Kunthavai Naacchiyaar Govt. Arts College for Women (Autonomous),

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DEPARTMENT OF PHYSICS

III B.Sc Physics

Numerical Methods

Code: **18K5PELP1**

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UNIT - I

SOLUTION OF LINEAR ALGEBRAIC EQUATION

Method of triangularisation - Gauss elimination method - Gauss Jordan method - Inverse of matrix - Iterative methods - Gauss Jacobis method - Gauss - seidal method.

Method of Triangularisation:

Consider the system of eqns,

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

These eqns can be written in matrix form as

$$AX = B \longrightarrow (1)$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$A = LU \longrightarrow (2)$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$LUX = B \longrightarrow (3)$$

$$UX = Y \longrightarrow (4)$$

$$LY = B \longrightarrow (5)$$

$y_1 = b_1, \quad l_{21}y_1 + y_2 = b_2, \quad l_{31}y_1 + l_{32}y_2 + y_3 = b_3$

These y_1, y_2, y_3 can be solved by forward substitution

Now eqn (4) becomes

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

x_1, x_2, x_3 can be solved by back substitution

$\because LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \rightarrow (6)$$

Equating the corresponding elements of the first row in both sides of (6),

$$u_{11} = a_{11}, \quad u_{12} = a_{12}, \quad u_{13} = a_{13}$$

Equating the remaining corresponding elements of the first column in both sides of (6),

$$l_{21} = \frac{a_{21}}{u_{11}} = \frac{a_{21}}{a_{11}}, \quad l_{31} = \frac{a_{31}}{a_{11}} = \frac{a_{31}}{a_{11}}$$

Second row in both sides of (6),

$$u_{22} = a_{22} - \frac{a_{21}}{a_{11}} \cdot a_{12}, \quad u_{23} = a_{23} - \frac{a_{21}}{a_{11}} \cdot a_{13}$$

Third row in both sides of (6),

$$l_{32} = \frac{a_{32} - \frac{a_{31}}{a_{11}} \cdot a_{12}}{u_{22}} \text{ and } u_{33} = a_{33} - \frac{a_{31}}{a_{11}} \cdot a_{13} - l_{32}u_{23}.$$

1. Solve the following eqns by the method of triangularisation.

$$2x+y+4z=12$$
$$8x-3y+2z=0$$
$$4x+11y-z=33$$

$$A=\begin{bmatrix}2&1&4\\8&-3&2\\4&11&-1\end{bmatrix} \quad L=\begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix} \quad U=\begin{bmatrix}u_{11}&u_{12}&u_{13}\\0&u_{22}&u_{23}\\0&0&u_{33}\end{bmatrix}$$

$$LU=A$$

$$\begin{bmatrix}u_{11}&u_{12}&u_{13}\\l_{21}u_{11}&l_{21}u_{12}+u_{22}&l_{21}u_{13}+u_{23}\\l_{31}u_{11}&l_{31}u_{12}+l_{32}u_{22}&l_{31}u_{13}+l_{32}u_{23}+u_{33}\end{bmatrix}=\begin{bmatrix}2&1&4\\8&-3&2\\4&11&-1\end{bmatrix}$$

Ist Row elements,

$u_{11}=2,\ u_{12}=1,\ u_{13}=4$

Ist column elements

$l_{21}=8/2=4,\ l_{31}=4=2$

2nd Row elements

$u_{22}=-7,\ u_{23}=-14$

3rd row elements

$l_{32}=-9/7,\ u_{33}=-27$

$$A=\begin{bmatrix}1&0&0\\4&1&0\\2&-9/7&1\end{bmatrix}\begin{bmatrix}2&1&4\\0&-7&-14\\0&0&-27\end{bmatrix} \text{ (i.e) } \begin{bmatrix}1&0&0\\4&1&0\\2&-9/7&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}12\\20\\33\end{bmatrix} \longrightarrow (1)$$

$$\begin{bmatrix}2&1&4\\0&-7&-14\\0&0&-27\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \longrightarrow (2)$$

Eqn (1) can be solved by forward substitution.

$x_1 = 12, \; x_2 = -28, \; x_3 = -27$

eqn (2) becomes,

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix} \rightarrow (3)$$

Solving (3) by back substitution,

$$x = 3, \; y = 2, \; z = 1$$

Gauss elimination Method:-

· Solve the following system by Gaussian elimination method.

$$x_1 - x_2 + x_3 = 1$$
$$-3x_1 + 2x_2 - 3x_3 = -6$$
$$2x_1 - 5x_2 + 4x_3 = 5$$

Solution:

step 1: write the given system in augmented matrix form

$$\begin{pmatrix} 1 & -1 & 1 & 1 & 1 \\ -3 & 2 & -3 & 1 & -6 \\ 2 & -5 & 4 & 1 & 5 \end{pmatrix}$$

Step 2: From the first column with non-zero Component (called the pivot column), select the component with the largest absolute value. This Component is called the pivot.

$$\begin{array}{r} \\ \text{pivot} \rightarrow \\ \\ \end{array} \begin{pmatrix} 1 & -1 & 1 & 1 & 1 \\ -3 & 2 & -3 & 1 & -6 \\ 2 & -5 & 4 & 1 & 5 \end{pmatrix}$$

step : 3: Rearrange the rows to move the pivot element to the top of First Column. Interchange the first and 2nd row.

$$\text{pivot} \rightarrow \left(\begin{array}{cccc} -3 & 2 & -3 & 1 & -6 \\ 1 & -1 & 1 & 1 & 1 \\ 2 & -5 & 4 & 1 & 5 \end{array}\right)$$

step 4: Make the pivot as 1, by dividing the Ist row by the Pivot.

$$\left(\begin{array}{ccc|c} 1 & -\frac{2}{3} & 1 & 2 \\ 1 & -1 & 1 & 1 \\ 2 & -5 & 4 & 5 \end{array}\right)$$

step 5:

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - 2R_1$

$$\left(\begin{array}{ccc|c} 1 & -\frac{2}{3} & 1 & 2 \\ 0 & -\frac{1}{3} & 0 & -1 \\ 0 & -\frac{11}{3} & 2 & 1 \end{array}\right)$$

step 6: delete the Ist row and Ist column and perform step 2-5 on the resulting Matrix.

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & -1/3 & 0 & -1 \\ 0 & -11/3 & 2 & 1 \end{pmatrix} \leftarrow \text{New pivot}$$

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & -11/3 & 2 & 1 \\ 0 & -1/3 & 0 & -1 \end{pmatrix}$$

R_3 and R_1 are interchanged to Move the pivot to the top of new submatrix.

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & -2/11 & -12/11 \end{pmatrix} \quad R_3 \to R_3 + 1/3 R_2$$

Step 7 : Delete Ist two Rows and Ist two columns. perform step 6 on the resulting matrix.

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & -2/11 & -12/11 \end{pmatrix} \leftarrow \text{New pivot.}$$

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & 1 & 6 \end{pmatrix} \quad R_3 \text{ is divided by the new pivot.}$$

Step 8: Use back substitution to find the soln to the system.

$x_3 = 6$

$x_2 = 3$

$x_1 = -2$

Checking

$x_1 - x_2 + x_3 = 1$

$-2 - 3 + 6 = 1$

Gauss-Jordan Method:-

This method is a modified form of Gaussian elimination method. The completion of the Gauss-Jordan method the eqns become

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

1. Using the Gauss-Jordan Method solve the following eqns.

$$10x+y+z=12$$
$$2x+10y+z=13$$
$$x+y+5z=7$$

Soln:

step 1: write the given system of eqns in augmented Matrix form.

$$\begin{pmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{pmatrix}$$

step 2: Make the element in the I^{st} Row and I^{st} column as 1

$$\begin{pmatrix} 1 & 1/10 & 1/10 & 12/10 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{pmatrix} \quad R_1 \to R_2 \div 10$$

step 3: Add multiples of the I^{st} row to the other rows to make all the other Comports in the I^{st} column equal to zero.

$$\begin{pmatrix} 1 & 1/10 & 1/10 & 12/10 \\ 0 & 49/5 & 4/5 & 106/10 \\ 0 & 9/10 & 49/10 & 58/10 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

step 4: Make the element in the 2^{nd} row and 2^{nd} column as 1

$$\begin{pmatrix} 1 & 1/10 & 1/10 & 12/10 \\ 0 & 1 & 4/49 & 53/49 \\ 0 & 9/10 & 49/10 & 58/10 \end{pmatrix} \quad R_2 \to R_2 \div 49/5$$

step 5: Add multiples of the 2nd row to the other rows to make all the other Components in the 2^{nd} column equal to zero.

$$\begin{pmatrix} 1 & 0 & 0.0918 & 1.0918 \\ 0 & 1 & 4/49 & \frac{52}{49} \\ 0 & 0 & 4.8365 & 4.8365 \end{pmatrix} \quad \begin{array}{l} R_3 \to R_3 - \frac{9}{10} R_2 \\ R_1 \to R_1 - \frac{1}{10} R_2, \end{array}$$

Step 6: Make the elements in the third row and third column as 1.

$$\begin{pmatrix} 1 & 0 & 0.0918 & 1.0918 \\ 0 & 1 & 4/49 & 52/49 \\ 0 & 0 & 1 & 1 \end{pmatrix} \quad R_3 \to R_3 \div 4.8363$$

Step 7: Add multiples of the 3rd row to the other rows to make the components in third column equal to zero

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \quad \begin{array}{l} R_1 \to R_1 - 0.0918 R_3 \\ R_2 \to R_2 - \frac{4}{49} R_3 \end{array}$$

The Matrix finally reduces to the form given by

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore x = 1, y = 1, z = 1$

Checking:

$$10x + y + z = 12$$
$$10(1) + 1 + 1 = 12$$

Inverse matrix- Gauss elimination and Gauss Jordan

Apart from the Gaussian elimination, there is an alternative method to calculate the inverse matrix. It is much less intuitive, and may be much longer than the previous one, but we can always use it because it is more direct.
Let's remember that given a matrix A, its inverse A^{-1} is the one that satisfies the following:
$A \cdot A^{-1} = I$
where I is the identity matrix, with all its elements being zero except those in the main diagonal, which are ones.
The inverse matrix can be calculated as follows:
$A^{-1} = \frac{1}{|A|}(A^{adj})^t$
Where:
$A^{-1} \rightarrow$ Inverse matrix
Given a square matrix A, which is non-singular (means the Determinant of A is nonzero); Then there exists a matrix
A^{-1}
which is called inverse of matrix A.
The inverse of a matrix is only possible when such properties hold:

1. The matrix must be a square matrix.
2. The matrix must be a non-singular matrix and,
3. There exist an Identity matrix I for which

$A\,A^{-1} = A^{-1}A = I$
In general, the inverse of n X n matrix A can be found using this simple formula:

$$A^{-1} = \frac{Adj\ (A)}{Det\ (A)}$$

$|A| \rightarrow$ Determinant
$A^{adj} \rightarrow$ Adjoint matrix
$A^t \rightarrow$ Transpose matrix

Methods for finding Inverse of Matrix:
Finding the inverse of a 2×2 matrix is a simple task, but for finding the inverse of larger matrix (like 3×3, 4×4, etc) is a tough task, So the following methods can be used:

1. Elementary Row Operation (Gauss-Jordan Method) **(Efficient)**
2. Minors, Cofactors and Ad-jugate Method (Inefficient)

Gauss-Jordan Method is a variant of Gaussian elimination in which row reduction operation is performed to find the inverse of a matrix.

Steps to find the inverse of a matrix using Gauss-Jordan method:
In order to find the inverse of the matrix following steps need to be followed:
1. Form the augmented matrix by the identity matrix.
2. Perform the row reduction operation on this augmented matrix to generate a row reduced echelon form of the matrix.

3. The following row operations are performed on augmented matrix when required:
   - Interchange any two row.
   - Multiply each element of row by a non-zero integer.
   - Replace a row by the sum of itself and a constant multiple of another row of the matrix.

The Jacobi and Gauss-Seidel Iterative Methods

The Jacobi Method Two assumptions made on Jacobi Method:

1. The system given by

$$a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \ldots a_{nn}x_n = b_n$$

Has a unique solution.

2. The coefficient matrix has no zeros on its main diagonal, namely, a_{11}, a_{22}, ………a_{nn} are nonzeros.

Main idea of Jacobi To begin, solve the 1st equation for , the 2nd equation for $x_2$and so on to obtain the rewritten equations:

$$x_1 = 1/\, a_{11}\, (b_1 - a_{12}x_2 - a_{13}x_3 - \ldots\ldots a_{1n}x_n)$$
$$x_2 = 1/\, a_{22}\, (b_2 - a_{21}x_1 - a_{23}x_3 - \ldots\ldots a_{2n}x_n)$$
$$\vdots$$
$$X_n = 1/\, a_{nn}\, (b_n - a_{n1}x_1 - a_{n3}x_2 - \ldots\ldots a_{nn}x_{n-1})$$

Then make an initial guess of the solution $x^{(0)} = (x_1^{(0)}, x_2^{(0)}, x_3^{(0)}, \ldots\ldots x_n^{(0)})$. Substitute these values into the right hand side the of the rewritten equations to obtain the first approximation, $(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, \ldots\ldots x_n^{(1)})$. This accomplishes one iteration.

In the same way, the second approximation $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}, \ldots\ldots x_n^{(2)})$ is computed by substituting the first approximation's x- vales into the right hand side of the rewritten equations.

By repeated iterations, we form a sequence of approximations $x^{(k)} = (x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \ldots\ldots x_n^{(k)})^t$, K= 1,2,3……

The Jacobi Method. For each $k \geq 1$, generate the components $x_i^{(k)}$of $x^{(k)}$ from $x^{(k-1)}$by

$$x_i^k = \frac{1}{aii} \sum_{j=1}^{n} (-a_{ij} x_j^{(k-1)} + b_i \quad , \qquad \text{for } i = 1,2,\ldots..n$$

Main idea of Gauss-Seidel, With the Jacobi method, the values of $x_i^{(k)}$ obtained in the k^{th} iteration remain unchanged until the entire $(k+1)^{th}$ iteration has been calculated. With the Gauss-Seidel method, we use the new values $x_i^{(k+1)}$as soon as they are known. For example, once we have computed $x_1^{(k+1)}$from the first equation, its value is then used in the second equation to obtain the new $x_2^{(k+1)}$ and so on.

UNIT-1 - SOLUTIONS OF LINEAR ALGEBRAIC EQUATION

Gauss Elimination Method

$$x_1 - x_2 + x_3 = 1$$
$$-3x_1 + 2x_2 - 3x_3 = -6$$
$$2x_1 - 5x_2 + 4x_3 = 5$$

solution :

step 1 : Write the given system in augmented matrix form.

$$\begin{pmatrix} -1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{pmatrix}$$

Step 2 : From the first column with non-zero components, select the component with the largest absolute value. component is called the pivot.

$$\text{pivot} \longrightarrow \begin{pmatrix} 1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{pmatrix}$$

Step 3 : Rearrange the rows to move the pivot elements to the top of column. Here we interchange the first the second row.

$$\text{pivot} \longrightarrow \begin{pmatrix} -3 & 2 & -3 & -6 \\ 1 & -1 & 1 & 1 \\ 2 & -5 & 4 & 5 \end{pmatrix}$$

<u>Step 4</u> : Make the pivot as 1, by dividing the first row by the pivot.

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 1 & -1 & 1 & 1 \\ 2 & -5 & 4 & 5 \end{pmatrix}$$

<u>Step 5</u> : Add multiples of the first row to the other rows to make all the other components in the pivot coloum equal to zero.

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & -1/3 & 0 & -1 \\ 0 & -11/3 & 2 & 1 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

<u>Step 6</u> : Delete the first row and column and perform steps 2-5 on the resulting matrix.

$$\begin{pmatrix} 0 & -2/3 & 1 & 2 \\ 0 & -1/3 & 0 & -1 \\ 0 & -11/3 & 2 & 1 \end{pmatrix} \quad \leftarrow \text{New pivot.}$$

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & -11/3 & 2 & 1 \\ 0 & -1/3 & 0 & -1 \end{pmatrix}$$ R_3 and R_2 are inter-changed.

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & -1/3 & 0 & -1 \end{pmatrix}$$ R_2 is divided by $-11/3$ to make the pivot as 1

$$\begin{pmatrix} 0 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & -2/11 & -12/11 \end{pmatrix} \quad R_3 \to R_3 + \tfrac{1}{3} R_2$$

Step 7 : Deleting first rows and first two columns. perform step 6 on the resulting matrix.

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & -2/11 & -12/11 \end{pmatrix} \longleftarrow \text{New pivot}$$

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & 1 & 6 \end{pmatrix}$$ R_3 is divided by the new pivot.

Step : 8 use back substitution to find the solution to the system.

Here, $x_3 = 6$

$$x_2 - \frac{6}{11} x_3 = \frac{-3}{11}$$

(i.e) $x_2 = \dfrac{-3}{11} + \dfrac{6}{11}(6) = 3$

$$x_1 - 2/3\, x_2 + x_3 = 2$$

(i.e) $x_1 = 2 + 2/3\, x_2 - x_3 = 2 + \dfrac{2}{3}(3) - 6 = -2$

Hence the solution is . $x_1 = -2$, $x_2 = 3$, $x_3 = 6$

checking : $x_1 - x_2 + x_3 = 1$

$$-2 - 3 + 6 = 1$$

$$\boxed{1 = 1}$$

Gauss-Jordan Method

$$2x_1 + 2x_2 - x_3 + x_4 = 4$$
$$4x_1 + 3x_2 - x_3 + 2x_4 = 6$$
$$8x_1 + 5x_2 - 3x_3 + 4x_4 = 12$$
$$3x_1 + 3x_2 - 2x_3 + 2x_4 = 6$$

Solution :

Step 1 : Write the given system of equation in augmented matrix form.

$$\begin{pmatrix} 2 & 2 & -1 & 1 & 4 \\ 4 & 3 & -1 & 2 & 6 \\ 8 & 5 & -3 & 4 & 12 \\ 3 & 3 & -2 & 2 & 6 \end{pmatrix}$$

Step 2 : Make the element in the first row and first column as 1.

$$\begin{pmatrix} 1 & 1 & -0.5 & 0.5 & 2 \\ 4 & 3 & -1 & 2 & 6 \\ 8 & 5 & -3 & 4 & 12 \\ 3 & 3 & -2 & 2 & 6 \end{pmatrix} \quad R_1 \to R_1 \div 2$$

Step : 3 Add multiple of the first row to the other rows to make all the other components in the first column equal to zero.

$$\begin{pmatrix} 1 & 1 & -0.5 & 0.5 & 2 \\ 0 & -1 & 1 & 0 & -2 \\ 0 & -3 & 1 & 0 & -4 \\ 0 & 0 & -0.5 & 0.5 & 0 \end{pmatrix} \quad \begin{array}{l} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 8R_1 \\ R_4 \to R_4 - 3R_1 \end{array}$$

Step 4 : Make the element in the second row to the other and second column

$$\begin{pmatrix} 1 & 1 & -0.5 & 0.5 & 2 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & -3 & 1 & 0 & -4 \\ 0 & 0 & -0.5 & 0.5 & 0 \end{pmatrix} \quad R_2 \to R_2(-1)$$

Step 5 :

$$\begin{pmatrix} 1 & 0 & -0.5 & 0.5 & 0 \\ 0 & 0 & -1 & 0 & 2 \\ 0 & 0 & -2 & 0 & 2 \\ 0 & 0 & -0.5 & 0.5 & 0 \end{pmatrix} \quad \begin{matrix} R_2 \to R_1 - R_2 \\ R_3 \to R_3 + 3R_2 \end{matrix}$$

Step 6 : Make the element in the third row and third column as 1.

$$\begin{pmatrix} 1 & 0 & 0.5 & 0.5 & 0 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & -0.5 & -0.5 & 0 \end{pmatrix} \quad R_3 \to R_3 + (-2)$$

Step 7 : Add multiples of the third row to the other rows to make all the other components in the third column equal to zero.

$$\begin{pmatrix} 1 & 0 & 0 & 0.5 & 0.5 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0.5 & -0.5 \end{pmatrix} \quad \begin{matrix} R_1 \to R_1 - 0.5R_3 \\ R_2 \to R_2 + R_3 \\ R_4 \to R_4 + 0.5R_3 \end{matrix}$$

Step 8 : Make the element in the fourth row and fourth column as 1.

$$\begin{pmatrix} 1 & 0 & 0 & 0.5 & 0.5 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \quad R_4 \to R_4 \div 0.5$$

Step 9 :

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \quad R_1 \to R_1 - 0.5R_4$$

Finally the matrix reduces to the form.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$$

therefore, $x_1 = 1$, $x_2 = 1$, $x_3 = -1$, $x_4 = -1$

checking : $8x_1 + 5x_2 - 3x_3 + 4x_4 = 12$

$$8(1) + 5(1) - 3(-1) + 4(-1) = 12$$

$$\boxed{12 = 12}$$

Jacobi's iteration method

$$30x - 2y + 3z = 75$$

$$x + 17y - 2z = 48$$

$$x + y + 9z = 15$$

Solution :

$$30x - 2y + 3z = 75$$

The Given equation can be written as

$$\left.\begin{aligned} x &= \tfrac{1}{30}(75 + 2y - 3z) \\ y &= \tfrac{1}{17}(48 - x + 2z) \\ z &= \tfrac{1}{9}(15 - x - y) \end{aligned}\right\} \longrightarrow (1)$$

First approximation

$$\left.\begin{aligned} x_1 &= \tfrac{1}{30}(75 + 2y_0 - 3z_0) \\ y_1 &= \tfrac{1}{17}(48 - x_0 + 2z_0) \\ z_1 &= \tfrac{1}{9}(15 - x_0 - y_0) \end{aligned}\right\} \longrightarrow (2)$$

put $x_0 = y_0 = z_0 = 0$ in equ (2)

$x_1 = 1/30\,(75) = 2.5$

$y_1 = 1/17\,(48) = 2.824$

$z_1 = 1/9\,(15) = 1.667$

second approximation

$$\left.\begin{aligned} x_2 &= 1/30\,(75+2y_1-3z_1) \\ y_2 &= 1/17\,(48-x_1+2z_1) \\ z_2 &= 1/9\,(15-x_1-y_1) \end{aligned}\right\} \longrightarrow (3)$$

put $x_1 = 2.5$, $y_1 = 2.824$, $z_1 = 1.667$

$x_2 = 1/30\,(75+2(2.824)-3(1.667)) = 2.522$

$y_2 = 1/17\,(48-2.5+2(1.667)) = 2.873$

$z_2 = 1/9\,(15-2.5-2.824) = 1.075$

third approximation,

$x_3 = 1/30\,(7.5+2y_2-3z_2)$

$y_3 = 1/17\,[48-x_2+2(z_2)]$

$z_3 = 1/9\,(15-x_2-y_2)$

put, $x_2 = 2.522$, $y_2 = 2.873$, $z_2 = 1.075$

$x_3 = 1/30\,(7.5+2(2.873)-3(1.075)) = 2.584$

$y_3 = 1/17\,(48-(2.522)+2(1.075)) = 2.802$

$z_3 = 1/9\,(15-2.522-2.873) = 1.0672$

fourth approximation,

$x_4 = 1/30\,(75+2y_3-3z_3)$

$y_4 = 1/17\,(48-x_3+2z_3)$

$z_4 = 1/9\,(15-x_3-y_3)$

put, $x_3 = 2.584$, $y_3 = 2.802$, $z_3 = 1.067$

$x_4 = 1/30\,(75 + 2(2.802) - 3(1.067) = 2.5801$

$y_4 = 1/17\,(48 - 2.584 + 2(1.067) = 2.797$

$z_4 = 1/9\,(15 - 2.584 - 2.802) = 1.0683$

fifth approximation

$$x_5 = 1/30\,(75 + 2y_4 - 3z_4)$$

$$y_5 = 1/17\,(48 - x_4 + 2z_4)$$

$$z_5 = 1/9\,(15 - x_4 - y_4)$$

put, $x_4 = 2.580$, $y_4 = 2.797$, $z_4 = 1.068$

$$x_5 = 1/30\left[75 + 2(2.797) - 3(1.068)\right] = 2.5796$$

$$y_5 = 1/17\,(48 - 2.580 + 2(1.068)] = 2.797$$

$$z_5 = 1/9\,(15 - 2.580 - 2.979) = 1.069$$

Gauss seidal iteration method

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

Solution :

$x + y$

The given equation is

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

Interchanging the equation,

$$27x + 6y + z = 85 \longrightarrow ①$$

$$6x + 15y + 2z = 72 \longrightarrow ②$$

$$x + y + 54z = 110 \longrightarrow ③$$

Clearly it the co-efficient of matrix is diagonally dominant. Hence, we apply Gauss seidal method from ①, ② and ③, we get,

$$x = \frac{1}{27}(85 - 6y + z) \longrightarrow ④$$

$$y = \frac{1}{15}(72 - 6x - 2z) \longrightarrow ⑤$$

$$z = \frac{1}{54}(110 - x - y) \longrightarrow ⑥$$

First iteration,

put, $y = z = 0$ in equ ④, we get

$$x = \frac{1}{27}(85 - 6(0) + 0) = 3.148$$

put, $x = 3.148$, $z = 0$ in equ ⑤, we get,

$$y = \frac{1}{15}(75 - 6(3.148) - 2(0)) = 3.5408 = 3.541$$

put, $x = 3.148$, $y = 3.541$ in equ ⑥, we get

$$z = \frac{1}{54}(110 - 3.148 - 3.541) = 1.91312$$

∴ from 1st iteration

$\boxed{x = 3.148}$, $\boxed{y = 3.5408}$, $\boxed{z = 1.91312}$

Second iteration:

Put, $y = 3.5408$, $z = 1.9132$ in equ ④

$$x = \frac{1}{27}(85 - 6(3.5408) + 1.9132) = 2.4322$$

put $x = 2.4322$, $z = 1.91312$ in equ ⑤

$$y = \frac{1}{15}(72 - 6(2.4322) - 2(1.91312)) = 3.5720$$

put, $x = 2.4322$, $y = 3.5720$ in equ ①

$$z = \frac{1}{54}(110 - 2.4322 - 3.5720) = 1.9258$$

∴ <u>from 2nd iteration</u>

$\boxed{x = 2.4322}$ $\boxed{y = 3.5720}$ $\boxed{z = 1.9258}$

<u>third iteration</u>,

put $y = 3.5720$, $z = 1.9258$ in equ ④

$$x = \frac{1}{27}(85 - 6(3.5720) + (1.9258)) = 2.4257$$

put, $x = 2.4257$, $z = 1.9258$ in equ ⑤

$$y = \frac{1}{15}(72 - 6(2.4257) - 2(1.9258)) = 3.5729$$

put, $x = 2.4257$, $y = 3.5729$ in equ ⑥

$$z = \frac{1}{54}(110 - 2.4257 - 3.5729) = 1.9259$$

∴ <u>from 3rd iteration</u>

$\boxed{x = 2.4257}$ $\boxed{y = 3.5729}$ $\boxed{z = 1.9259}$

<u>fourth iteration</u>,

put, $y = 3.5729$, $z = 1.9259$ in equ ④

$$x = \frac{1}{27}(85 - 6(3.5729) + 1.9259) = 2.4255$$

put $x = 2.4255$, $z = 1.9259$ in equ ⑤

$$y = \frac{1}{15}(72 - 6(2.4255) - 2(1.9259)) = 3.5730$$

put, $x = 2.4255$, $y = 3.5730$

$$z = \frac{1}{54}(110 - 2.4255 - 3.5730) = 1.9259$$

<u>fron fourth iteration is</u>,

$\boxed{x = 2.4255}$, $\boxed{y = 3.5730}$ $\boxed{z = 1.9254}$

Put, $y = 3.5730$, $z = 1.9259$ in equ (4)

$$x = \frac{1}{27}(85 - 6(3.5730) + 1.9259) = 2.42547$$
$$= 2.4255$$

put, $x = 2.4255$, $\cancel{y} = 1.9259$ in equ (5)

$$y = \frac{1}{15}(72 - 6(2.4255) - 2(1.9259)) = 3.5730$$

put, $x = 2.4255$, $y = 3.5730$

$$z = \frac{1}{54}(110 - 2.4255 - 3.5730) = 1.9259$$

$\therefore$ fifth iteration is,

$\boxed{x = 2.4255}$ $\boxed{y = 3.5730}$ $\boxed{z = 1.9250}$

The fourth and fifth interation these values of

$x = 2.425$

$y = 3.5730$

$z = 1.9259$ is accuretly corrected value.

UNIT - II

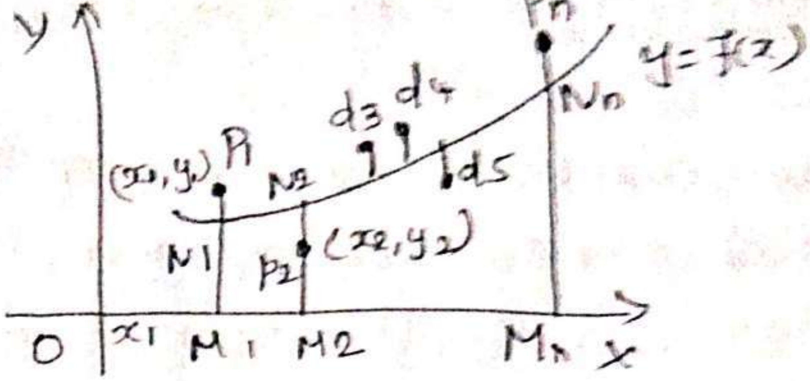
CURVE FITTING

Curve fitting - method of least squares - Fitting of straight line - Fitting of a parabola - Fitting of an exponential curve - Linear regression - regression Co-efficient for linear regression.

The general problem of finding equations of approximating curves which fit a given data is called curve fitting.

The principle of Least squares:-

we have described (i) the graphical method and (ii) the method of group averages to determine the constants that occur in the equation chosen to represent a given data. In the graphical method of fitting a straight line $y=a+bx$ to a given data, the constant b is the slop, which can be calculated with the help of any two points on the line. In the method of group averages, different groupings of the observations can be made.



Let (x_1, y_1) (x_2, y_2) $\cdots$ (x_n, y_n) be n sets of observations of related data and $y = f(x) \longrightarrow ①$

when $x = x_1$, the observed (experimental) value of y

$$= y_1 = P_1M_1$$

$$y = f(x_1) = N_1M_1$$

This known as the expected value of y or calculated value of y. The expression $d_1 = y_1 - f(x_1)$ which is the difference between the observed and calculated values of y is called a residual (or error). Residuals $d_2, d_3, \cdots d_n$ for all the remaining observations.

$$d_1 = y_1 - f(x_1) = P_1M_1 - N_1M_1 = P_1N_1$$

$$d_2 = y_2 - f(x_2) = P_2N_2$$

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$$d_n = y_n - f(x_n) = P_nN_n$$

$$E = d_1^2 + d_2^2 + \cdots + d_n^2$$

$$= [y_1 - f(x_1)]^2 + [y_2 - f(x_2)]^2 + \cdots + [y_n - f(x_n)]^2$$

"The best representative curve to the set of points is that for which E, the sum of the squares of the residuals is a minimum. This is known as the least square criterion or the principle of least squares.

We shall subsequently consider the fitting of the following types of curves.

(i) A straight line $y = ax + b$

(ii) A parabola $y = ax^2 + bx + c$

(iii) The exponential curve $y = ae^{bx}$

(iv) The curve $y = ax^b$.

<u>Fitting a straight line:-</u>

Let $(x_1, y_1), (x_2, y_2) \cdots (x_n, y_n)$ be n sets of observations of related data and $y = ax + b$ the equation to the line of best fit for them.

residual. $d_r = y_r - (ax_r + b)$

$$E = \Sigma \{y_r - (ax_r + b)\}^2$$

E - minimum, the conditions are,

$$\frac{\partial E}{\partial a} = 0, \quad \frac{\partial E}{\partial b} = 0$$

Partially differentiating E with respect to a.

$$\frac{\partial E}{\partial a} = -2[x_1(y_1 - ax_1 - b) + x_2(y_2 - ax_2 - b) + \cdots + x_n(y_n - ax_n - b)]$$

$$a\sum_{r=1}^{n} x_r^2 + b\sum_{r=1}^{n} x_r = \sum_{r=1}^{n} x_r y_r \longrightarrow (1)$$

partially differentiating E with respect to b

$$\frac{\partial E}{\partial b} = -2\left[(y_1 - ax_1 - b) + (y_2 - ax_2 - b) + \cdots + (y_n - ax_n - b)\right]$$

$$a\sum_{r=1}^{n} x_r + nb = \sum_{r=1}^{n} y_r \longrightarrow (2).$$

Eqn (1) & (2) are called the Normal eqns.
Dropping off the suffixes, they can be written as.

$$a\sum x^2 + b\sum x = \sum xy \longrightarrow \text{I}$$

$$a\sum x + nb = \sum y \longrightarrow \text{II}.$$

---

1. Use the method of least squres to fit a Straight line to the following data.

x	0	5	10	15	20
y	7	11	16	20	26

Estimate the value of y when $x = 25$.

Solution

Let the straight line fit be $y = ax + b \longrightarrow (1)$

The Normal eqns are,

$$a\sum x^2 + b\sum x = \sum xy$$

$$a\sum x + 5b = \sum y$$

x	y	xy	x^2
0	7	0	0
5	11	55	25
10	16	160	100
15	20	300	225
20	26	520	400
50	80	1035	750

the Normal eqns become,

$$750a + 50b = 1035$$
$$50a + 5b = 80$$

Solving these, we have $a = 0.94$, $b = 6.6$

putting these values in ①, the line of best fit is,

$$y = 0.94x + 6.6 \longrightarrow ②$$

putting $x = 25$ in ②

$$y = 0.94 \times 25 + 6.6 = 30.1$$

when $x = 25$, the expected value of $y = 30.1$

Fitting a parabola :-

Let $(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$ be n sets of observations of related data and $y = ax^2 + bx + c$ The eqn to the parabola of best fit for them.

$$d_r = y_r - (ax_r^2 + bx_r + c)$$
$$E = \sum [y_r - (ax_r^2 + bx_r + c)]^2$$

E - minimum, the conditions are,

$$\frac{\partial E}{\partial a} = 0, \quad \frac{\partial E}{\partial b} = 0, \quad \frac{\partial E}{\partial c} = 0$$

partially differentiating E with respect to a

$$\frac{\partial E}{\partial a} = -2\left[x_1^2(y_1 - ax_1^2 + bx_1 - c) + x_2^2(y_2 - ax_2^2 - bx_2 - c) + \cdots + x_n^2(y_n - ax_n^2 - bx_n - c)\right]$$

$$a\sum_{r=1}^{n} x_r^4 + b\sum_{r=1}^{n} x_r^3 + c\sum_{r=1}^{n} x_r^2 = \sum_{r=1}^{n} x_r^2 y_r \longrightarrow (1)$$

partially differentiating E with respect to b

$$\frac{\partial E}{\partial b} = -2\left[x_1(y_1 - ax_1^2 - bx_1 - c) + x_2(y_2 - ax_2^2 - bx_2 - c) + \cdots + x_n(y_n - ax_n^2 - bx_n - c)\right]$$

$$a\sum_{r=1}^{n} x_r^3 + b\sum_{r=1}^{n} x_r^2 + c\sum_{r=1}^{n} x_r = \sum_{r=1}^{n} x_r y_r \longrightarrow (2)$$

partially differentiating E with respect to c.

$$\frac{\partial E}{\partial c} = -2\left[(y_1 - ax_1^2 - bx_1 - c) + (y_2 - ax_2^2 - bx_2 - c) + \cdots + (y_n - ax_n^2 - bx_n - c)\right]$$

$$a\sum_{r=1}^{n} x_r^2 + b\sum_{r=1}^{n} x + nc = \sum_{r=1}^{n} y_r \longrightarrow (3)$$

dropping off the suffixes. The eqns (1), (2), (3) can be written as,

$$a\sum x^4 + b\sum x^3 + c\sum x^2 = \sum x^2 y \longrightarrow \text{I}$$

$$a\sum x^3 + b\sum x^2 + c\sum x = \sum xy \longrightarrow \text{II}$$

$$a\sum x^2 + b\sum x + nc = \sum y \longrightarrow \text{III}$$

By solving these normal eqns, we get the values of a, b, c and hence the eqn to the best fitting parabola.

1. A parabola of a form $y = ax^2+bx+c$ to their given data below

x	0	2	4	6	8	10
y	1	3	13	31	57	91

solution:

The eqn of fitting on a parabola,

$$y = ax^2+bx+c \longrightarrow ①$$

The Normal eqns are,

$$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = \Sigma x^2 y \longrightarrow ②$$

$$a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = \Sigma xy \longrightarrow ③$$

$$a\Sigma x^2 + b\Sigma x + nc = \Sigma y \longrightarrow ④$$

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
2	3	4	8	16	6	12
4	13	16	64	256	52	208
6	31	36	216	1296	186	1116
8	57	64	512	4096	456	3648
10	91	100	1000	10000	910	9100
$\Sigma x = 30$	$\Sigma y = 196$	$\Sigma x^2 = 220$	$\Sigma x^3 = 1800$	$\Sigma x^4 = 15664$	Σxy 1610	$\Sigma x^2y = 14084$

$$15664a + 1800b + 220c = 14084 \rightarrow (5)$$
$$1800a + 220b + 30c = 1610 \rightarrow (6)$$
$$220a + 30b + 6c = 196 \rightarrow (7)$$

Solving these, $a=1$, $b=-1$, $c=1$

Hence, the best fitting parabola is

$$y = ax^2 + bx + c$$
$$y = x^2(1) + (-1)x + 1$$
$$\boxed{y = x^2 - x + 1}$$

<u>Fitting an Exponential Curve:-</u>

Let $(x_1, y_1), (x_2, y_2) \dots (x_n, y_n)$ be n sets of observations of related data and $y = ae^{bx}$ the eqn to the curve fitting them well. Taking logarithms of both sides of the eqn,

$$\log_{10} y = \log_{10} a + bx \log_{10} e$$
$$\log_{10} y = Y, \quad \log_{10} a = A, \quad b \log_{10} e = B$$
$$Y = A + Bx$$

Hence we can fit a straight line to the data x and Y by the method of least squres.

From the values of A and B, we can find a and b.

1. Using the principle of least squares, fit an eqn of the form $y = ae^{bx}$ to the data,

x	1	2	3	4
y	1.65	2.70	4.50	7.35

solution:

$$y = ae^{bx}$$

Taking logarithms, $\log_{10} y = \log_{10} a + bx \log_{10} e$

$$\log_{10} y = Y, \quad \log_{10} a = A, \quad b\log_{10} e = B$$

$$Y = A + Bx \longrightarrow (1)$$

The Normal eqns are,

$$A\Sigma x + B\Sigma x^2 = \Sigma xY \longrightarrow (2)$$

$$4A + B\Sigma x = \Sigma Y \longrightarrow (3)$$

x	y	$Y = \log_{10} y$	x^2	xY
1	1.65	0.2175	1	0.2175
2	2.70	0.4314	4	0.8628
3	4.50	0.6532	9	1.9596
4	7.35	0.8663	16	3.4652
10	16.2	2.1684	30	6.5051

Eqn (2) & (3) become

$$10A + 30B = 6.5051 \longrightarrow (4)$$

$$4A + 10B = 2.1684 \longrightarrow (5)$$

Solving (4) & (5), $A=0$, $B=0.2168$

$\log_{10} a = 0$ (or) $a=1$

$b\log_{10} e = B = 0.2168$

$b = \frac{0.2168}{\log_{10} e} = 0.2168 \times \log_e 10 = 0.2168 \times 2.3026$

$= 0.4994$

$$\boxed{y = e^{0.4994x}}$$

Regression:-

Regression is the measure of the average relationship between two or more variable in terms of the original units of the data.

Regression eqn of x on y

$$x-\bar{x} = r\frac{\sigma x}{\sigma y}(y-\bar{y})$$

$r\frac{\sigma x}{\sigma y}$: The regression Co-efficient of x of y on

bxy (or) $\frac{\Sigma xy}{\Sigma y^2}$

$$bxy = r\frac{\sigma x}{\sigma y} = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} \times \frac{\sqrt{\Sigma x^2}}{\sqrt{\Sigma y^2}}$$

$$r\frac{\sigma x}{\sigma y} = \frac{\Sigma xy}{\Sigma y^2} \rightarrow (1)$$

$r\frac{\sigma y}{\sigma x}$: The regression co-efficient of y on x (or) byx

$$\frac{\Sigma xy}{\Sigma x^2} = byx = r\frac{\sigma y}{\sigma x} = \frac{\Sigma xy}{\sqrt{\Sigma x^2 . \Sigma y^2}} \times \frac{\sqrt{\Sigma y^2}}{\sqrt{\Sigma x^2}}$$

$$\therefore \frac{r\sigma_y}{\sigma_x} = \frac{\Sigma xy}{\Sigma x^2} \rightarrow (2).$$

Linear Regression:

Linear Regression is an approach for modelling the relationship between a scalar dependent variable and one (or) more explanatory variable (or) independent variable denoted x. In the case of one explanatory is called Simple Linear Regression.

1. Calculate the first two regression of X on Y and Y on X from the data given below, taking deviation from actual means of X and Y

price	10	12	13	12	16	15
Amount demanded	40	38	43	45	37	43

Estimate the likely demanded when the price is 20.

Solution

X	$X-\bar{X}=x$	x^2	Y	$Y-\bar{Y}=y$	y^2	xy
10	-3	9	40	-1	1	3
12	-1	1	38	-3	9	3
13	0	0	43	2	4	0
12	-1	1	45	4	16	-4
16	3	9	37	-4	16	-12
15	2	4	43	2	4	4
$\Sigma fx = 78$	$\Sigma x = 0$	$\Sigma x^2 = 24$	$\Sigma fy = 246$	$\Sigma y = 0$	$\Sigma y^2 = 50$	$\Sigma xy = -6$

Regression eqn of X on Y.

$$X-\bar{X} = r\frac{\sigma x}{\sigma y}(Y-\bar{Y}) \rightarrow (1)$$

$\bar{X} = 78/6 \quad \bar{X} = 13$

$\bar{Y} = 41$

$$r\frac{\sigma x}{\sigma y} = \frac{\Sigma xy}{\Sigma y^2}$$

$$\boxed{r\frac{\sigma x}{\sigma y} = 0.12}$$

$$\frac{r\sigma y}{\sigma x} = \frac{\Sigma xy}{\Sigma x^2}$$

$$\boxed{\frac{r\sigma y}{\sigma x} = -0.25}$$

Subst. the value in formula:

$$x - \bar{x} = r\frac{\sigma x}{\sigma y}(y - \bar{y})$$

$$x = -0.12y + 17.9$$

Regression eqn of y on x.

$$y - \bar{y} = r\frac{\sigma y}{\sigma x}(x - \bar{x})$$

$$y = -0.25x + 44.25$$

Applying $x = 20$ in y

$$\boxed{y = 39.25}$$

when the price is Rs. 20, the likely demanded is 39.25.

Unit- III: Solution of Algebraic, transcendental and differential equations

Bisection method- Method of successive approximation – Regulafalsi Method – Newton – Raphson method - Taylor series method- Euler's method Runge Kutta method (II and IV order)

Bisection Method

In mathematics, the bisection method is a root-finding method that applies to any continuous functions for which one knows two values with opposite signs. The method consists of repeatedly bisecting the interval defined by these values and then selecting the subinterval in which the function changes sign, and therefore must contain a root. It is a very simple and robust method, but it is also relatively slow. Because of this, it is often used to obtain a rough approximation to a solution which is then used as a starting point for more rapidly converging methods. The method is also called the interval halving method, the binary search method or the dichotomy method.

The method is applicable for numerically solving the equation $f(x) = 0$ for the real variable x, where f is a continuous function defined on an interval $[a, b]$ and where $f(a)$ and $f(b)$ have opposite signs. In this case a and b are said to bracket a root since, by the intermediate value theorem, the continuous function f must have at least one root in the interval (a, b).

At each step the method divides the interval in two by computing the midpoint $c = (a+b) / 2$ of the interval and the value of the function $f(c)$ at that point. Unless c is itself a root (which is very unlikely, but possible) there are now only two possibilities: either $f(a)$ and $f(c)$ have opposite signs and bracket a root, or $f(c)$ and $f(b)$ have opposite signs and bracket a root.[5] The method selects the subinterval that is guaranteed to be a bracket as the new interval to be used in the next step. In this way an interval that contains a zero of f is reduced in width by 50% at each step. The process is continued until the interval is sufficiently small.

Explicitly, if $f(a)$ and $f(c)$ have opposite signs, then the method sets c as the new value for b, and if $f(b)$ and $f(c)$ have opposite signs then the method sets c as the new a. (If $f(c)=0$ then c may be taken as the solution and the process stops.) In both cases, the new $f(a)$ and $f(b)$ have opposite signs, so the method is applicable to this smaller interval

The input for the method is a continuous function f, an interval $[a, b]$, and the function values $f(a)$ and $f(b)$. The function values are of opposite sign (there is at least one zero crossing within the interval). Each iteration performs these steps:

1. Calculate c, the midpoint of the interval, $c = a + b/2$.
2. Calculate the function value at the midpoint, $f(c)$.
3. If convergence is satisfactory (that is, $c - a$ is sufficiently small, or $|f(c)|$ is sufficiently small), return c and stop iterating.
4. Examine the sign of $f(c)$ and replace either $(a, f(a))$ or $(b, f(b))$ with $(c, f(c))$ so that there is a zero crossing within the new interval.

When implementing the method on a computer, there can be problems with finite precision, so there are often additional convergence tests or limits to the number of iterations. Although *f* is continuous, finite precision may preclude a function value ever being zero. For example, consider $f(x) = x - \pi$; there will never be a finite representation of x that gives zero. Additionally, the difference between a and b is limited by the floating point precision; i.e., as the difference between a and b decreases, at some point the midpoint of $[a, b]$ will be numerically identical to (within floating point precision of) either a or b.

Method of successive approximation (Iterative Method)

In computational mathematics, an iterative method is a mathematical procedure that uses an initial value to generate a sequence of improving approximate solutions for a class of problems, in which the n^{th} approximation is derived from the previous ones. A specific implementation of an iterative method, including the termination criteria, is an algorithm of the iterative method. An iterative method is called convergent if the corresponding sequence converges for given initial approximations. A mathematically rigorous convergence analysis of an iterative method is usually performed; however, heuristic-based iterative methods are also common.

In contrast, direct methods attempt to solve the problem by a finite sequence of operations. In the absence of rounding errors, direct methods would deliver an exact solution Iterative methods are often the only choice for nonlinear equations. However, iterative methods are often useful even for linear problems involving many variables (sometimes of the order of millions), where direct methods would be prohibitively expensive (and in some cases impossible) even with the best available computing power.

If an equation can be put into the form f(x) = x, and a solution x is an attractive fixed point of the function f, then one may begin with a point x1 in the basin of attraction of x, and let Xn+1 = f(Xn) for $n \geq 1$, and the sequence $\{Xn\}n \geq 1$ will converge to the solution x. Here Xn is the nth approximation or iteration of x and Xn+1 is the next or n + 1 iteration of x. Alternately, superscripts in parentheses are often used in numerical methods, so as not to interfere with subscripts with other meanings. (For example, X(n+1) = f(X(n)).) If the function f is continuously differentiable, a sufficient condition for convergence is that the spectral radius of the derivative is strictly bounded by one in a neighbourhood of the fixed point. If this condition holds at the fixed point, then a sufficiently small neighbourhood (basin of attraction) must exist.

Regula Falsi Method

In mathematics, the regula falsi, method of false position, or false position method is a very old method for solving an equation with one unknown, that, in modified form, is still in use. In simple terms, the method is the trial and error technique of using test ("false") values for the variable and then adjusting the test value according to the outcome. This is sometimes also referred to as "guess and check". Versions of the method predate the advent of algebra and the use of equations.

Modern versions of the technique employ systematic ways of choosing new test values and are concerned with the questions of whether or not an approximation to a solution can be obtained, and if it can, how fast can the approximation be found.

Two basic types of false position method can be distinguished historically, simple false position and double false position.

Simple false position is aimed at solving problems involving direct proportion. Such problems can be written algebraically in the form: determine x such that

$$ax = b$$

if a and b are known. The method begins by using a test input value x′, and finding the corresponding output value b′ by multiplication: ax′ = b′. The correct answer is then found by proportional adjustment, $x = \frac{b}{b'} x'$

Double false position is aimed at solving more difficult problems that can be written algebraically in the form: determine x such that

$$f(x) = ax + C = 0,$$

if it is known that

$$f\ (x1) = b1, \qquad f(x2) = b2.$$

Double false position is mathematically equivalent to linear interpolation. By using a pair of test inputs and the corresponding pair of outputs, the result of this algorithm given by,

$$x = \frac{b1x2 - b2x1}{b1 - b2}$$

Newton- Raphson Method

In numerical analysis, Newton's method, also known as the Newton–Raphson method, named after Isaac Newton and Joseph Raphson, is a root-finding algorithm which produces successively better approximations to the roots (or zeroes) of a real-valued function. The most basic version starts with a single-variable function f defined for a real variable x, the function's derivative f′, and an initial guess x0 for a root of, f. If the function satisfies sufficient assumptions and the initial guess is close, then

$$g(x) = x - \frac{f(x)}{f'(x)}$$

is a better approximation of the root than x0 Geometrically, (x1, 0) is the intersection of the x-axis and the tangent of the graph of , f at (x0, f(x0)): that is, the improved guess is the unique root of the linear approximation at the initial point. The process is repeated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

until a sufficiently precise value is reached. This algorithm is first in the class of Householder's methods, succeeded by Halley's method. The method can also be extended to complex functions and to systems of equations.

Taylor Series Method

To derive these method, we start with a Taylor Expansion:

$$y\,(t + \Delta t) \approx y(t)+\Delta t y'\,(t) + 1/2\ \Delta t^2\ y''\,(t) + ... + 1/r!\ y^{(r)}\,(t)\Delta t^r$$

Let's say we want to truncate this at the second derivative and base a method on that. The scheme is, then:

$$y_{n+1} = y_n + f_n\Delta t + f'_{tn}/2\ \Delta t^2.$$

The Taylor series method can be written as

$$y_{n+1} = y_n + \Delta t\ F\,(t_n, y_n, \Delta t)$$

where $F = f + 1/2\Delta t f'$. If we take the LTE for this scheme, we get (as expected)

$$LT\,E(t) = y\,(t_n + \Delta t) - y(t_n)\ \Delta t - f\,(t_n, y(t_n)) - 1/2\ \Delta t f'\,(t_n, y(t_n)) = O\,(\Delta t^2)$$

Of course, we designed this method to give us this order, so it shouldn't be a surprise!

So the LTE is reasonable, but what about the global error? Just as in the Euler Forward case, we can show that the global error is of the same order as the LTE. How do we do this? We have two facts,

$$y(t_{n+1}) = y(t_n)+\Delta t\ F\,(t_n, y(t_n), \Delta t),$$

and

$$y_{n+1} = y_n + \Delta t F(t_n, y_n, \Delta t)$$

where $F = f + 1/2\Delta t f'$. Now we subtract these twos

$$\begin{aligned}|y(t_{n+1}) - y_{n+1}| &= |y(t_n) - y_n + \Delta t(F(t_n, y(t_n)) - F(t_n, y_n)) + \Delta t LT\,E| \\ &\le |y(t_n) - y_n| + \Delta t\ |F(t_n, y(t_n)) - F(t_n, y_n)| + \Delta t\ |LT\,E|\ .\end{aligned}$$

Now, if F is Lipschitz continuous, we can say

$$e_{n+1} \le (1 + \Delta t L)e_n + \Delta t\ |LT\,E|.$$

Of course, this is the same proof as for Euler's method, except that now we are looking at F, not f, and the LT E is of higher order. We can do this no matter which Taylor series method we use, how many terms we go forward before we truncate.

Advantages and Disadvantages of the Taylor Series Method:

Advantages a) One step, explicit

b) can be high order

c) easy to show that global error is the same order as LTE

disadvantages Needs the explicit form of derivatives of f

Euler's Method

If we truncate the Taylor series at the first term

$$y\,(t + \Delta t) = y(t)+\Delta t y'\,(t) + 1/2\ \Delta t^2\ y''\,(\tau),$$

we can rearrange this and solve for y' (t)

$$y'(t) = y(t + \Delta t) - y(t) / \Delta t + O(\Delta t).$$

Now we can attempt to solve (1.1) by replacing the derivative with a difference

$$y((n + 1)\Delta t) \approx y(n\Delta t) + \Delta t\, f(n\Delta t, y(n\Delta t))$$

Start with y(0) and step forward to solve for any time.

What's good about this? If the O term is something nice looking, this quantity decays with Δt, so if we take Δt smaller and smaller, this gets closer and closer to the real value. Also, even though this may be a good approximation for y' (t) it may not converge to the right solution. To answer these questions, we look at this scheme in depth. Terminology: From now on, we'll call yn the numerical approximation to the solution y(nΔt); tn = nΔt. Euler's method can then be written

$$y_{n+1} = y_n + \Delta t\, f(t_n, y_n) \qquad n = 1, ..., N - 1$$

Runge-Kutta Method

To avoid the disadvantage of the Taylor series method, we can use Runge-Kutta methods. These are still one step methods, but they depend on estimates of the solution at different points. They are written out so that they don't look messy:

Second Order Runge-Kutta Methods:

$$k_1 = \Delta t\, f(t_i, y_i)$$

$$k_2 = \Delta t\, f(t_i + \alpha\Delta t, y_i + \beta k1)$$

$$y_{i+1} = y_i + ak_1 + bk_2$$

let's see how we can chose the parameters a, b, α, β so that this method has the highest order LT E possible. Take the Taylor expansions to express the LTE:

$$k_1(t) = \Delta t\, f(t, y(t))$$

$$k_2(t) = \Delta t f(t + \alpha\Delta t, y + \beta k_1(t))$$

$$= \Delta t\, (f(t, y(t)) + f_t(t, y(t))\alpha\Delta t + f_y(t, y(t))\beta k_1(t) + O(\Delta t^2))$$

$$LTE(t) = y(t + \Delta t) - y(t) / \Delta t - (a / \Delta t)\, f(t, y(t))\Delta t - (b / \Delta t)\, (ft(t, y(t))\alpha\Delta t + fy(t, y(t)\beta k_1(t)$$

$$+ f(t, y(t))\,\Delta t + O(\Delta t^2)$$

$$= y(t + \Delta t) - y(t) \;/\; (\Delta t\;) - af(t, y(t)) - bf(t, y(t)) - bf_t(t, y(t))\alpha$$

$$- bf_y(t, y(t)\; \beta f(t, y(t)) + O(\Delta t^2)$$

$$= y'(t) + 1/2\Delta t y''(t) - (a + b)f(t,y(t)) - \Delta t(b\alpha f_t(t,y(t)) + b\beta f(t, y(t))f_y(t,y(t)) + O(\Delta t^2)$$

$$= (1 - a - b)f + (1/2 - b\alpha)\Delta t f_t + (1/2 - b\beta)\Delta t f_y f + O(\Delta t^2)$$

So we want $a = 1 - b$, $\alpha = \beta = 1/2b$.

Fourth Order Runge-Kutta Methods:

$$k_1 = \Delta t\, f(t_i, y_i)$$

$$k_2 = \Delta t\, f(t_i + 1/2\,\Delta t, y_i + 1/2\, k_1)$$

$$k_3 = \Delta t\, f(t_i + 1/2\,\Delta t, y_i + 1/2\, k_2)$$

$$k_4 = \Delta t\ f(t_i + \Delta t,\ y_i + k_3)$$

$$y_{i+1} = y_i + 1/6\ (k_1 + k_2 + k_3 + k_4)$$

The second order method requires 2 evaluations of f at every timestep, the fourth order method requires 4 evaluations of f at every timestep. In general: For an rth order RungeKutta method we need S(r) evaluations of f for each timestep, where

$$S(r) = \begin{cases} r & for\ r \leq 4 \\ r+1 & for\ r = 5\ and\ r = 6 \\ \geq r+2 & for\ r \geq \end{cases}$$

Practically speaking, people stop at r = 5. Advantages of Runge-Kutta Methods

1. One step method – global error is of the same order as local error.
2. Don’t need to know derivatives of f.
3. Easy for” Automatic Error Control”.

UNIT-3 SOLUTION OF ALGEBRAIC, TRANSCENDENDAL AND DIFFERENTAL EQUATION

Bisection Method

1. $f(x) = \cos x - xe^x$ perform five itruation of the bisection method to upon the root of the equation.

Solution :

$f(x) = \cos x - xe^x$

$f(0) = 1 = +ve$

$f(1) = -2.17797 = -ve$

The root lies between 0 and 1

$x_0 = \frac{0+1}{2} = 0.5$, $f(x_0) = \cos x - xe^x = 0.05322 = +ve$

The root lies between 0.5 and 1

$x_1 = \frac{0.5+1}{2} = 0.75$, $f(x_1) = \cos x - xe^x = -0.85606 = -ve$

The root lies between 0.5 and 0.75

$x_2 = \frac{0.5+0.75}{2} = 0.625$, $f(x_2) = \cos x - xe^x = -0.35669 = -ve$

The root lies between 0.625 and 0.5

$x_3 = \frac{0.625+0.5}{2} = 0.5625$, $f(x_3) = \cos x - xe^x = -0.14129 = -ve$

The root lies between 0.5625 and 0.5

$x_4 = \frac{0.5625+0.5}{2} = 0.53125$, $f(x_4) = \cos x - xe^x = -0.0415 = -ve$

The root lies between 0.53125 and 0.5

$x_5 = \frac{0.53125+0.5}{2} = 0.51562$, $f(x_5) = \cos x - xe^x = 0.00649 = +ve$

Average root lies between $= \frac{x_4+x_5}{2} = \frac{0.53125+0.51562}{2}$

$= 0.52343$

Itruative Method

Find the root of the equation, $x^3+x^2-100=0$ upto 4 decimal sucessive.

sol :

$$f(x) = x^3+x^2-100$$

$$f(3) = 27+9-100 = -64 = -ve$$

$$f(4) = 64+16-100 = -20 = -ve$$

$$f(5) = 125+25-100 = 50 = +ve$$

$$x^3+x^2-100 = 0$$

$$x(x^2+x)-100=0$$

$$x^2(x+1)-100=0$$

$$x^2 = \frac{100}{x+1}$$

$$x = \frac{10}{\sqrt{(x+1)}}$$

$$\varphi(x) = \frac{10}{\sqrt{(x+1)}}$$

$$\varphi'(x) = 10(x+1)^{-1/2} = \cancel{10}^{5}(-1/\cancel{2})(x+1)^{-1/2-1} = (-5)(x+1)^{-3/2}$$

$$\varphi'(x) = \frac{-5}{(x+1)^{3/2}}$$

$$\varphi'(4) = \frac{-5}{(4+1)^{3/2}} = \frac{-5}{(5)^{3/2}} = -0.44$$

$$\varphi'(5) = \frac{-5}{(5+1)^{3/2}} = \frac{-5}{(6)^{3/2}} = -0.34$$

$$x_0 = 4.2$$

$$x_1 = \varphi(x_0)$$

$$x_1 = \frac{10}{\sqrt{x_0+1}} = \frac{10}{\sqrt{4.2+1}} = 4.38529$$

$$x_2 = \varphi(x_1)$$

$$x_2 = \frac{10}{\sqrt{x_1+1}} = \frac{10}{\sqrt{4.3852+1}} = 4.30918$$

$x_3 = \varphi(x_2)$

$$= \frac{10}{\sqrt{x_2+1}} = \frac{10}{\sqrt{4.30918+1}} = 4.33995$$

$x_4 = \varphi(x_3)$

$$= \frac{10}{\sqrt{x_3+1}} = \frac{10}{\sqrt{4.33995+1}} = 4.32743$$

$x_5 = \varphi(x_4)$

$$= \frac{10}{\sqrt{x_4+1}} = \frac{10}{\sqrt{4.32743+1}} = 4.33252$$

$x_6 = 4.33045$, $x_7 = 4.33129$, $x_8 = 4.33095$, $x_9 = 4.33109$

$x_{10} = 4.33103$, $x_{11} = 4.33106$, $x_{12} = 4.33105$

values of x_{11}, x_{12} all collected the same and the values are same.

$\therefore$ The root of the equation $= 4.3311$ took correct to four decimal places.

Regulafalsi Method

Find the root of $xe^x = 3$ by regula falis method 3 decimal place root 1 and 1.5.

Sol: $f(x) = xe^x - 3$

$f(1) = -0.2817$ $(-)$

$f(1.5) = 3.7225$ $(+)$

$a = 1$ $\quad$ $b = 1.5$

$f(a) = -0.2817$ $\quad$ $f(b) = 3.7225$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{1(3.7225) - 1.5(-0.2817)}{(3.7225) - (-0.2817)}$$

$x_1 = 1.0351$

$f(x) = xe^x - 3$

$f(1.0351) = -0.0857 = -ve$

The root lies between 1.0351 and 1.5

$$x_2 = \frac{1.0351(3.7225) - 1.5(-0.0857)}{3.7225 - (-0.0857)} = 1.0455$$

$f(1.0455) = -0.0257 = -ve$

The root lies between 1.0455 and 1.5

$a = 1.0455 \qquad b = 1.5$

$f(a) = -0.0257 \quad f(b) = 3.7225$

$x_3 = 1.0486$

$f(1.0486) = xe^x - 3 = -0.0076 = -ve$

The root lies between 1.0486 and 1.5

$a = 1.0486 \qquad b = 1.5$

$f(a) = -0.0076 \quad f(b) = 3.7225$

$x_4 = 1.0495$

$f(1.0495) = xe^x - 3 = -0.0023$

$\therefore$ Hence the ual root of the equation $= 1.049$.

Newton Raphson Method

Evaluate $\sqrt{12}$ to four decimal of newton Raphson method.

Sol : $x = \sqrt{12}$

$x^2 = 12$

$x^2 - 12 = 0$

$f(x) = x^2 - 12$

$f'(x) = 2x$

$f(x) = x^2 - 12$

$f(0) = -12 = -ve$

$f(1) = 1 - 12 = -11 = -ve$

$f(2) = 4-12 = -8 = -ve$

$f(3) = 9-12 = -3 = -ve$

$f(4) = 16-12 = 4 = +ve$

$x_0 = 3 \quad n = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{(9-12)}{6} = 3 - \frac{(-3)}{6}$$

$\boxed{x_1 = 3.5}$

$x_1 = 3.5, \quad n = 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_1 - \frac{f(x_1)}{f'(x_1)} = 3.5 - \frac{(0.25)}{7}$$

$\boxed{x_2 = 3.4643}$

$x_2 = 3.4643 \quad n=2$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 3.4643 - \frac{0.0013}{6.9286}$$

$\boxed{x_3 = 3.4641}$

$x_3 = 3.4641. \quad n = 3$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 3.4641 - \frac{(-0.00001)}{6.9282}$$

$\boxed{x_4 = 3.4641}$

$\therefore$ Hence the real root of the equation $= 3.4641$

Euler's Method

Solve $dy/dx = 1 + xy$ with $y(0) = 2$, using Euler's Method also find using $y(0.1)$, $y(0.2)$, $y(0.3)$.

Sol: The Euler's formula.

$$y_{m+1} = y_m + h f(x_m, y_m)$$

$$dy/dx = f(x, y) = f(1 + xy)$$

$x_0 = 0$, $y_0 = 2$, $h = 0.1$, $n = 0$

$$y_1 = y_0 + (0.1) f(0.2)$$
$$= y_0 + (0.1)[1 + (0)(2)]$$
$$= 2 + 0.1$$
$$\boxed{y_1 = 2.1}$$

$$x_1 = x_0 + h = 0 + 0.1$$
$$\boxed{x_1 = 0.1}$$

$x_1 = 0.1$, $y_1 = 2.1$, $h = 0.1$, $n = 1$

$$y_2 = y_1 + h f(x_1, y_1)$$
$$= 2.1 + 0.1 [1 + (0.1)(2.1)]$$
$$= 2.1 + 0.1 (3.2)$$
$$\boxed{y_2 = 2.221}$$

$$x_2 = x_1 + h$$
$$= 0.1 + 0.1$$
$$\boxed{x_2 = 0.2}$$

$$y_3 = y_2 + h f(x_2, y_2)$$
$$= 2.221 + 0.1 [1 + (0.2)(2.221)]$$
$$= 2.221 + 0.14442$$
$$\boxed{y_3 = 2.3654}$$
$$x_3 = x_2 + h = 0.3$$

conclusion:

$y(0.1) = 2.1$
$y(0.2) = 2.221$
$y(0.3) = 2.3654$

Rungekutta Method

R.K Method II & IV order. Using runge-kutta method of 4^{th} order $dy/dx = \frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1$ at $x=0.2, 0.4$

sol:

$$y' = \frac{y^2-x^2}{y^2+x^2}$$

$x_0 = 0 \quad y_0 = 1 \quad h = 0.2 \quad x = 0.2, \ y = ?$

$$K_1 = hf(x_0, y_0) = 0.2(1) = 0.2$$

$$\boxed{K_1 = 0.2}$$

$$K_2 = hf(x_0 + h/2, y_0 + K_1/2) = 0.2(0+0.1, 1+0.1)$$
$$= 0.2(0.1, 1.1)$$
$$= 0.2(0.9836)$$

$$\boxed{K_2 = 0.1967}$$

$$K_3 = hf(x_0 + h/2, y_0 + K_2/2) = 0.2(0+0.1, 1+0.0984)$$
$$= 0.2(0.1, 1.0984)$$
$$= 0.2(0.9836)$$

$$\boxed{K_3 = 0.1967}$$

$$K_4 = hf(x_0 + h, y_0 + K_3) = 0.2(0+0.2, 1+0.1967)$$
$$= 0.2(0.2, 1.1967)$$
$$= 0.2(0.9457)$$

$$\boxed{K_4 = 0.1891}$$

$$\Delta y = 1/6 (K_1 + 2K_2 + 2K_3 + K_4)$$
$$= 1/6 (0.2 + 0.3934 + 0.3934 + 0.1891)$$
$$= 1/6 (1.1759)$$
$$\Delta y = 0.19598$$
$$y_1 = y_0 + \Delta y = 1 + 0.19598$$
$$\boxed{y_1 = 1.19598}$$

$x_2 = 0.4 \quad y_2 = ?$

$x_1 = 0.2, \quad y_1 = 1.19598, \; h = 0.2$

$k_1 = hf(x_1, y_1) = 0.2(0.2, 1.19598)$
$= 0.2(0.9456)$

$\boxed{k_1 = 0.1891}$

$k_2 = hf(x_1 + h/2, y_1 + k_1/2) = 0.2(0.2+0.1, 1.19598 + 0.0946)$
$= 0.2(0.3, 1.2906)$
$= 0.2(0.8975)$

$\boxed{k_2 = 0.1794}$

$k_3 = hf(x_1 + h/2, y_1 + k_2/2) = 0.2(0.2+0.1, 1.19598 + 0.0897)$
$= 0.2(0.3, 1.2857)$
$= 0.2(0.8967)$

$\boxed{k_3 = 0.1793}$

$k_4 = hf(x_1 + h, y_1 + k_3) = 0.2(0.2 + 0.2, 1.19598 + 0.1793)$
$= 0.2(0.4, 1.3753)$
$= 0.2(0.8440)$

$\boxed{k_4 = 0.1688}$

$\Delta y = 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$
$= 1/6\,(0.1891 + 0.3588 + 0.3586 + 0.1688)$
$= \frac{1.0753}{6}$

$\Delta y = 0.1792$

$y_2 = y_1 + \Delta y$
$= 1.19598 + 0.1792$

$\boxed{y_2 = 1.3752}$

Taylor Series Method

Find the value of $y(0.1)$ correct of four decimal places from $dy/dx = x^2 - y$ $\quad y(0) = 1$ with $h = 0.1$ using Taylor Series Method.

Sol : $\quad y' = x^2 - y$

$x_0 = 0 \quad y_0 = 1 \quad h = 0.1$

$$y_1 = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{y^4}{4!} y_0^{IV} + \ldots\ldots$$

$y' = x^2 - y$	$y_0' = x_0^2 - y_0$ $= (0)^2 - 1 = -1$
$y'' = 2x - y'$	$y_0'' = 2x_0 - y_0'$ $= 2(0) - (-1) = 1$
$y''' = 2 - y''$	$y_0''' = 2 - y_0''$ $= 2 - 1 = 1$
$y^{IV} = -y'''$	$y_0^{IV} = -y_0'''$ $= -(1) = -1$

$$y_1 = 1 + \frac{0.1}{1}(-1) + \frac{(0.1)^2}{2}(1) + \frac{(0.1)^3}{6}(1) + \frac{(0.1)^4}{24}(-1) + \ldots\ldots$$

$$= 1 + (0.1)(-1) + 0.005 + 0.00016 + 0.000004(-1)$$

$$\boxed{y_1 = 0.905156}$$

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