pi}\right) 2\pi n I \frac{\sin^2\theta \cdot d\theta}{\sin\theta} \qquad \because \frac{a^2}{x^2} = \sin^2\theta$$

$$= \left(\frac{1}{4\pi}\right) 2\pi n I \sin\theta \, d\theta$$

$\therefore$ The magnetic intensity at p due to the whole solenoid

$$F = \left(\frac{1}{4\pi}\right) \int_{\theta_1}^{\theta_2} 2\pi n I \sin\theta \, d\theta$$

$[\angle RPX = \theta_1, \angle SPX = \theta_2]$

$$= \left(\frac{1}{4\pi}\right) 2\pi n I \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= \left(\frac{1}{4\pi}\right) 2\pi n I \left[\cos\theta_1 - \cos\theta_2\right] \quad \text{--- (1)}$$

SI units, the magnetic intensity at

$$p = \frac{1}{4\pi} \cdot 2\pi n I \left[\cos\theta_1 - \cos\theta_2\right]$$

$$= \frac{nI}{2} \left[\cos\theta_1 - \cos\theta_2\right]$$

Flux density $B = \frac{\mu_0 n I}{2} \left[\cos\theta_1 - \cos\theta_2\right]$

Special cases

(i) When the solenoid is infinitely long and the point p is well within the solenoid, then, $\theta_1 = 0$; $\theta_2 = 180$ (or) π

$\cos 0 = 1$
$\cos 180 = -1$

From (1) $n = \frac{N}{l}$

$$F = \frac{1}{4\pi} \cdot \frac{4\pi NI}{l} = \frac{NI}{l} \text{ amp - turns}/m$$

Flux density $B = \frac{\mu_0 NI}{l}$ webers/m^2

(ii) When p lies at the centre of one end of a long solenoid

$\theta_1 = 90^\circ$ $\theta_2 = 180^\circ$ $n = \frac{N}{l}$

From equation (i)

$\cos 90 = 0$
$\cos 180 = -1$

magnetic intensity.

$$F = \frac{1}{4\pi} \frac{2\pi NI}{l} = \frac{NI}{2l} \text{ amp - turns}/m$$

Flux density $B = \frac{\mu_0 NI}{2l}$ webers/m^2

12·7 Moving Coil Ballistic Galvanometer

The construction of a moving coil ballistic galvanom
similar to that of a dead beat galvanometer, except for the mo
tions discussed in article 12·5.

Theory. (*i*) If a current I is passed through the coil on n turns (length l, breadth b) and if the magnetic flux density is B, then the force experienced by the length side conductor of the coil $= BInl$

$\therefore$ Force $= BInl$

If this current flows for a small interval of time dt, then,

$$\text{impulse} = BnlIdt$$

$\therefore$ Change in momentum $= BnlIdt$

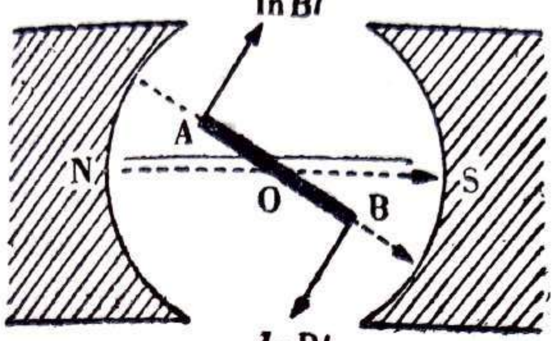


Fig. 12·7

Change in momentum for a charge q that flows through the coil

$$= \int BnlIdt = Bnl\int Idt = Bnlq \quad \left(\because \int Idt = q\right)$$

$\therefore$ Moment of momentum $= Bnlq \times b = nABq$

But moment of momentum $=$ Angular momentum $= \mathcal{J}\omega$

$\therefore \quad \mathcal{J}\omega = nABq$ ... (*i*) [where $A = l \times b$ the face area of coil]

(*ii*) Work done in twisting the suspension fibre by an angle $\theta = \frac{1}{2}C\theta^2$. This work done is equal to the KE of the oscillating system $\frac{1}{2}\mathcal{J}\omega^2$, when the angular displacement is θ.

$$\therefore \quad \frac{1}{2}\mathcal{J}\omega^2 = \frac{1}{2}C\theta^2 \quad \text{or} \quad \mathcal{J}\omega^2 = C\theta^2 \quad \ldots(ii)$$

(*iii*) The time period of oscillation of the oscillating system executing torsional oscillations is

$$t = 2\pi\sqrt{\frac{\mathcal{J}}{C}} \quad \text{or} \quad t^2 = \frac{4\pi^2\mathcal{J}}{C}$$

$$\mathcal{J} = \frac{t^2C}{4\pi^2} \quad \ldots(iii)$$

Multiplying equations (*ii*) and (*iii*), $\quad \mathcal{J}^2\omega^2 = \dfrac{C^2t^2\theta^2}{4\pi^2}$

or

$$\mathcal{J}\omega = \frac{C\,t\,\theta}{2\pi} \quad \ldots(iv)$$

Equating (*i*) and (*iv*) $\quad nABq = \dfrac{C\,t\,\theta}{2\pi}$

$$q = \frac{t}{2\pi} \cdot \frac{C}{nAB} \cdot \theta \quad \ldots(v)$$

Here $\frac{C}{nAB}$ is the current reduction factor of the galvanometer

and $\frac{t}{2\pi}\ \frac{C}{nAB}$ is the ballistic reduction factor (K).

$$\therefore \qquad q = K\theta$$

In equation (v), q is the charge that flows through the ballistic galvanometer, t the time period of oscillation of the coil, C the couple per unit twist of the suspension fibre, n the number of turns of the coil, A the face area of the coil, B the flux density and θ the throw observed in the ballistic galvanometer.

12·9 Correction for Damping in Ballistic Galvanometers

In the case of an ordinary moving coil galvanometer, a con-
tant current is passed through the coil and hence the deflection
constant. The pointer gives a constant reading. Ballistic galva-
meters measure charge in the form of sudden discharge and due
the impulse, a sudden kick is given to the coil. · Hence it is only
first throw that is effective in measuring the charge that flo
through the coil. After the first throw, the coil oscillates in
magnetic field with continuously decreasing amplitude. Due
electromagnetic induction in the coil, air resistance etc., there
decrease in amplitude. Let θ be the actual deflection in the absen
of damping and θ_1, θ_2, θ_3 etc. be the successive observed throw
the right and left continuously (Fig. 12·8).

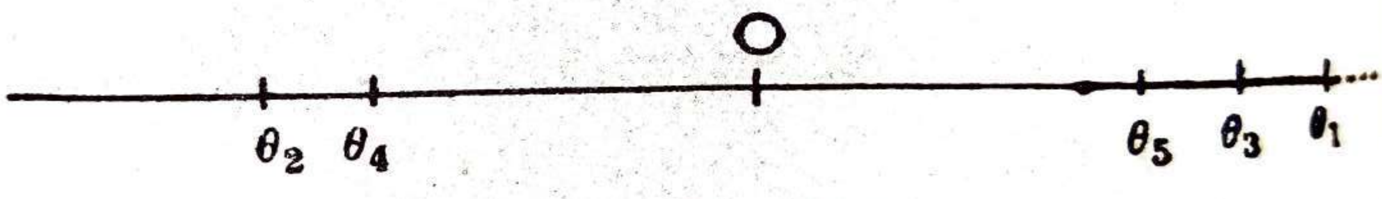


Fig. 12·8

It will be found that $\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = d$, where d is called the decrement.

Let d be equal to e^{λ} so that $\lambda = \log_e d$. Here λ is called the logarithmic decrement. Each complete vibration comprises of two swings (*i.e.*, from extreme right to left θ_1 to θ_2 and from extreme left to right θ_2 to θ_3).

$$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = d^2 = e^{2\lambda} \text{ (for two swings)}$$

Similarly for four swings, $\frac{\theta_1}{\theta_5} = d^4 = e^{4\lambda}$ and so on.

Let θ be the true first throw in the absence of damping which is higher than the observed first throw θ_1. The motion of the coil from the mean position to extreme right corresponds to half a swing,

$$\therefore \frac{\theta}{\theta_1} = d^{1/2} = e^{\lambda/2} = \left(1 + \frac{\lambda}{2}\right) \text{ approx.}$$

$$\theta = \theta_1 \left[1 + \frac{\lambda}{2}\right]$$

Substituting this value in equation (v) of article 12·7

$$q = \frac{t}{2\pi} \cdot \frac{C}{nAB} . \theta_1 \left[1 + \frac{\lambda}{2}\right] \qquad ...(i)$$

where θ_1 is the observed first throw and λ is the logarithmic decrement.

Calculation of λ. The successive throws $\theta_1, \theta_2, \theta_3$ etc. are noted.

Then, $\frac{\theta_1}{\theta_{11}} = d^{10} = e^{10\lambda}$

Taking logarithms on both sides

$$\log_e \left(\frac{\theta_1}{\theta_{11}}\right) = 10\lambda \text{ or } \lambda = \frac{1}{10} . \log_e \left(\frac{\theta_1}{\theta_{11}}\right)$$

$$\lambda = \frac{1}{10} \times 2{\cdot}3026 \times \log_{10}\left(\frac{\theta_1}{\theta_{11}}\right) \qquad ...(ii)$$

From equation (i),

$$q = \frac{t}{2\pi} \cdot \frac{C}{nAB} . \theta_1 \left[1 + \frac{2{\cdot}3026}{20} \times \log_{10} \frac{\theta_1}{\theta_{11}}\right]$$

$$= \frac{t}{2\pi} \cdot \frac{C}{nAB} . \theta_1 \left[1 + 0{\cdot}11513 \log_{10} \frac{\theta_1}{\theta_{11}}\right] \qquad ...(iii)$$

13·21 Kirchhoff's Laws

For steady currents flowing through a network of conductors, the following two laws known as *Kirchhoff's Laws* are applicable.

(*i*) **First Law.** *The algebraic sum of currents meeting at any junction in a circuit is zero.*

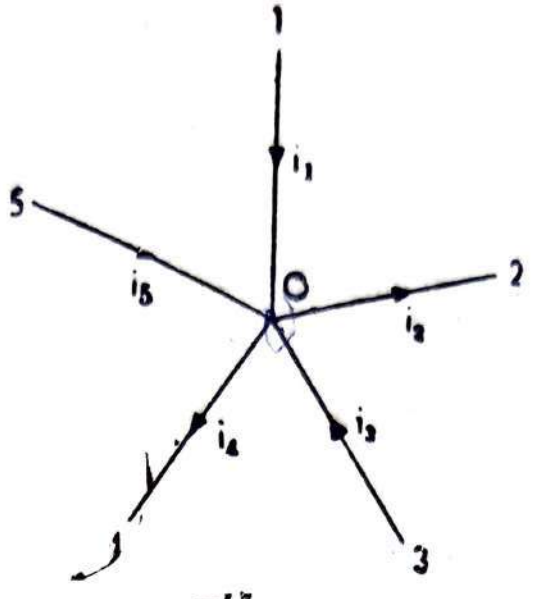


Fig. 13·24

Let 1, 2, etc. be the conductors meeting at the point O of an electrical circuit and i_1, i_2 etc. be the currents passing though them (Fig. 13·24). Taking the currents flowing towards the point as +*ve* and those flowing away from the point as − *ve*, the algebraic sum of the currents is $i_1 - i_2 + i_3 - i_4 + i_5$ which is equal to zero according to the first law. In general $\Sigma i = 0$. Taking the current as the rate of flow of charge, the total inflow of charge towards a point must be the same as the total outflow of charge in the same time as there is no accumulation of charge at any point in a circuit.

(*ii*) **Second Law.** In any closed mesh (or path) of an electrical circuit, the algebraic sum of the products of the currents and resistances of the various branches of the mesh is equal to the total emf of the mesh (or path).

Consider the electrical circuit given in Fig. 13·25. The values of the currents and resistances are indicated in the diagram.

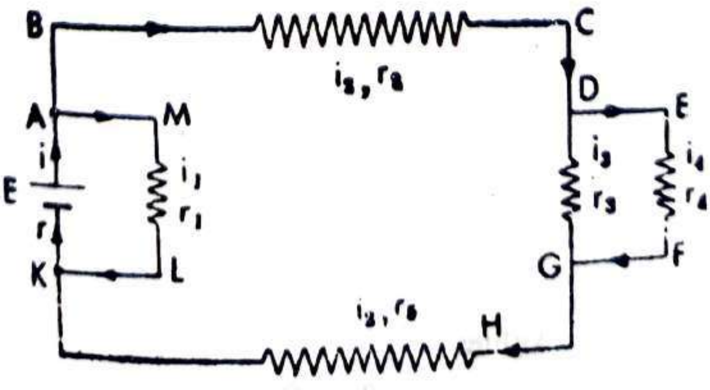


Fig. 13·25

Applying the second law

(*i*) to the mesh *ABCDGHKA*,

$$ir + i_2r_2 + i_3r_3 + i_3r_5 = E, \qquad \text{or } \Sigma\, i\, r = \Sigma E$$

(**Note.** For any mesh, the product is taken as positive for the current in one direction and negative in the opposite direction. Further if the direction of the current due to the cell is the same as the assumed + *ve* direction, the emf of the cell is taken as +ve, otherwise −*ve*.)

(*ii*) For the mesh *AMLKA*, $\quad ir + i_1r_1 = E$

(*iii*) For the mesh *DEFGD*, $\quad i_4r_4 - i_3r_3 = 0$

(as there is no source of emf in the mesh)

(The direction of current through r_4 is taken as positive and the current through r_3 which is in the opposite direction is taken as negative).

13·22 Application of Kirchhoff's Laws to Wheatstone's Bridge

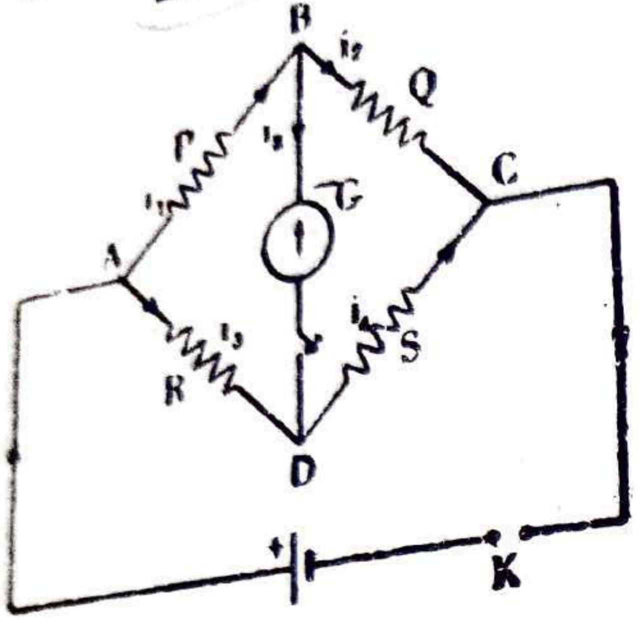


Fig. 13·26 represents the Wheatstone's bridge circuit where P, Q, R and S are connected to form a mesh. A cell is connected between the points A and C and a galvanometer is connected between the points B and D. The currents through the various branches are indicated in the figure. The current through the galvanometer is i_g and the resistance of the galvanometer is G.

Fig. 13·26

Applying the first law:—

At the junction B, $$i_1 - i_2 - i_g = 0 \quad ...(i)$$

At the junction D, $$i_3 + i_g - i_4 = 0 \quad ...(ii)$$

Applying the second law to the meshes $ABDA$ and $ABCDA$

$$i_1P + i_gG - i_3R = 0 \quad ...(iii)$$

$$i_1P + i_2Q - i_4S - i_3R = 0 \quad ...(iv)$$

When the galvanometer shows zero deflection, the points B and D are at the same potential and $i_g = 0$.

Substituting this value in (i), (ii), and (iii),

$$i_1 = i_2 \quad ...(v)$$

$$i_3 = i_4 \quad ...(vi)$$

and $$i_1P = i_3R \quad ...(vii)$$

Substituting the values of (v) and (vi) in equation (iv)

$$i_1P + i_1Q - i_3S - i_3R = 0 \quad ...(viii)$$

$$i_1(P+Q) = i_3(R+S)$$

Dividing (*viii*) by (*vii*),

$$\frac{i_1(P+Q)}{i_1P}=\frac{i_3(R+S)}{i_3R}$$

$$\frac{P+Q}{P}=\frac{R+S}{R}\ ,\ \frac{Q}{P}=\frac{S}{R}$$

$\therefore$

$$\frac{P}{Q}=\frac{R}{S} \quad \text{...(ix)}$$

or

Thus, if the values of the resistances P, Q, R and S are ...r zero deflection of the galvanometer, then $\frac{P}{Q}=\frac{R}{S}$. This is the ...ndition for the Wheatstone's bridge circuit when the galvanometer ...eflection is zero.

13·32 Carey Foster Bridge

A *Carey Foster Bridge* is principally the same as a metre bridge except that two more gaps are provided as shown in Fig. 13·48. This bridge is used to measure the difference between two nearly equal resistances and knowing the value of one, the other can be calculated. In this bridge, the end resistances are eliminated in calculations, which is an advantage and hence it can conveniently be used to measure a given *low* resistance.

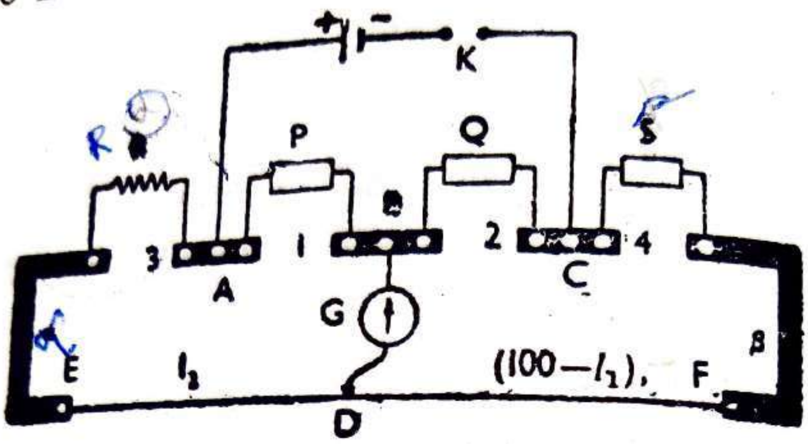
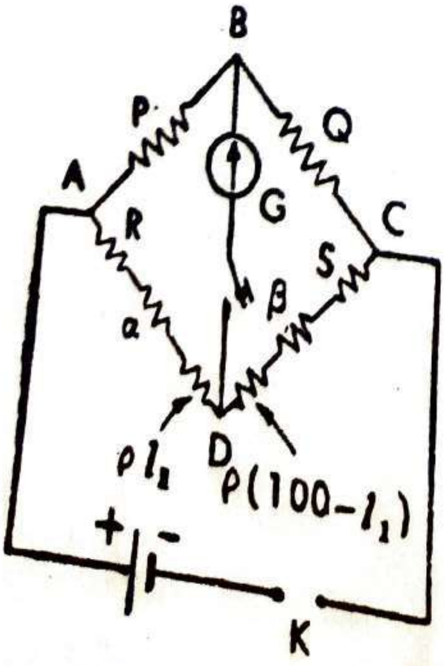


Fig. 13·48

P and *Q* are two resistance boxes connected in the inner gaps 1 and 2, *R* is the unknown low resistance and *S* is a fractional resistance box. Let the length of the bridge wire be 100 cm and α and β the end resistances on the sides of *R* and *S* respectively. The galvanometer *G* is connected between the points *B* and *D*. The cell is connected through a key between the points *A* and *C*.

Keeping suitable values of *P* and *Q*, the resistance *R* is placed in the left gap and *S* in the right gap and the balance length l_1 is measured from the point *E*. *R* and *S* are interchanged and the balancing length l_2 is noted. Figs. 13·49 and 13·50 represent the equivalent Wheatstone's bridge circuit in the two cases. Let ρ be the resistance per unit length of the bridge wire.



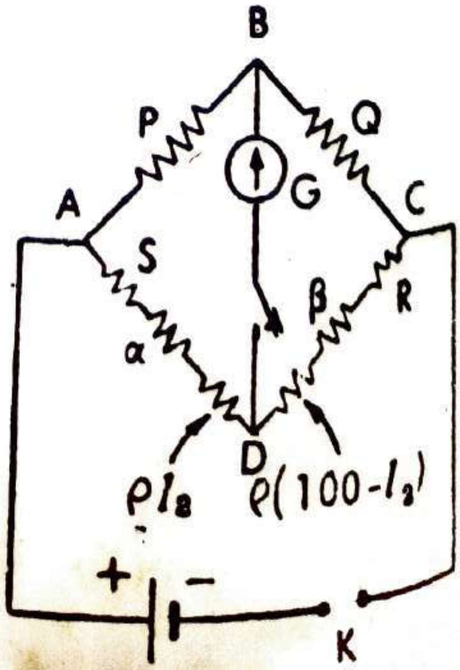


Fig 13·49

For zero deflection of the galvanometer, in the first case

$$\frac{P}{Q}=\frac{R+\alpha+l_1\rho}{S+\beta+(100-l_1)\rho} \quad ...(i)$$

In the second case

$$\frac{P}{Q}=\frac{S+\alpha+l_2\rho}{R+\beta+(100-l_2)\rho} \quad ...(ii)$$

Equating the right hand sides of (*i*) and (*ii*)

$$\frac{R+\alpha+l_1\rho}{S+\beta+(100-l_1)\rho}=\frac{S+\alpha+l_2\rho}{R+\beta+(100-l_2)\rho}$$

Adding *one* to both sides

$$\frac{R+\alpha+l_1\rho+S+\beta+100\rho-l_1\rho}{S+\beta+(100-l_1)\rho}=\frac{S+\alpha+l_2\rho+R+\beta+100\rho-l_2\rho}{R+\beta+(100-l_2)\rho}$$

$$\therefore \quad \frac{R+S+\alpha+\beta+100\rho}{S+\beta+(100-l_1)\rho}=\frac{R+S+\alpha+\beta+100\rho}{R+\beta+(100-l_2)\rho} \quad ...(iii)$$

The numerators of equation (*iii*) are equal. Therefore, the denominators are equal.

$$\therefore \quad S+\beta+100\rho-l_1\rho = R+\beta+100\rho-l_2\rho$$

or
$$S-l_1\rho = R-l_2\rho$$

or
$$R-S = \rho(l_2-l_1) \quad ...(iv)$$

or
$$R = S+\rho\,(l_2-l_1) \quad ...(v)$$

Thus, knowing the values of l_1 and l_2, the difference $R-S$ can be calculated, provided ρ the resistance per unit length of the bridge wire is known (equation *iv*). Further, if the value of S is known, R can be calculated (equation *v*).

(*ii*) **Determination of** ρ. To determine the resistance per unit length of the bridge wire, the resistance R is replaced by a thick copper strip (*i.e.*, $R = 0$) and the balancing length l_1' is determined. Now keeping S in the left gap and the copper strip in the right gap the balancing length l_2' is determined with the same values of P and Q.

From equation (*v*)

$$0 = S+\rho(l_2'-l_1')$$

or

$$\rho = \frac{S}{(l_1'-l_2')} \quad ...(vi)$$

The experiment is repeated with different values of S and the mean value of ρ is taken.

Calibration of the bridge wire. In equation (*v*), $\rho(l_2-l_1)$ measures the resistance of the bridge wire between the two balance points.

Thus, $R-S$ = Resistance of the bridge wire between the two balance points.

Initially, using known values of R and S, the resistance for various portions of the bridge wire is determined. The balance points can be shifted to various positions of the wire by suitably altering the values of P and Q.

A graph is drawn between the length of the wire along the X-axis and the resistance of the wire along the Y-axis. This calibration is necessary when the bridge wire is not uniform.

13·35 Potentiometer

Principle. A potentiometer is a device used to measure potential difference. It consists of a uniform wire *AB* of length usually 10 metres stretched on a wooden board by the side of a metre scale. The wires of length one metre each are joined in series and connected between the points *A* and *B*. The wires used have a low temperature coefficient of resistance. A steady current is passed through the wire *AB* with the help of a constant source of *E.M.F.* (Fig. 13·54).

Let the resistance per unit length of the potentiometer wire be ρ and the steady current passing through the wire be *I* amperes.

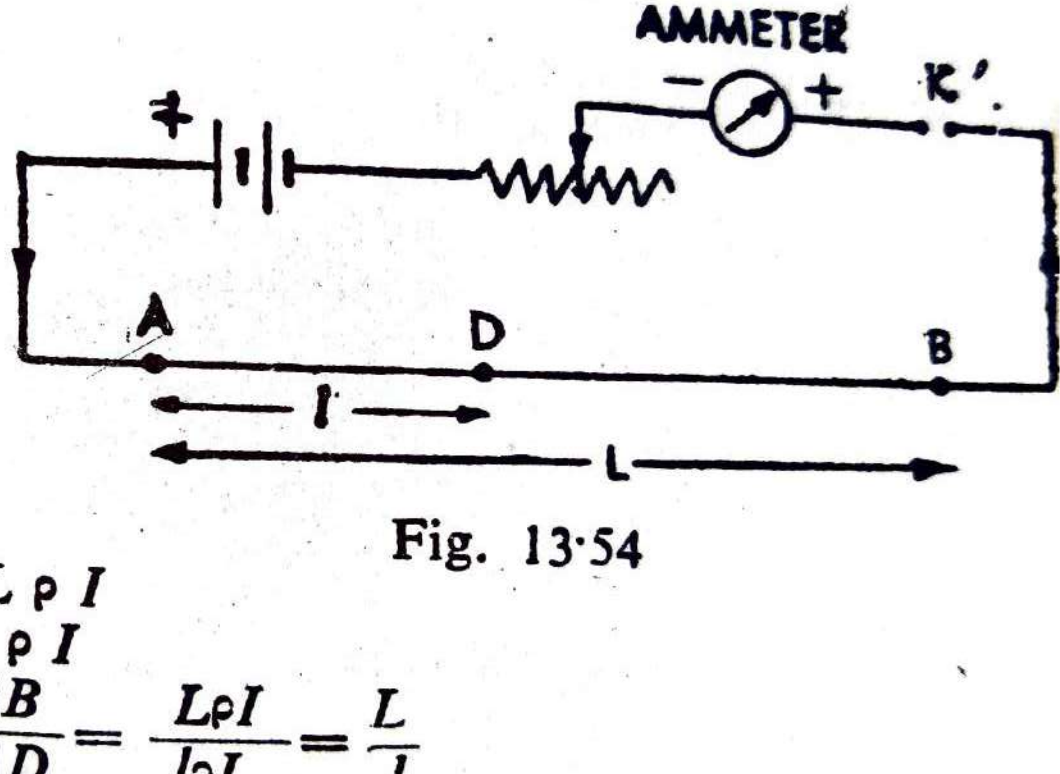


Fig. 13·54

$AB = L$ cm

$AD = l$ cm

PD across $AB = L \rho I$

PD across $AD = l \rho I$

$$\therefore \quad \frac{\text{PD across } AB}{\text{PD across } AD} = \frac{L\rho I}{l\rho I} = \frac{L}{l}$$

$$\therefore \quad \text{PD across } AD = \frac{l}{L} \times \text{PD across } AB$$

Thus, for a steady current passing through the potentiometer wire AB, the potential difference across any length is proportional to the length of the wire.

41 Calibration of Ammeter and Voltmeter

(1) **Ammeter.** The principle is the same as the method of measuring current discussed in article **13·38** The current through the resistance r is gradually increased with the help of the rheostat Rh (Fig. 13·61). The ammeter reading is noted and the corresponding value $I = \frac{le}{r}$ is calculated. A calibration graph is drawn between the observed values and the calculated values.

(2) V...

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1. **Brij lal N.Subrahmanyam, 1999, Electricity & Magnetism - Ratan Prakashan Mandir, Delhi.**