Methods for Economic Analysis-I

(Mathematical Methods)

18KP1EC03

UNIT : I

Constants and Variables are the two types of symbols in algebra. variable: a quantity represented by a symbol that can take different possible values. constant: a quantity whose value is fixed, even if we do not know its numerical amount

Constant: A symbol which has a fixed numerical value is called a constant. For example: 2, 5, 0, -3, -7, 2/7, 7/9 etc., are constants. Number of days in a week represents a constant. In the expression 5x + 7, the constant term is 7.

Variables: A variable represents a concept or an item whose magnitude can be represented by a number, i.e. measured quantitatively. Variables are called variables because they vary, i.e. they can have a variety of values. Thus a variable can be considered as a quantity which assumes a variety of values in a particular problem. Many items in economics can take on different values. Mathematics usually uses letters from the end of the alphabet to represent variables. Economics however often uses the first letter of the item which varies to represent variables. Thus p is used for the variable price and q is used for the variable quantity.

Parameter, in mathematics, a variable for which the range of possible values identifies a collection of distinct cases in a problem. Any equation expressed in terms of parameters is a parametric equation. The general equation of a straight line in slope-intercept form, $y = mx + b$, in which m and b are parameters, is an example of a parametric equation. When values are assigned to the parameters, such as the slope $m = 2$ and the y-intercept $b = 3$, and substitution is made, the resulting equation, $y = 2x + 3$, is that of a specific straight line and is no longer parametric.
In the set of equations $x = 2t + 1$ and $y = t^2 + 2$, t is called the parameter. As the parameter varies over a given domain of values, the set of solutions, or points (x, y), describes a curve in the plane. The use of parameters often enables descriptions of very simple curves for which it is difficult to write down a single equation in x and y.In statistics, the parameter in a function is a variable whose value is sought by means of evidence from samples. The resulting assigned value is the estimate, or statistic.

Intercepts co efficient
The intercept (often labeled the constant) is the expected mean value of Y when all X=0. Start with a regression equation with one predictor, X. If X sometimes equals 0, the intercept is simply the expected mean value of Y at that value. If so, and if X never = 0, there is no interest in the intercept.

Functions - function is: a relation from a set of inputs to a set of possible outputs where each input is related to exactly one output. We can write the statement that f is a function from X to Y using the function notation f:X→Y. function: a systematic relationship between pairs of values of the variables, written y = f(x). A function tries to define these relationships. It tries to give the relationship a mathematical form. An equation is a mathematical way of looking at the relationship between concepts or items. These concepts or items are represented by what are called variables.
Function, an expression, rule, or law that defines a relationship between one variable (the independent variable) and another variable (the dependent variable). Functions are ubiquitous in mathematics and are essential for formulating physical relationships in the sciences. The modern definition of function was first given in 1837 by the German mathematician Peter Dirichlet: This relationship is commonly symbolized as $y = f(x)$. In addition to $f(x)$, other abbreviated symbols such as $g(x)$ and $P(x)$ are often used to represent functions of the independent variable x, especially when the nature of the function is unknown or unspecified.

Inverse function - the inverse function of the formula that converts Celsius temperature to Fahrenheit temperature is the formula that converts Fahrenheit to Celsius. Applying one formula and then the other yields the original temperature. Inverse procedures are essential to solving equations because they allow mathematical operations to be reversed (e.g. logarithms, the inverses of exponential functions, are used to solve exponential equations). Whenever a mathematical procedure is introduced, one of the most important questions is how to invert it. Thus, for example, the trigonometric functions gave rise to the inverse trigonometric functions. an inverse function (or anti-function[1]) is a function that "reverses" another function: if the function f applied to an input x gives a result of y, then applying its inverse function g to y gives the result x, and vice versa, i.e., $f(x) = y$ if and only if $g(y) = x$. inverse function of f is also denoted as As an example, consider the real-valued function of a real variable given by $f(x) = 5x - 7$. Thinking of this as a step-by-step procedure (namely, take a number x, multiply it by 5, then subtract 7 from the result), to reverse this and get x back from some output value, say y, we would undo each step in reverse order. In this case, it means to add 7 to y, and then divide the result by 5.
In functional notation, this inverse function would be given by,
With $y = 5x - 7$ we have that $f(x) = y$ and $g(y) = x$.

General Function - Function, in mathematics, an expression, rule, or law that defines a relationship between one variable (the independent variable) and another variable (the dependent variable). Functions are ubiquitous in mathematics and are essential for formulating physical relationships in the sciences.

Specific Functions - specialized functions in sub-disciplines of mathematics. For example, in linear algebra and functional analysis, linear forms and the vectors they act upon are denoted using a dual pair to show the underlying duality. In logic and the theory of computation, the function notation of lambda calculus is used to explicitly express the basic notions of function abstraction and application. In category theory and homological algebra, networks of functions are described in terms of how they and their compositions commute with each other using commutative diagrams that extend and generalize the arrow notation for functions.

Equation - In algebra, an equation can be defined as a mathematical statement consisting of an equal symbol between two algebraic expressions that have the same value. The most basic and common algebraic equations in math consist of one or more variables. For instance, $3x + 5 = 14$ is an equation, in which $3x + 5$ and 14 are two expressions separated by an 'equal' sign. In an algebraic equation, the left-hand side is equal to the right-hand side. Here, for example, $5x + 9$ is the expression on the left-hand side, which is equal to the expression 24 on the right-hand side.
A)Linear Equation or First degree Equation
B)Quadratic Equation or Second degree Equation
1.Pure Quadratic Equation
2.Adfected Quadratic Equation
C)Cubic Equation or Third degree Equation
D) Quartic Equation
E)Simultaneous Equation

Economic Functions: revenue function, demand function, supply function, saving function, cost function, utility function, profit function, production function

UNIT : II

Differential calculus
calculus is the branch of mathematics of change motion and growth in related variables it is a science of fluctuations therefore any economic populace has a role to pay play when we consider how the sales is

affected when there is change in the price are how the total cost price extra are affected when there is change in output and so on.
branches of Calculus
A) differential calculus
B) integral calculus

rules of differentiation
rule 1: polynomial functions rule or power function rule
rule 2 : constant function rule
A) derivative of a constant
B) derivative of the product of a constant and a function
rule 3 : linear function rule
rule 4 addition rule or derivative of a sum or sum rule
rule 5 subtraction rule
rule 6 product rule for multiplication rule for derivative of a product
rule 7 audient roll are division rule are derivative of the quadrant of two functions
rule 8 chain rule or function of a function rule are derivative of a composite function
differential calculus of two variables 1. partial differentiation and 2.total differentiation

Maxima and Minima (the respective plurals of maximum and minimum) of a function, known collectively as extreme (the plural of extremum), are the largest and smallest value of the function, either within a given range (the *local* or *relative* extrema), or on the
entire domain (the *global* or *absolute* extrema). Pierre de Fermat was one of the first mathematicians to propose a general technique, ad equality, for finding the maxima and minima of functions.As defined in set theory, the maximum and minimum of a set are the greatest and least elements in the set, respectively. Unbounded infinite sets, such as the set of real numbers, have no minimum or maximum.If the domain of a function for which an extremum is to be found consists itself of functions (i.e. if an extremum is to be found of a functional), then the extremum is found using the calculus of variations. Minima of a function the function of X is said to have attained its minimum value or minimum at X equal to b is the function stops to decrease and begins to increase at X equal to b e f within bracket x 2 is a minimum value of a function if it is the lowest of all its values for values of X in some neighborhood of be the maxima and minima of the function are called the extreme values of the function

Uses of Derivative in Economics Increasing and Decreasing Functions Concavity and Convexity Relative Extreme. Increasing and Decreasing Function If a graph is going up is that its slope is positive. If the graph is going down, then the slope will be negative. Since slope and derivative are synonymous, we can relate increasing and decreasing with the derivative of a function. A function is increasing on an interval if for any x^1 and x^2 in the interval then Increasing and Decreasing Function $x^1 < x^2$ implies $f(x^1) <$ x $f(x^2)$ A function is decreasing on an interval if for any x^1 and x^2 in the interval then $f(x^2) < x^2$implies $f(x^1) > f(x^2)$ differentiable function on the interval (a,b) then Theorem on Derivatives and Increasing/Decreasing Functions < If f '(x)=0 for x in (a,b), then f is decreasing there. > If f '(x) = 0 for x in (a,b), then f is constant.0 for x in (a,b), then f is increasing there. convex if every line segment joining two points on its graph is never below the graph. Let f be a function of a single variable defined on an interval. Then f is concave if every line segment joining two points on its graph is never above the graph General definitions If has a minimum at c if f (c) Let f be a function defined on the interval (a,b) containing the point c. Then The extrema of a function f are the values where f is either a maximum or a minimum.

Maximization : human behavior is somehow oriented toward a maximization of some desired end has appeared in a great range of social science theory. Maximization is, of course, a fundamental concept in economics, for a central axiom of that discipline is that human wants are unlimited, but that we constantly strive to maximize our satisfactions. More specifically, ail of microeconomics, the study of how an entrepreneur or a firm should behave, assumes that he or it is trying to maximize money profit. Such questions as what will happen to profit if price is increased, or how a decrease in production will effect the ratio of income to costs, are at the heart of a great deal of economic theorizing, and they assume that the end in view is to make as much money as possible. Of course we know, and to give them their due I believe that economists know also, that not even entrepreneurs always strive to maximize money profit, but that sometimes they may prefer something else-leisure, conceivably even good human relations-rather than more money. These entrepreneurs are trying to maximize something, but only states that they sometimes have to choose between money and some other desired end. Economics, however, is by no means the only branch of social science that has looked upon man as though he were maximizing something.

A process that companies undergo to determine the best output and price levels in order to maximize its return. The company will usually adjust influential factors such as production costs, sale prices, and output levels as a way of reaching its profit goal. There are two main profit maximization methods used, and they are Marginal Cost-Marginal Revenue Method and Total Cost-Total Revenue Method. Profit maximization is a good thing for a company, but can be a bad thing for consumers if the company starts to use cheaper products or decides to raise prices. economics that holds the belief that when individuals purchase a good or a service, they strive to obtain the most amount of value possible, while at the same time spending the least amount of money possible. When combined, the consumer is attempting to derive the greatest amount of value from their available funds.

Study Material : Introduction to Mathematical Methods – D.Bose

UNIT – III

INTEGRATION

The reverse or inverse process of "Differentiation" is called " indefinite Integration" or "Integral Calculus" or "Anti differentiation" .

Rule – I $\int dx=x+c$

Rule – II $\int K\,dx=K\int dx = Kx + C$ k is constant

1) $\int 5dx = 5 \int dx = 5x+C$

2) $\int 9dx = 9 \int dx=9x+c$

Rule – III $\int x^n dx= x^{n+1}/n+1 +C$ $n\neq -1$

1)$\int x^5 dx = x^{5+1}/5+1 +c = X^6/6 +c$

2)$\int x^7 dx = x^{7+1}/7+1 +c =x^8/8+c$

Rule – IV Integral of sum or Difference

$\int(dx_1 + dx_2 + \ldots +dx_n) =\int dx_1 + \int dx_2 + \ldots +\int dx_n + c$

$\int(dx_1 - dx_2 - \ldots - dx_n) =\int dx_1 - \int dx_2 - \ldots -\int dx_n + c$

1) $\int(x^5-x+1)dx$

$= \int x^5\, dx-\int xdx+\int 1\, dx$

$= x^6/6 - x^2/2 + x + c$

2) $\int(8x^3- 3x^2 +x-1)dx$

$= \int 8x^3dx-\int 3x^2\, dx+\int xdx-\int 1dx$

$= 8\int x^3dx-3\int x^2\, dx+\int xdx-1\int dx$

$=8\, x^4/4 - 3x^3/3 + x^2/2-x + c$

$= 2\, x^4 - x^3+ x^2/2-x + c$

Rule – V Integral of a Multiple by a Constant

1)$\int 4x^8dx = 4\int x^8dx = 4\ x^{8+1}/8+1 +c = 4\, x^9/9 + c$

2) $\int 4x^3dx = 4\, x^4/4 + c = x^4+ c$

Rule – VI Integration by substitution

$\int f(x)dx = \int[f(u)du/dx]dx = \int f(u)\,.\, du=F(u)+c$

1)Evaluate $\int 4x^2(x^3+5)^3dx$

$U = x^3+5$

$du/dx = 3x^2$

$3x^2dx = du$

$dx=du/3x^2$

$\int 4x^2(x^3+5)^3dx = \int 4x^2.u^3dx$ $(U = x^3+5)$

$=\int 4x^2.u^3.\ du/3x^2$ $(dx=du/3x^2)$

$=\int 4/3\ u^3du = 4/3\int u^3du$

$4/3\ u^{3+1}/3+1 +c = 4/3\ u^4/4+c$

$1/3\ u^4 +c$

$1/3\ (x^3+5)^4+c$ $(U = x^3+5)$

$(x^3+5)^4/3 + c$

Definite integration or Definite integral

The anti derivative of a function achieved an indefinite result (i e..no definite numerical values) this process as indefinite integration. Integral to find the area between Two curves and the area between the curve and the X – axis, definite numerical result I e …a number or a value independent of the constant 'C' and not a function as for the indefinite Integral. Therefore , this process as Definite integration

$\int_a^b y\, dx$ or $\int_a^b f(x)dx$

The definite Integral of y or f(x)from x=a to x=b

Then the value of definite Integral from a to b

$\int_a^b y\, dx$ or $\int_a^b f(x)dx = [F(x)]_a^b =F(b) - F(a)$ …b>a

Where $\int_a^b y\,dx$ or $\int_a^b f(x)dx$ is the definite Integral of y or f(x) an is the area bound by y=f(x) the X – axis and the curves x = a and x = b a is called the lower limit of the Integral and b is called the upper limit of the Integral.

1) $\int_1^2 x^2 dx = [\frac{x2+1}{2+1}]_1^2 = [\frac{x3}{3}]_1^2 = [2^3/3 - 1^3/3] = 8/3 - 1/3 = 7/3$

2) $\int_2^3 3x dx = [3 \frac{x1+1}{1+1}]_2^3 = [3 \frac{x2}{2}]_2^3 = [3 \frac{3^2}{2} - 3\frac{2^2}{2}] = \frac{27}{2} - \frac{12}{2} = \frac{15}{2}$

3) $\int_2^3 4x dx = [4 \frac{x^{1+1}}{1+1}]_2^3 = [4\frac{x2}{2}]_2^3 = [4 \frac{3^2}{2} - 4\frac{2^2}{2}] = \frac{36}{2} - \frac{16}{2} = \frac{20}{2} = 10$

4) $\int_2^3 (x^2 + 5x + 7)dx$

$= [\frac{x2+1}{2+1} + \frac{5x1+1}{2} + 7x]_2^3$

$= [\frac{3^3}{2} + 5\frac{3^2}{2} + 7(3)]$ -

$[\frac{2^3}{3} + 5\frac{2^2}{2} + 7(2)]$

$= [\frac{27}{3} + \frac{45}{2} + 21] - [\frac{8}{3} + \frac{20}{2} + 14]$

$= [\frac{54+135+126}{6}] - [\frac{16+60+84}{6}] = \frac{315}{6} - \frac{160}{6} = \frac{155}{6} = 25\frac{5}{6}$

Consumer surplus : Consumer surplus is the difference between the price that a consumer is willing to pay for a commodity rather than go without it and the actual price he pays for the commodity

Consumer surplus= potential price - actual price

The marginal utility of money is constant and all the consumers have the same utility function a demand curve or a demand function for a commodity represents the amount of that commodity that will be bought by people at a given price 'P'

P= f(x) be the demand function for a commodity. Suppose a consumer purchases x0 quantity at p_0 price

The total expenditure of the consumer = p_0x_0

Consumer surplus = $\int_0^{X0} f(x)dx$ - $P_0 X_0$

1)If the Demand function is P =25-3x-3x^2 and the demand x_0 is 2 what will be Consumer surplus

P =25-3x-3x^2 $x_0 = 2$

$P_0 = 25-3(2)-3(2)^2 = 25-6-12=25-18=7$

$P_0 = 7$

$X_0 = 2$

$= \int_0^2 (25 - 3x - 3x2)dx$ - *(7×2)*

$= [25\times - \frac{3x2}{2} - x^3]_0^2 - 14$

$= [25(2) - \frac{3(2)2}{2} - (2)^3] - 14$

= (50-6-8)-14

= 36-14 =22

Consumer surplus = 22

Producer surplus

Marginal utility of money is constant and all the producers have the same production function. P=f(x) be the supply function or supply curve represents the amount of commodity that can be supply give price P example market price suppose a producer sells a quantity x_0 at price p_0

P_0=f(x)

Producer revenue = p_0x_0 … (1)

$\int_0^{X0} f(x)dx$ … (2)

Producer surplus = $P_0 X_0 - \int_0^{X0} f(x)dx$

1)The supply function for a commodity $P = x^2-x+5$ where x denotes supply. find the Producer surplus when the Price is Rs.11

$P = x^2-x+5$ $P = 11$

$x^2-x+5 = 11$

$x^2-x = 11 -5$

$x^2-x = 6$

$x^2-x - 6 = 0$

$x(x-3)+2(x-3)=0$

$x^2-3x+2x-6 = 0$ Therefore

$(x-3)(x+2)=0$

$P_0 = 11$

$X_0 = 3$

Producer surplus = $P_0 X_0 - \int_0^{X0} P\,dx$

$= 11\times 3 - \int_0^3 (x2 + x + 5)dx$

$= 33 - [\frac{x3}{3} - \frac{x2}{2} + 5x]^3_0$

$= 33 - [\frac{33}{3} - \frac{x2}{2} + 5x]^3_0$

$= 33 - (3)^3/3 - (3)^2/2 + 5(3) - 0$

$= 33 - [27/3 - 9/2 + 15] - 0$

$= 33 - 9 + 4.5 - 15$

$= 13.5$

Producer surplus = 13.5

COST FUNCTION

In differentiation total cost (C) of producing an output x Marginal cost(MC),that first order derivative of the Total Cost (C) MC=dc/dx

Example : If $MC = 3-2x-x^2$ find the Total Cost.

Total Cost = $\int(3-2x-x^2)dx = 3x - x^2 - x3/3 + C$

REVENUE FUNCTION

Total Revenue (TR or R)of producing an output x ,Marginal Revenue (MR) ,that is the first order derivative of the Total Revenue (TR or R) MR = dr/dx . TR or R = $\int MR.dx = \int\frac{dR}{dx}\,dx$

Example : If MR function MR = 100-4Q find the Total Revenue function

TR = $\int(100-4Q)dQ = 100Q - 4Q^2/2 + C = 100Q-2Q^2 + C$

UNIT – IV MATRICES

A matrix is a collection of numbers arranged into a fixed number of rows and columns. Rows × columns . Each number that makes up a matrix is called an element of the matrix. The elements in a matrix have specific locations. Matrix number are written in square or rectangular brackets[ ] or parentheses ( ) or pair double bars ‖ ‖

Types of Matrices

Different types of Matrices and their forms are used for solving numerous problems. Some of them are as follows:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

1) Row Matrix
A row matrix has only one row but any number of columns. A matrix is said to be a row matrix if it has only one row. For example, A=[1] $_{1\times 1}$ A=[1 2] $_{1\times 2}$ A=[1 2 3] $_{1\times 3}$ A=[1 2 3 4] $_{1\times 4}$ is a row matrix of order. In general,
A = $[a_{ij}]_{1\times n}$ is a row matrix of order 1 × n.
2) Column Matrix
A column matrix has only one column but any number of rows. A matrix is said to be a column matrix if it has only one column.
$\begin{bmatrix}1\\2\end{bmatrix}$ a column matrix of order 1× 2.
In general, B = $[b_{ij}]_{m\times 1}$ is a column matrix of order m × 1.
3) Square Matrix
A square matrix has the number of columns equal to the number of rows. A matrix in which the number of rows is equal to the number of columns is said to be a square matrix. Thus an m × n matrix is said to be a square matrix if m = n and is known as a square matrix of order 'n'. For example,A = $\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}_{2\times 2}$ is a square matrix of order 2. In general, A = $[a_{ij}]$ m × m is a square matrix of order m.
4) Rectangular Matrix
A matrix is said to be a rectangular matrix if the number of rows is not equal to the number of columns. For example,

A= $\begin{bmatrix}1 & 2 & 3 & 4 & 5\\6 & 7 & 8 & 9 & 10\end{bmatrix}_{2\times 5}$ A= $\begin{bmatrix}1 & 5\\2 & 6\\3 & 7\\4 & 8\end{bmatrix}_{4\times 2}$

5) Diagonal matrix
A square matrix B = [bij] m × m is said to be a diagonal matrix if all its non-diagonal elements are zero, that is a matrix B =$[b_{ij}]_{m\times m}$ is said to be a diagonal matrix if $b_{ij} = 0$, when i ≠ j. For example, A = $\begin{matrix}5 & 0 & 0\\0 & 6 & 0\\0 & 0 & 2\end{matrix}$
are diagonal matrices of order 1, 2, 3, respectively.
6) Scalar Matrix
A diagonal matrix is said to be a scalar matrix if all the elements in its principal diagonal are equal to some non-zero constant. A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is, a square matrix B = $[b_{ij}]_{n\times n}$ is said to be a scalar matrix if

- $b_{ij} = 0$, when i ≠ j
- $b_{ij} = k$, when i = j, for some constant k. For example, A = $\begin{matrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{matrix}$ are scalar matrices of order 1, 2 and 3, respectively.

7) Zero or Null Matrix
A matrix is said to be zero matrix or null matrix if all its elements are zero.
For Example, A=[0] A = $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ A = $\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$ are all zero matrices of the order 1, 2 and 3 respectively. We denote zero matrix by O.
8) Unit or Identity Matrix
If a square matrix has all elements 0 and each diagonal elements are non-zero, it is called identity matrix and denoted by I.
Equal Matrices: Two matrices are said to be equal if they are of the same order and if their corresponding elements are equal to the square matrix A = $[a_{ij}]_{n\times n}$ is an identity matrix if

- $a_{ij} = 1$ if i = j
- $a_{ij} = 0$ if i ≠ j

We denote the identity matrix of order n by I_n. When the order is clear from the context, we simply write it as I. For example,A=[1] A = $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ A = $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ are identity matrices of order 1, 2 and 3, respectively. Observe that a scalar matrix is an identity matrix when k = 1. But every identity matrix is clearly a scalar matrix.

9) Upper Triangular Matrix

A square matrix in which all the elements below the diagonal are zero is known as the upper triangular matrix. For example, A=$\begin{bmatrix}1 & 2 & 3\\0 & 1 & 2\\0 & 0 & 1\end{bmatrix}$

10) Lower Triangular Matrix

A square matrix in which all the elements above the diagonal are zero is known as the upper triangular matrix. For example, A= $\begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\3 & 2 & 1\end{bmatrix}$

determinant is a scalar value that can be computed from the elements of a square matrix and encodes certain properties of the linear transformation described by the matrix. The determinant of a matrix A is denoted $\det(A)$, $\det A$, or $|A|$

A) First Order Matrix
B) Second Order Matrix
C) Third Order Matrix
D) Fourth Order Matrix
E) Row Matrix
F) Column Matrix
G) Triangular Matrix
H) Product of Matrix

Rank of Matrix The rank of a matrix is defined as (a) the maximum number of linearly independent column vectors in the matrix or (b) the maximum number of linearly independent row vectors in the matrix. Both definitions are equivalent. For an r x c matrix, If r is less than c, then the maximum rank of the matrix is r.levels in various industries that would be required by particular levels of demand for final goods.

Cramer's rule is an explicit formula for the solution of a system of linear equations with as many equations as unknowns, valid whenever the system has a unique solution.equations in two variables and three variables, and by multiple methods: substitution, addition, Gaussian elimination, using the inverse of a matrix, and graphing. Some of these methods are easier to apply than others and are more appropriate in certain situations. In this section, we will study two more strategies for solving systems of equations.

A determinant is a real number that can be very useful in mathematics because it has multiple applications, such as calculating area, volume, and other quantities. Here, we will use determinants to reveal whether a matrix is invertible by using the entries of a square matrix to determine whether there is a solution to the system of equations. Perhaps one of the more interesting applications, however, is their use in cryptography. Secure signals or messages are sometimes sent encoded in a matrix. The data can only be decrypted with an invertible matrix and the determinant. For our purposes, we focus on the determinant as an indication of the invertibility of the matrix. Calculating the determinant of a matrix involves following the specific patterns that are outlined in this section.

Input-Output analysis; Input-output analysis is a mathematical technique for studying the production structure of an economy on the assumption of mutual interdependence of the various sectors of the economy. The primary purpose of the input-output analysis is to calculate the out-put

UNIT – V LINEAR PROGRAMING

linear programming is a method of optimising operations with some constraints. The main objective of linear programming is to maximize or minimize the numerical value. It consists of linear functions which are subjected to the constraints in the form of linear equations or in the form of inequalities.

Linear programming is considered as an important technique which is used to find the optimum resource utilisation. The term "linear programming" consists of two words such as linear and programming. The word "linear" defines the relationship between multiple variables with degree one. The word "programming" defines the process of selecting the best solution from various alternatives.

Linear Programming is widely used in Mathematics and some other field such as economics, business, telecommunication, and manufacturing fields. In this article, let us discuss the definition of linear programming, its components, a simplex method with linear programming problems.

Linear programming (LP) or **Linear Optimisation** may be defined as the problem of maximizing or minimizing a linear function which is subjected to linear constraints. The constraints may be equalities or inequalities. The optimisation problems involve the calculation of profit and loss. Linear programming problems are an important class of optimisation problems, that helps to find the feasible region and optimise the solution in order to have the highest or lowest value of the function.

Linear programming is the method of considering different inequalities relevant to a situation and calculating the best value that is required to be obtained in those conditions.

Some of the assumption taken while working with linear programming are:

- The number of constraints should be expressed in the quantitative terms
- The relationship between the constraints and the objective function should be linear
- The linear function (i.e., objective function) is to be optimized
- Components of Linear Programming The basic components of the LP are as follows:
- Decision Variables
- Constraints
- Data
- Objective Functions

Characteristics of Linear Programming

The following are the five characteristics of the linear programming problem:

Constraints – The limitations should be expressed in the mathematical form, regarding the resource.

Objective Function – In a problem, the objective function should be specified in a quantitative way.

Linearity – The relationship between two or more variables in the function must be linear. It means that the degree of the variable is one.

Finiteness – There should be finite and infinite input and output numbers. In case, if the function has infinite factors, the optimal solution is not feasible.

Non-negativity – The variable value should be positive or zero. It should not be a negative value.Linear Programming Simplex Method

To solve linear programming models, the simplex method is used to find the optimal solution to a problem. It involves slack variables, tableau and pivot variables for the optimisation of a problem. The algorithm used here is

- Change of variables and normalise the sign of independent terms
- Normalise restrictions
- Match the objective functions to zero
- Write the initial tableau of the simplex method
- Stopping condition
- Input and output variable choices
- Again update tableau.
- Continue the iteration until to get the optimal solution

Linear Programming Applications

A real-time example would be considering the limitations of labours and materials and finding the best production levels for maximum profit in particular circumstances. It is part of a vital area of mathematics known as optimisation techniques. The applications of LP in some other fields are

- Engineering – It solves design and manufacturing problems as it is helpful for doing shape optimisation
- Efficient Manufacturing – To maximise profit, companies use linear expressions
- Energy Industry – It provides methods to optimise the electric power system.
- Transportation Optimisation – For cost and time efficiency.

Importance of Linear Programming

Linear programming is broadly applied in the field of optimisation for many reasons. Many functional problems in operations analysis can be represented as linear programming problems. Some special problems of linear programming are such as network flow queries and multi-commodity flow queries are deemed to be important to have produced much research on functional algorithms for their solution.

Plenty of algorithms for different types of optimisation difficulties work by working on LP problems as sub-problems.Linear Programming Example

Let us see an example here and understand the concept of linear programming in a better way.

Example: Calculate the maximal and minimal value of $z = 5x + 3y$ for the following constraints.

$x + 2y \leq 14$

$3x - y \geq 0$

$x - y \leq 2$

Solution: The three inequalities indicate the constraints. The area of the plane that will be marked is the feasible region.

The optimisation equation $(z) = 5x + 3y$. You have to find the (x,y) corner points that give the largest and smallest values of z.

To begin with, first solve each inequality.

$x + 2y \leq 14 \Rightarrow y \leq -(1/2)x + 7$

$3x - y \geq 0 \Rightarrow y \leq 3x$

$x - y \leq 2 \Rightarrow y \geq x - 2$

Here is the graph for the above equations.

Now pair the lines to form a system of linear equations to find the corner points.

$y = -(½) x + 7$

$y = 3x$

Solving the above equations, we get the corner points as (2, 6)

$y = -1/2 x + 7$

$y = x - 2$

Solving the above equations, we get the corner points as (6, 4)

$y = 3x$

$y = x - 2$

Solving the above equations, we get the corner points as (-1, -3)

For linear systems, the maximum and minimum values of the optimisation equation lie on the corners of the feasibility region. Therefore, to find the optimum solution, you only need to plug these three points in $z = 3x + 4y$

(2, 6) :

$z = 5(2) + 3(6) = 10 + 18 = 28$

(6, 4):

$z = 5(6) + 3(4) = 30 + 12 = 42$

(−1, −3):

$z = 5(-1) + 3(-3) = -5 -9 = -14$

Hence, the maximum of z = 42 lies at (6, 4) and the minimum of z = -14 lies at (-1, -3)

References

An Introduction to Mathematical Methods – D.Bose

Websites - www.google.com