

SEMESTER : 1  
CORE COURSE : 1

Inst Hour	: 5
Credit	: 5
Code	: 18K1M01

### BASICS, DIFFERENTIAL CALCULUS AND TRIGONOMETRY

#### UNIT 1:

Sets – Mappings – Equivalence Relations – The Integers – Binary Operations – Partially Ordered Sets.

(Chapter 1: Sections 1.1 to 1.6 of Text Book 1).

Methods of Successive Differentiation – Leibnitz's Theorem for the  $n^{\text{th}}$  derivative -Increasing & Decreasing functions, Maxima and Minima for one variable.

(Chapter 3: Sections 1.1-1.6, 2.1, 2.2, Chapter IV Sections 2.1, 2.2 &

Chapter 5: Sections 1.1-1.5 of Text Book 2)

#### UNIT 2:

Curvature – Radius of curvature in Cartesian & in Polar Coordinates – Centre of curvature – Evolutes & Involutives

(Chapter 10: Sections 2.1 –2.6 of Text Book 2)

#### UNIT 3:

Expansions of  $\sin(nx)$ ,  $\cos(nx)$ ,  $\tan(nx)$  – Expansions of  $\sin^n x$ ,  $\cos^n x$  – Expansions of  $\sin(x)$ ,  $\cos(x)$ ,  $\tan(x)$  in powers of  $x$  and related problems.

(Chapter 1: Sections 1.2 to 1.4 of Text Book 3)

#### UNIT 4:

Hyperbolic functions – Relation between hyperbolic & Circular functions- Inverse hyperbolic functions.

(Chapter 2: Sections 2.1 & 2.2 of Text Book 3)

#### UNIT 5:

Logarithm of a complex number -Summation of Trigonometric series-Difference method- Angles in arithmetic progression method –Gregory's Series

(Chapter 3 & Chapter 4 : Sections 4.1,4.2 & 4.4 of Text Book 3)

#### Text Book(s)

- [1] M. L. Santiago, Modern Algebra, Tata McGraw – Hill Publishing Company Limited, 2001.
- [2] S.Narayanan, T.K.Manickavasagam Pillai, Calculus Volume I, S.V Publications - 2004
- [3] S. Arumugam, Issac & Somasundaram, Trigonometry and Fourier Series New Gamma Publications – 1999 Edition

#### Books for Reference

- [1] S.Arumugam & others, Calculus Volume I.
- [2] S.Narayanan, Trigonometry
- [3] Ajit Kumar, S.Kumaresan, Bhaba Kumar Sarma, A Foundation Course in Mathematics, Narosa Publishing House.

#### Question Pattern (Both in English & Tamil Version)

Section A :  $10 \times 2 = 20$  Marks, 2 Questions from each Unit.

Section B :  $5 \times 5 = 25$  Marks, EITHER OR ( a or b) Pattern, One question from each Unit.

Section C :  $3 \times 10 = 30$  Marks, 3 out of 5, One Question from each Unit.

Generalization (विशेषण)  $n^{\text{th}}$  order derivative :-

①  $y = e^{ax}$  माना  $y_n$  - मी वरिष्ठतम आनीक

Soln :-

$$y_1 = a e^{ax}$$

$$y_2 = a^2 e^{ax}$$

$$y_n = a^n e^{ax}$$

②  $y = (ax+b)^m$ , माना  $y_n$  मी वरिष्ठतम आनीक

श्रीतः

$$y_1 = m(ax+b)^{m-1} \cdot a$$

$$y_2 = m(m-1)(ax+b)^{m-2} \cdot a^2$$

$$\vdots$$

$$y_n = m(m-1)(m-2) \dots (m-n+1)(ax+b)^{m-n} \cdot a^n$$

③  $y = (ax+b)^{-1}$ , माना  $y_n$  - मी वरिष्ठतम आनीक

श्रीतः

$$y_1 = (-1)a(ax+b)^{-2}$$

$$y_2 = (-1)(-2)(ax+b)^{-3} \cdot a^2$$

$$\vdots$$

$$y_n = (-1)^n n! a^n (ax+b)^{-n-1}$$

④  $y = \log(ax+b)$  माना  $y_n$  - आनीक

श्रीतः :-

$$y_1 = \frac{1}{(ax+b)} \cdot a = a \cdot (ax+b)^{-1}$$

$$y_2 = a \cdot (-1)(ax+b)^{-2} \cdot a = a^2 \cdot (-1)(ax+b)^{-2}$$

$$\vdots$$

$$y_n = a^n (-1)^{n-1} (n-1)! (ax+b)^{-n}$$

⑤  $y = \sin(ax+b)$  माना  $y_n$  - श्रीतः आनीक

श्रीतः

$$y_1 = a \cos(ax+b) = a \sin(\pi/2 + ax+b)$$

$$y_2 = a^2 \cos(\pi/2 + ax+b) = a^2 \sin(2\pi/2 + ax+b)$$

$$\vdots$$

$$y_n = a^n \sin(n\pi/2 + ax+b)$$



⑥  $y = \cos(ax+b)$ , स्मरनि  $y_n$  - अनमिठ.

डिपिः

$$y_1 = -\sin(ax+b) \cdot a = a \cdot \cos(\pi/2 + ax+b)$$

$$y_2 = a^2 \cos(\pi/2 + ax+b)$$

$$y_n = a^n \cos(\pi/2 + ax+b)$$

⑦  $y = e^{ax} \sin(bx+c)$  स्मरनि  $y_n$  लडुडु अनमिठ.

डिपिः

$$y_1 = e^{ax} \cos(bx+c) + a \cdot e^{ax} \sin(bx+c)$$

$$a = r \cos \theta \quad ; \quad b = r \sin \theta$$

$$r = \sqrt{a^2 + b^2} \quad \theta = \tan^{-1}(b/a)$$

$$a^2 + b^2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta = r^2$$

$$y_1 = e^{ax} (r \sin \theta \cos(bx+c) + r \cos \theta \sin(bx+c))$$

$$= r \cdot e^{ax} (\sin(\theta + bx+c))$$

$$y_2 = r^2 \cdot e^{ax} (\sin(2\theta + bx+c))$$

$$y_n = r^n \cdot e^{ax} (\sin(n\theta + bx+c))$$

⑧  $y = \cos 3x$  स्मरनि,  $y_n$  - लडुडु अनमिठ.

डिपिः

$$y = \cos 3x$$

$$[y_n = a^n \cos(\frac{n\pi}{2} + ax+b)]$$

$$y_n = 3^n \cos(\frac{n\pi}{2} + 3x)$$

⑨  $y = \sin^2 x$ , स्मरनि  $y_n$  - अनमिठ.

डिपिः

$$y = \sin^2 x$$

$$y_n = -\frac{1}{2} [2^n \cos(n\pi/2 + 2x)]$$

$$[\sin^2 x = \frac{1 - \cos 2x}{2}]$$

$$= -\frac{1}{2} 2^{n-1} \cos(n\pi/2 + 2x)$$

⑩  $y = \sin 4x \cos x$ , स्मरनि  $y_n$  - लडुडु अनमिठ.

डिपिः

$$y = \sin 4x \cos x$$

$$[2 \sin A \cos B = \sin(A+B) + \sin(A-B)]$$

$$= \frac{1}{2} (\sin 5x + \sin 3x)$$

$$y_n = \frac{1}{2} [5^n \sin(n\pi/2 + 5x) + 3^n \sin(n\pi/2 + 3x)]$$



11)  $y = \frac{3}{(2x-1)(x+1)}$  ආකාරයේ,  $y_n$ -හි ලේඛනය සොයන්න.

Soln :-  $y = \frac{3}{(2x-1)(x+1)}$

$$\frac{3}{(2x-1)(x+1)} = \frac{A}{(2x-1)} + \frac{B}{(x+1)}$$

$$3 = A(x+1) + B(2x-1)$$

$$x = -1 \Rightarrow B = -1 ; x = 1/2 \Rightarrow A = 2$$

$$y = \frac{2}{2x-1} - \frac{1}{x+1}$$

$$\therefore y_n = \frac{2(-1)^n \cdot 2^n n!}{(2x-1)^{n+2}} - \frac{(-1)^n n!}{(x+1)^{n+1}}$$

12)  $\frac{x^2}{(x+2)(x-1)^2}$  -හි  $n$  වැනි වශයෙන් සොයන්න.

ආකාරය:  $y = \frac{x^2}{(x+2)(x-1)^2}$

$$\frac{x^2}{(x+2)(x-1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x^2 = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

$$x = 1 \Rightarrow B = 1/3 ; x = -2 \Rightarrow C = 4/9 ; x = 0 \Rightarrow A = 5/9$$

$$y = \frac{5}{9} \frac{1}{(x-1)} + \frac{1}{3} \cdot \frac{1}{(x-1)^2} + \frac{4}{9} \cdot \frac{1}{(x+2)}$$

$$y_n = \frac{5}{9} \frac{(-1)^n n!}{(x-1)^{n+1}} + \frac{(-1)^n n!}{3(x-1)^{n+2}} + \frac{4}{9} \frac{(-1)^n n!}{(x+2)^{n+2}}$$

13)  $y = \frac{1}{x^2+a^2}$  ආකාරයේ,  $y_n$ -හි ලේඛනය සොයන්න.

ආකාරය:  $\frac{1}{x^2+a^2} = \frac{1}{(x+ia)(x-ia)}$

$$\frac{1}{x^2+a^2} = \frac{A}{x+ia} + \frac{B}{x-ia}$$

$$1 = A(x-ia) + B(x+ia)$$

$$x = ia \Rightarrow B = 1/2ia ; x = -ia \Rightarrow A = -1/2ia$$

$$y = \frac{-1/2ia}{(x+ia)} + \frac{1/2ia}{(x-ia)}$$

$$y_n = \frac{-1(-1)^n n! i^n}{2ia(x+ia)^{n+1}} + \frac{(-1)^n n! i^n}{2ia(x-ia)^{n+1}}$$



(11)  $\log \left( \frac{x+a}{x-a} \right)$  - ന്നി  $n^{\text{ആം}}$  ഘാതത്തിന്റെ  $n^{\text{ആം}}$  കണക്കാക്കുക.

കുറിപ്പ്:

$$\log \left( \frac{x+a}{x-a} \right) = \log(x+a) - \log(x-a)$$

$$\begin{aligned} D^n \left[ \log \left( \frac{x+a}{x-a} \right) \right] &= D^n [\log(x+a)] - D^n [\log(x-a)] \\ &= \frac{(-1)^{n-1} (n-1)!}{(x+a)^n} - \frac{(-1)^{n-1} (n-1)!}{(x-a)^n} \end{aligned}$$

(12)  $\cos^3 2x$  - ന്നി  $n^{\text{ആം}}$  ഘാതത്തിന്റെ  $n^{\text{ആം}}$  കണക്കാക്കുക.

കുറിപ്പ്:

$$D^n [\cos^3 2x] = D^n \left[ \frac{3 \cos 2x + \cos 6x}{4} \right]$$

$$= \frac{3}{4} D^n (\cos 2x) + \frac{1}{4} D^n (\cos 6x)$$

$$= \frac{3}{4} \cos \left( 2x + n\pi/2 \right) + \frac{1}{4} 6^n \cos \left( 6x + n\pi/2 \right)$$

(15)  $\cos x \cos 2x \cos 3x$  - ന്നി  $n^{\text{ആം}}$  ഘാതത്തിന്റെ  $n^{\text{ആം}}$  കണക്കാക്കുക.

കുറിപ്പ്:

$$y = \cos x \cos 2x \cos 3x$$

$$\left[ \cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)] \right]$$

$$= \frac{1}{2} \cos 2x [\cos 6x + \cos 2x]$$

$$= \frac{1}{2} [\cos 8x + \cos 4x] + \frac{1}{2} \cos^2 2x$$

$$= \frac{1}{4} (\cos 2x + \cos 6x) + \frac{1}{4} (1 + \cos 4x)$$

$$= \frac{1}{4} + \frac{1}{4} (\cos 2x + \cos 4x + \cos 6x)$$

$$D^n (\cos x \cos 2x \cos 3x) = \frac{1}{4} \left\{ 2^n \cos \left( \frac{n\pi}{2} + 2x \right) + 4^n \cos \left( \frac{n\pi}{2} + 4x \right) + 6^n \cos \left( \frac{n\pi}{2} + 6x \right) \right\}$$

(16)  $\cos^5 \theta \sin^7 \theta$  - ന്നി  $n^{\text{ആം}}$  ഘാതത്തിന്റെ  $n^{\text{ആം}}$  കണക്കാക്കുക.

കുറിപ്പ്:

$$x = \cos \theta + i \sin \theta \quad ; \quad 1/x = \cos \theta - i \sin \theta$$

$$x + 1/x = 2 \cos \theta \quad ; \quad x - 1/x = 2i \sin \theta$$

$$x^n = \cos n\theta + i \sin n\theta \quad ; \quad 1/x^n = \cos n\theta - i \sin n\theta$$

$$x^n + 1/x^n = 2 \cos n\theta \quad ; \quad x^n - 1/x^n = 2i \sin n\theta$$

$$2^5 \cos^5 \theta = (x + 1/x)^5 \quad ; \quad 2^7 \sin^7 \theta = (x - 1/x)^7$$

$$\begin{aligned} \therefore 2^{12} \cos^5 \theta \sin^7 \theta &= (x + 1/x)^5 (x - 1/x)^7 \\ &= (x + 1/x)^5 (x - 1/x)^5 (x - 1/x)^2 \\ &= (x^2 - 1/x^2)^5 (x - 1/x)^2 \end{aligned}$$



$$= \left( x^{10} - 5x^6 + 10x^2 - \frac{10}{x^2} + \frac{5}{x^6} - \frac{1}{x^{10}} \right) (x^2 - 2 + 1/x^2)$$

$$= \left[ x^{12} - 2x^{10} + 2x^6 - x^4 - 4x^8 + 8x^6 - 8x^2 + 4 + 6x^{14} - 12x^2 + 12/x^2 - 6/x^4 - 4 + 8/x^2 - 8/x^6 + 4/x^8 + 1/x^4 - 2/x^6 + 2/x^{10} - 1/x^{12} \right]$$

$$= (x^{12} - 1/x^{12}) - 2(x^{10} - 1/x^{10}) - 4(x^8 - 1/x^8) + 10(x^6 - 1/x^6) + 5(x^4 - 1/x^4) - 20(x^2 - 1/x^2)$$

$$= 2i \sin 120 - 2(2i \sin 100) - 4(2i \sin 80) + 10(2i \sin 60) + 5(2i \sin 40) - 20(2i \sin 20)$$

2i - 2sin இரண்டு பக்கமும் 2இல்

$$- 2'' \cos 50 \sin 70 = \sin 120 - 2 \sin 100 - 4 \sin 80 + 10 \sin 60 + 5 \sin 40 - 20 \sin 20$$

$$D^n (\cos^5 \theta \sin^7 \theta) = -1/2'' \{ 12^n \sin(n\pi/2 + 120) - 10^n 2 \sin(n\pi/2 + 100) - 8^n 4 \sin(n\pi/2 + 80) + 6^n 10 \sin(n\pi/2 + 60) + 4^n 5 \sin(n\pi/2 + 40) - 2^n 20 \sin(n\pi/2 + 20) \}$$

அளக்கொடுக்கான கணக்கில் சமன்பாட்டைத் தீர்க்கும்படி :-

①  $xy = ae^x + be^{-x}$  எனின்,  $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy = 0$  என கிண்புடி  
 தீர்வு :-  $xy = ae^x + be^{-x}$

'x' மூலம் வகுத்துக் கொடுக்கும் அளக்கொடுக்கான கணக்கைப் பெறும்.

$$x \frac{dy}{dx} + y = ae^x - be^{-x}$$

மேலும் 'x' மூலம் வகுத்துக் கொடுக்கும் அளக்கொடுக்கான கணக்கைப் பெறும்

$$x \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = ae^x + be^{-x}$$

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = xy$$

$$\therefore x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy = 0$$

②  $y = \sin(m \sin^{-1} x)$  எனின்,  $(1-x^2)y'' - xy' + M^2y = 0$  என கிண்புடி

தீர்வு :-  $y = \sin(m \sin^{-1} x)$

$$\sin^{-1} y = m \sin^{-1} x$$

'x' மூலம் வகுத்துக் கொடுக்கும் அளக்கொடுக்கான கணக்கைப் பெறும்

$$\frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = m \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2} \frac{dy}{dx} = m \sqrt{1-y^2}$$

இருபுறமும் வர்க்கப்படுத்துவது

$$(1-x^2) \left( \frac{dy}{dx} \right)^2 = M^2 (1-y^2)$$

மேலும் 'x' மூலம் வகுத்துக் கொடுக்கும் அளக்கொடுக்கான கணக்கைப் பெறும்

$$(1-x^2) 2 \left( \frac{dy}{dx} \right) \left( \frac{d^2y}{dx^2} \right) + \left( \frac{dy}{dx} \right)^2 \cdot (-2x) = -M^2 2y \frac{dy}{dx}$$

$2 \frac{dy}{dx} - 2sin$  இரண்டு பக்கமும் 2இல்



$$(1-x^2) d^2y/dx^2 - x dy/dx = -m^2 y$$

$$(1-x^2) y_2 - x y_1 + m^2 y = 0$$

①  $x = \sin \theta, y = \cos p \theta$  எனில்  $(1-x^2) y_2 - x y_1 + p^2 y = 0$  என  
 தீர்வு:

$$dx/d\theta = \cos \theta; dy/d\theta = -p \sin \theta; dy/dx = \frac{-p \sin \theta}{\cos \theta}$$

$$x = \sin \theta \Rightarrow x^2 = \sin^2 \theta \Rightarrow 1-x^2 = 1-\sin^2 \theta \Rightarrow 1-x^2 = \cos^2 \theta$$

$$\sqrt{1-x^2} = \cos \theta$$

$$y = \cos p \theta \Rightarrow y^2 = \cos^2 p \theta \Rightarrow 1-y^2 = 1-\cos^2 p \theta \Rightarrow 1-y^2 = \sin^2 p \theta$$

$$\sqrt{1-y^2} = \sin p \theta$$

$$dy/dx = \frac{-p \sqrt{1-y^2}}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} dy/dx = -p \sqrt{1-y^2}$$

இதையெழுத்து வர்க்கப்படுத்துவது

$$(1-x^2) (d^2y/dx^2)^2 = p^2 (1-y^2)$$

'x' மீது வகையிடும் போது காரணவெண்கள்

$$(1-x^2) 2 (dy/dx) (d^2y/dx^2) + (dy/dx)^2 (-2x) = -p^2 2y dy/dx$$

2 (dy/dx) - என இரண்டு பக்கமும் வகையிட

$$(1-x^2) d^2y/dx^2 - x (dy/dx) = -p^2 y$$

$$(1-x^2) y_2 - x y_1 + p^2 y = 0$$

இருபுறமும் தேற்றம் - n-ஆம் வகையிடும் போது :-

இங்கு x எனப்படும் U மற்றும் V என சுருக்கம்,

n எனப்படும் வகையிடும் போது எனில்,

$$D^n(UV) = U_n V + n C_1 U_{n-1} V_1 + \dots + n C_r U_{n-r} V_r + \dots + U V_n$$

①  $y = \sin(m \sin^{-1} x)$  எனில்  $(1-x^2) y_{n+2} - (2n+1) x y_{n+1} + (m^2 - n^2) y_n = 0$

தீர்வு:

$$y = \sin(m \sin^{-1} x) \Rightarrow \sin^{-1} y = m \sin^{-1} x \rightarrow \textcircled{2}$$

↳ ①

'x' - மீது வகையிடும் போது (2) - வகையிடும் போது காரணவெண்கள்

$$\frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = m \cdot \frac{1}{\sqrt{1-x^2}}$$

இதையெழுத்து வர்க்கப்படுத்துவது

$$(1-x^2) (dy/dx)^2 = m^2 (1-y^2)$$

இருபுறமும் 'x' மீது வகையிடும் போது காரணவெண்கள்

$$(1-x^2) d^2y/dx^2 - x dy/dx = -m^2 y$$

$$(1-x^2) y_2 - x y_1 + m^2 y = 0$$

$$(1-x^2) y_2 - x y_1 + m^2 y = 0 \rightarrow \textcircled{3}$$

இருபுறமும் தேற்றம் (3) - ன் பக்கப்படுத்துவது



$$(1-x^2)y_{n+2} + nc_1 y_{n+1} (-2x) + nc_2 (-2)y_n -$$

$$(xy_{n+1} + nc_1 y_n) + m^2 y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - 2nxy_{n+1} + \frac{n(n-1)}{2} (-2)y_n -$$

$$(xy_{n+1}) - (ny_n) + m^2 y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

②  $y = a \cos(\log x) + b \sin(\log x)$  எனில்  
 $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$  என நினைக்க.

நினைவு :-  $y = a \cos(\log x) + b \sin(\log x)$

'x' மூலம் வகையிடும் போது கிடைக்கும்

$$y_1 = -a \sin(\log x) \cdot \frac{1}{x} + b \cos(\log x) \cdot \frac{1}{x}$$

$$xy_1 = -a \sin(\log x) + b \cos(\log x)$$

மீண்டும் 'x' மூலம் வகையிடும் போது கிடைக்கும்.

$$xy_2 + y_1 = -a \cos(\log x) \cdot \frac{1}{x} - b \sin(\log x) \cdot \frac{1}{x}$$

$$xy_2 + y_1 = -\frac{1}{x}(a \cos(\log x)) + b \sin(\log x)$$

$$x^2 y_2 + xy_1 = -y$$

$$\Rightarrow x^2 y_2 + xy_1 + y = 0$$

வகையிடும் போது கிடைக்கும்

$$x^2 y_{n+2} + nc_1 y_{n+1} 2x + nc_2 y_n (2) + xy_{n+1} + nc_1 y_n + y_n = 0$$

$$x^2 y_{n+2} + 2nxy_{n+1} + \frac{n(n-1)}{2} (2)y_n + xy_{n+1} + ny_n + y_n = 0$$

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

③  $y = (x + \sqrt{1+x^2})^m$ , எனில்  
 $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$  என நினைக்க.

நினைவு :-  $y = (x + \sqrt{1+x^2})^m \rightarrow ①$

'x' மூலம் வகையிடும் போது கிடைக்கும்

$$y_1 = m(x + \sqrt{1+x^2})^{m-1} + \left(1 + \frac{1}{2\sqrt{1+x^2}} \cdot 2x\right)$$

$$\sqrt{1+x^2} y_1 = m(x + \sqrt{1+x^2})^{m-1}$$

$$\sqrt{1+x^2} y_1 = my \quad (\text{மேல்க்கண்ட (1) - ன் வகையிடும் போது})$$

மீண்டும் வகையிடும் போது

$$(1+x^2)y_1^2 = m^2 y^2$$

'x' மூலம் வகையிடும் போது கிடைக்கும்

$$(1+x^2)2y_1 y_2 + y_1^2 (2x) = 2m^2 y y_1$$



2y, එහි වෙනස්කම

$$(1+x^2)y_2 + xy_1 - m^2y = 0$$

ව්‍යුත්පන්න වෙනස්කම උපයෝජනය

$$(1+x^2)y_{n+2} + n c_1 y_{n+1} (2x) + n c_2 y_n (2) + xy_{n+1} + y_n - m^2y_n = 0$$

$$(1+x^2)y_{n+2} + 2nx y_{n+1} + \frac{n(n-1)}{2} 2y_n + xy_{n+1} + ny_n - m^2y_n = 0$$

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

④  $y^{1/m} + y^{-1/m} = 2x$ , එහි,

$$(x^2-1)y_{n+2} + (n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

ඒය :-  $y^{1/m} + y^{-1/m} = 2x \rightarrow ①$

'x' ධන වෙනස්කම ව්‍යවහාරය කරමු

$$y^{1/m} = z ; y^{-1/m} = 1/z$$

$$① \Rightarrow z + 1/z = 2x ; \frac{z^2+1}{z} = 2x$$

$$z^2+1 = 2zx \Rightarrow z^2 - 2zx + 1 = 0$$

$$z = \frac{2x \pm \sqrt{4x^2-4}}{2} = \frac{2(x \pm \sqrt{x^2-1})}{2} = x \pm \sqrt{x^2-1}$$

$$y^{1/m} = (x \pm \sqrt{x^2-1}) \Rightarrow y = (x \pm \sqrt{x^2-1})^m \rightarrow ②$$

(2) - ධන 'x' ධන වෙනස්කම ව්‍යවහාරය කරමු

$$y_1 = M (x \pm \sqrt{x^2-1})^{m-1} \left(1 + \frac{2x}{2\sqrt{x^2-1}}\right)$$

$$= M (x \pm \sqrt{x^2-1})^{m-1} \left(1 + \frac{x}{\sqrt{x^2-1}}\right)$$

$$= M (x \pm \sqrt{x^2-1})^{m-1} \left(\frac{x + \sqrt{x^2-1}}{\sqrt{x^2-1}}\right)$$

$$y_1 = \frac{M (x \pm \sqrt{x^2-1})^m}{\sqrt{x^2-1}}$$

$$(\sqrt{x^2-1})y_1 = M (x \pm \sqrt{x^2-1})^m$$

$$(\sqrt{x^2-1})y_1 = My$$

වෙනස්කම ව්‍යවහාරය

$$(x^2-1)y_1^2 = M^2y^2$$

'x' ධන වෙනස්කම ව්‍යවහාරය කරමු

$$(x^2-1)2y_1y_2 + y_1^2(2x) = M^22yy_1$$

ව්‍යුත්පන්න වෙනස්කම උපයෝජනය

$$(x^2-1)y_{n+2} + n c_1 y_{n+1} (2x) + n c_2 y_n (2) + xy_{n+1} + y_n - m^2y_n = 0$$

$$(x^2-1)y_{n+2} + 2nx y_{n+1} + \frac{n(n-1)}{2} (2) y_n + xy_{n+1} + ny_n - m^2y_n = 0$$

$$(x^2-1)y_{n+2} + (n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$



විශාලතම අගයයන් සොයන්න :-

- ①  $f(x) = x^3 + 3x^2 + 6$  අන්තර්  $x > 2$  අන්තර්වලයේ ඉහළම අගයයන් සොයන්න. ඉහළම අගයයන් සොයන්න.

ඉන්න :-  $f(x) = x^3 - 3x^2 + 6$

$$f'(x) = 3x^2 - 6x = 3x(x-2)$$

$$f'(x) = 0 \Rightarrow 3x(x-2) = 0 \Rightarrow x-2 = 0 \Rightarrow x = 2$$

$x > 2$ , අන්තර්වලයේ ඉහළම අගයයන්,  $f'(x)$  අනුපාතයන්

$\therefore f(x)$  අන්තර් වශයෙන් අගයයන්,  $x > 2$  අන්තර්වලයේ සොයන්න.

$$f(2) = 8 - 12 + 6 = 2$$

$$f'(2) = 8 - 12 + 6 = 2$$

$\therefore x > 2$ ,  $f(x)$  අනුපාතයන්.

- ②  $x$  - හි අගය සොයන්න  $2x^3 - 9x^2 + 12x + 4$  හි ඉහළම අගයයන් සොයන්න.

ඉන්න :-

$$f(x) = 2x^3 - 9x^2 + 12x + 4$$

$$f'(x) = 6x^2 - 18x + 12$$

3 අගය සොයන්න

$$\Rightarrow 3x^2 - 6x + 4 = 0$$

$$\Rightarrow (x-1)(x-2) = 0$$

$$x-1 = 0 \quad (\text{or}) \quad x-2 = 0$$

$$\boxed{x=1} \quad (\text{or}) \quad \boxed{x=2}$$

' $x$ ' හි අගය සොයන්න 1 හා 2 අගය සොයන්න.  $f'(x)$  අගය සොයන්න අනුපාතයන් සොයන්න.  $f(x)$  අගය සොයන්න අගයයන් සොයන්න.

- ③  $x > 0$ , අගයයන්  $x - \frac{1}{2}x^2 < \log(1+x) < x$  අගයයන් සොයන්න.

ඉන්න :-

$$f(x) = x - \frac{1}{2}x^2 < \log(1+x) < x$$

$$f(x) = x - \frac{1}{2}x^2 - \log(1+x)$$

$$f'(x) = 1 - x - \frac{1}{1+x}$$

$$= \frac{1(1+x) - x(1+x) - 1}{(1+x)}$$

$$= \frac{1+x-x-x^2-1}{(1+x)}$$



$$= \frac{-x^2}{1+x}$$

'x' හිඟිඟුමකට වුනද  $f'(x)$  ට ඟහිඟුමකට.

$\therefore f(x)$  ට ඟුමුඟුම ඟහිඟුම.

$f(x)$  ට ඟහිඟුමකට,  $x=0$  ඟහිඟුම වුනද 0

$f(x)$  ට ඟහිඟුමකට.

$$x - \frac{1}{2}x^2 - \log(1+x)$$

$$\therefore x - \frac{1}{2}x^2 < \log(1+x)$$

$$F(x) = \log(1+x) < x$$

$$F(x) = \log(1+x) - x$$

$$F'(x) = \frac{1}{1+x} - 1$$

$$= \frac{1-1-x}{1+x} = \frac{-x}{1+x}$$

$\therefore$  'x' හිඟිඟුමකට වුනද,  $f(x)$  ට ඟහිඟුමකට.

$\therefore f(x)$  ට ඟුමුඟුම ඟහිඟුම.

$$F(0) = 0$$

$x > 0$ ,  $F(x)$  ට ඟහිඟුමකට.

$$\therefore \log(1+x) - x < 0$$

$$\therefore \log(1+x) < x$$

ඉඟුමකට,  $x - \frac{1}{2}x^2 < \log(1+x) < x$ .



பகுதி - II

வளைவற்ற வளைவுகள் வளைவற்றுடன் தொடர்பு :-

இரு வட்டங்கள்  $xy = 2$  வளைவு எடுத்துக் கொள்ள. வளைவற்ற இரு வட்டங்களின் வளைவு அளவு வளைவற்ற சீர்தரம்  $\rho$  ஆக இருக்கின்றது. இப்படி சீர்தரம்  $\rho$  வளைவற்ற வளைவு அளவு  $\rho$  அதிகமான திசை மாற்றம் இருக்கும். வேறு மாதிரியான சீர்தரம்  $\rho$  வளைவற்ற வளைவு அளவு  $\rho$  குறைவானது.

ஒரு வளைவு வளைவற்றுடன் தொடர்பு :-

ஒரு வளைவற்ற சீர்தரம்  $\rho$  வளைவற்றுடன் தொடர்பு  $\rho$  அதிகமானது. சீர்தரம்  $\rho$  வளைவற்ற வளைவு அளவு  $\rho$  அதிகமானது. சீர்தரம்  $\rho$  வளைவற்ற வளைவு அளவு  $\rho$  குறைவானது. சீர்தரம்  $\rho$  வளைவற்ற வளைவு அளவு  $\rho$  அதிகமானது.

வளைவற்ற வளைவுகள் தொடர்பு :-

கார்ட்டீசியன் தொடர்பு :-

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2}$$

(அளவு)

$$\rho = \frac{[1 + y_1^2]^{3/2}}{y_2}$$

①  $x^4 + y^4 = 2$  என்ற வளைவற்றுக்கு  $(1,1)$  என்ற புள்ளியில் தொடர்பு வளைவு அளவு காண.

தீர்வு :-  $x^4 + y^4 = 2$

'x' னை வளைவற்றுக்கு வளைவற்றுக்கு காணவேண்டும்

$$4x^3 + 4y^3 \frac{dy}{dx} = 0$$

$$4y^3 \frac{dy}{dx} = -4x^3$$

$$\frac{dy}{dx} = -\frac{4x^3}{4y^3} = -\frac{x^3}{y^3}$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = -1$$



$$\frac{d^2y}{dx^2} = - \frac{[3y^3x^2 - 3x^3y^2 \cdot dy/dx]}{y^6}$$

$$= - \frac{3y^2 [yx^2 + x^3]}{y^6} = - \frac{3}{y^4} (yx^2 + x^3)$$

$$\left(\frac{d^2y}{dx^2}\right)_{(1,1)} = - \frac{3}{1} (1+1) = -6$$

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2} = \frac{[1 + (-1)^2]^{3/2}}{-6}$$

$$= \frac{(1+1)^{3/2}}{-6} = \frac{2^{3/2}}{-6} = \frac{2^1 \cdot 2^{1/2}}{-6} = \frac{2^{1/2}}{-3}$$

$$\therefore \rho = \sqrt{2}/3$$

② 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024 2025 2026 2027 2028 2029 2030 2031 2032 2033 2034 2035 2036 2037 2038 2039 2040 2041 2042 2043 2044 2045 2046 2047 2048 2049 2050 2051 2052 2053 2054 2055 2056 2057 2058 2059 2060 2061 2062 2063 2064 2065 2066 2067 2068 2069 2070 2071 2072 2073 2074 2075 2076 2077 2078 2079 2080 2081 2082 2083 2084 2085 2086 2087 2088 2089 2090 2091 2092 2093 2094 2095 2096 2097 2098 2099 2100 2101 2102 2103 2104 2105 2106 2107 2108 2109 2110 2111 2112 2113 2114 2115 2116 2117 2118 2119 2120 2121 2122 2123 2124 2125 2126 2127 2128 2129 2130 2131 2132 2133 2134 2135 2136 2137 2138 2139 2140 2141 2142 2143 2144 2145 2146 2147 2148 2149 2150 2151 2152 2153 2154 2155 2156 2157 2158 2159 2160 2161 2162 2163 2164 2165 2166 2167 2168 2169 2170 2171 2172 2173 2174 2175 2176 2177 2178 2179 2180 2181 2182 2183 2184 2185 2186 2187 2188 2189 2190 2191 2192 2193 2194 2195 2196 2197 2198 2199 2200 2201 2202 2203 2204 2205 2206 2207 2208 2209 2210 2211 2212 2213 2214 2215 2216 2217 2218 2219 2220 2221 2222 2223 2224 2225 2226 2227 2228 2229 2230 2231 2232 2233 2234 2235 2236 2237 2238 2239 2240 2241 2242 2243 2244 2245 2246 2247 2248 2249 2250 2251 2252 2253 2254 2255 2256 2257 2258 2259 2260 2261 2262 2263 2264 2265 2266 2267 2268 2269 2270 2271 2272 2273 2274 2275 2276 2277 2278 2279 2280 2281 2282 2283 2284 2285 2286 2287 2288 2289 2290 2291 2292 2293 2294 2295 2296 2297 2298 2299 2300 2301 2302 2303 2304 2305 2306 2307 2308 2309 2310 2311 2312 2313 2314 2315 2316 2317 2318 2319 2320 2321 2322 2323 2324 2325 2326 2327 2328 2329 2330 2331 2332 2333 2334 2335 2336 2337 2338 2339 2340 2341 2342 2343 2344 2345 2346 2347 2348 2349 2350 2351 2352 2353 2354 2355 2356 2357 2358 2359 2360 2361 2362 2363 2364 2365 2366 2367 2368 2369 2370 2371 2372 2373 2374 2375 2376 2377 2378 2379 2380 2381 2382 2383 2384 2385 2386 2387 2388 2389 2390 2391 2392 2393 2394 2395 2396 2397 2398 2399 2400 2401 2402 2403 2404 2405 2406 2407 2408 2409 2410 2411 2412 2413 2414 2415 2416 2417 2418 2419 2420 2421 2422 2423 2424 2425 2426 2427 2428 2429 2430 2431 2432 2433 2434 2435 2436 2437 2438 2439 2440 2441 2442 2443 2444 2445 2446 2447 2448 2449 2450 2451 2452 2453 2454 2455 2456 2457 2458 2459 2460 2461 2462 2463 2464 2465 2466 2467 2468 2469 2470 2471 2472 2473 2474 2475 2476 2477 2478 2479 2480 2481 2482 2483 2484 2485 2486 2487 2488 2489 2490 2491 2492 2493 2494 2495 2496 2497 2498 2499 2500 2501 2502 2503 2504 2505 2506 2507 2508 2509 2510 2511 2512 2513 2514 2515 2516 2517 2518 2519 2520 2521 2522 2523 2524 2525 2526 2527 2528 2529 2530 2531 2532 2533 2534 2535 2536 2537 2538 2539 2540 2541 2542 2543 2544 2545 2546 2547 2548 2549 2550 2551 2552 2553 2554 2555 2556 2557 2558 2559 2560 2561 2562 2563 2564 2565 2566 2567 2568 2569 2570 2571 2572 2573 2574 2575 2576 2577 2578 2579 2580 2581 2582 2583 2584 2585 2586 2587 2588 2589 2590 2591 2592 2593 2594 2595 2596 2597 2598 2599 2600 2601 2602 2603 2604 2605 2606 2607 2608 2609 2610 2611 2612 2613 2614 2615 2616 2617 2618 2619 2620 2621 2622 2623 2624 2625 2626 2627 2628 2629 2630 2631 2632 2633 2634 2635 2636 2637 2638 2639 2640 2641 2642 2643 2644 2645 2646 2647 2648 2649 2650 2651 2652 2653 2654 2655 2656 2657 2658 2659 2660 2661 2662 2663 2664 2665 2666 2667 2668 2669 2670 2671 2672 2673 2674 2675 2676 2677 2678 2679 2680 2681 2682 2683 2684 2685 2686 2687 2688 2689 2690 2691 2692 2693 2694 2695 2696 2697 2698 2699 2700 2701 2702 2703 2704 2705 2706 2707 2708 2709 2710 2711 2712 2713 2714 2715 2716 2717 2718 2719 2720 2721 2722 2723 2724 2725 2726 2727 2728 2729 2730 2731 2732 2733 2734 2735 2736 2737 2738 2739 2740 2741 2742 2743 2744 2745 2746 2747 2748 2749 2750 2751 2752 2753 2754 2755 2756 2757 2758 2759 2760 2761 2762 2763 2764 2765 2766 2767 2768 2769 2770 2771 2772 2773 2774 2775 2776 2777 2778 2779 2780 2781 2782 2783 2784 2785 2786 2787 2788 2789 2790 2791 2792 2793 2794 2795 2796 2797 2798 2799 2800 2801 2802 2803 2804 2805 2806 2807 2808 2809 2810 2811 2812 2813 2814 2815 2816 2817 2818 2819 2820 2821 2822 2823 2824 2825 2826 2827 2828 2829 2830 2831 2832 2833 2834 2835 2836 2837 2838 2839 2840 2841 2842 2843 2844 2845 2846 2847 2848 2849 2850 2851 2852 2853 2854 2855 2856 2857 2858 2859 2860 2861 2862 2863 2864 2865 2866 2867 2868 2869 2870 2871 2872 2873 2874 2875 2876 2877 2878 2879 2880 2881 2882 2883 2884 2885 2886 2887 2888 2889 2890 2891 2892 2893 2894 2895 2896 2897 2898 2899 2900 2901 2902 2903 2904 2905 2906 2907 2908 2909 2910 2911 2912 2913 2914 2915 2916 2917 2918 2919 2920 2921 2922 2923 2924 2925 2926 2927 2928 2929 2930 2931 2932 2933 2934 2935 2936 2937 2938 2939 2940 2941 2942 2943 2944 2945 2946 2947 2948 2949 2950 2951 2952 2953 2954 2955 2956 2957 2958 2959 2960 2961 2962 2963 2964 2965 2966 2967 2968 2969 2970 2971 2972 2973 2974 2975 2976 2977 2978 2979 2980 2981 2982 2983 2984 2985 2986 2987 2988 2989 2990 2991 2992 2993 2994 2995 2996 2997 2998 2999 3000

ඉතිරි :-

$$y = c \cosh x/c$$

$$y_1 = c \sinh x/c \cdot 1/c$$

$$y_1 = \sinh x/c$$

$$y_2 = \cosh x/c \cdot 1/c$$

$$y_2 = 1/c \cosh x/c$$

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2}$$

$$= \frac{[1 + (\sinh x/c)^2]^{3/2}}{1/c \cdot \cosh x/c}$$

$$= \frac{c [\cosh^2 x/c]^{3/2}}{\cosh x/c} = \frac{c [\cosh x/c]^3}{\cosh x/c}$$

$$= c [\cosh x/c]^2$$

$$= c [y/c]^2$$

$$= c [y^2/c] = y^2/c \rightarrow \textcircled{1}$$



$$\begin{aligned}
 &= y [1 + (dy/dx)^2]^{1/2} \\
 &= c [\cosh x/c] [1 + (\sinh x/c)^2]^{1/2} \\
 &= c [\cosh x/c] [\cosh^2 x/c]^{1/2} \\
 &= c [\cosh x/c] \sqrt{\cosh^2 x/c} \\
 &= c [\cosh x/c] \cosh x/c \\
 &= y \times y/c \\
 &= y^2/c \rightarrow (2)
 \end{aligned}$$

① & ② - ni Gornunak

∴  $\frac{d^2y}{dx^2} = \frac{d^2y}{ds^2} \left( \frac{ds}{dx} \right)^2$

③  $\frac{1}{\rho}$   $\frac{d^2y}{ds^2}$   $\frac{ds}{dx}$   $\frac{ds}{dx} = \frac{1}{\sqrt{1 + y'^2}}$   $\frac{d^2y}{ds^2} = \frac{d}{ds} \left( \frac{dy}{ds} \right) \cdot \frac{ds}{dx}$   $\frac{1}{\rho} = \frac{x'y'' - y'x''}{(x')^2} \cdot \frac{1}{x'}$   $\frac{1}{\rho} = \frac{x'y'' - y'x''}{(x')^3} \cdot \frac{1}{\sqrt{1 + (y'/x')^2}}$   $\frac{1}{\rho} = \frac{x'y'' - y'x''}{(x')^3 \sqrt{1 + (y'/x')^2}}$

$$\frac{dy}{dx} = \frac{dy/ds}{dx/ds} = \frac{y'}{x'}$$

$$\begin{aligned}
 \frac{d^2y}{dx^2} &= \frac{d}{ds} \left( \frac{dy}{ds} \right) \cdot \frac{ds}{dx} \\
 &= \frac{d}{ds} \left( \frac{y'}{x'} \right) \cdot \frac{1}{dx/ds} \\
 &= \frac{x'y'' - y'x''}{(x')^2} \cdot \frac{1}{x'}
 \end{aligned}$$

$$\frac{d^2y}{ds^2} = \frac{x'y'' - y'x''}{(x')^3}$$

$$\frac{1}{\rho} = \frac{d^2y/ds^2}{[1 + (dy/dx)^2]^{3/2}}$$

$$= \frac{\frac{x'y'' - y'x''}{(x')^3}}{[1 + (y'/x')^2]^{3/2}}$$

$$= \frac{\frac{x'y'' - y'x''}{(x')^3}}{\frac{(x'^2 + y'^2)^{3/2}}{(x')^{3/2}}}$$



$$= \frac{x'y'' - y'x''}{(x')^3} \cdot \frac{(x')^3}{(x'^2 + y'^2)^{3/2}}$$

$$\frac{1}{\rho} = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}}$$

Q) Find the radius of curvature  $\rho$  of the curve  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$  at the point where the curve crosses the x-axis.

Soln :-

$$x = a(\theta + \sin \theta) \quad y = a(1 - \cos \theta)$$

$$x' = a(1 + \cos \theta) \quad y' = a \sin \theta$$

$$x'' = a(-\sin \theta) \quad y'' = a \cos \theta$$

$$x''' = -a \cos \theta$$

$$\frac{1}{\rho} = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}}$$

$$= \frac{a(1 + \cos \theta)(a \cos \theta) - a(1 + \cos \theta)(-a \sin \theta)}{[(a(1 + \cos \theta))^2 + (a \sin \theta)^2]^{3/2}}$$

$$= \frac{a^2 \cos \theta + a^2 \cos^2 \theta + a^2 \sin^2 \theta}{[a^2(1 + 2 \cos \theta + \cos^2 \theta + \sin^2 \theta)]^{3/2}}$$

$$= \frac{a^2 \cos \theta + a^2}{a^3(2 + 2 \cos \theta)^{3/2}} = \frac{a^2(1 + \cos \theta)}{a^3[2(1 + \cos \theta)]^{3/2}}$$

$$= \frac{a^2(1 + \cos \theta)}{a^3[2(1 + \cos \theta)]^{3/2}} = \frac{2 \cdot \cos^2 \theta / 2}{a(2 \cdot 2 \cos^2 \theta / 2)}$$

$$= \frac{2 \cos^2 \theta / 2}{a(2 \cos^2 \theta / 2)^{3/2}}$$

$$= \frac{1}{4a \cos \theta / 2}$$

$$\rho = 4a \cos \theta / 2$$



தேர்வு :-

ஒரே தரத்தில் 3-ஓ அளவிலான வளைவற்ற (வளைவற்றையின் ஆரம்)  $x$  லாஜியல்  $y$  அளவிலான அளவற்றகளுடன்,  $x$  லாஜியல்  $y$  ஓ அளவற்றகளிலானது லாஜியல் வேகம்  $\rho$  ஆகிறது

$$\rho = \frac{[1 + (dx/dy)^2]^{3/2}}{d^2x/dy^2}$$

இந்த வாய்ப்பாட்டை  $dy/dx$  லேயுடன் ஆக இடுகிற வேகம் பயன்படுகிறது.

①  $y^2 = x^3 + 8$  என்கிற வளைவற்றை  $(-2, 0)$  என்கிற அளவற்றகளுடன் ஆரம் வளைவற்ற ஆரம் காண்க.  
 தீர்வு :-

$$y^2 = x^3 + 8 \Rightarrow 2y \frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2y}$$

$$\left(\frac{dy}{dx}\right)_{(-2,0)} = \frac{3(-2)^2}{2(0)} = \frac{12}{0} = \infty$$

நிலைக்கு கிடைக்காத வேகம்  $dy/dx$  at  $(-2, 0)$  இல்  $\infty$  ஆகிறது,  $\rho$  - னை காண்பதற்காக,  $\rho$  லின் வாய்ப்பாடு பயன்படுகிறது.

$$\rho = \frac{[1 + (dx/dy)^2]^{3/2}}{d^2x/dy^2} ; \frac{dx}{dy} = \frac{2y}{3x^2} ; \left(\frac{dx}{dy}\right)_{(-2,0)} = \frac{0}{12} = 0$$

$$\frac{d^2x}{dy^2} = \frac{3x^2 \cdot 2 - 2y \cdot 6x \cdot dx/dy}{9x^4}$$

$$= \frac{3(-2)^2 \cdot 2 - 2(0)6(-2) \cdot 0}{9(-2)^4} = \frac{12}{72} = 1/6$$

$$\rho = \frac{[1 + (dx/dy)^2]^{3/2}}{d^2x/dy^2} = \frac{[1 + 0]^{3/2}}{1/6} = 6$$

$\rho = 6$

வளைவற்றையின் ஊடகத்தின் ஆரம் :-

$P(x, y)$  என்கிற அளவற்றை  $(C(x, y))$  ஆகிறது வளைவற்றையின் ஊடகம் என்கிறது



$$\bar{x} = x - \frac{y_1}{y_2} (1 + y_1^2)$$

$$\bar{y} = y + \frac{(1 + y_1^2)}{y_2}$$

ചത്തമവാക്യം തിരുത്തൽ സമവാക്യം :-

$$(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2$$

- ①  $xy = 2$  എന്ന ചത്തമവാക്യം  $(2, 1)$  എന്ന ഹിനിയുടെ ചത്തമവാക്യം തിരുത്തൽ സമവാക്യം കണ്ടെത്തുക.

കിഴിവ് :-

$$xy_1 + y(1) = 0$$

$$y_1 = -y/x \Rightarrow y_1(2,1) = -1/2$$

$$y_2 = - \left[ \frac{xy_1 - y(1)}{x^2} \right] \Rightarrow y_2(2,1) = - \left[ \frac{2(-1/2) - 1}{4} \right]$$

$$y_2 = 2/4 = 1/2$$

$$x = \frac{2 - (-1/2)[1 + (-1/2)^2]}{1/2} = \frac{12+1}{4} = \frac{13}{4}$$

$$y = \frac{1 + (1 + 1/4)}{1/2} = \frac{1 + 5/4}{1/2} = \frac{1 + 5}{2} = 7/2$$

$\therefore$  ചത്തമവാക്യത്തിന്റെ തിരുത്തൽ  $(x, y) = (13/4, 7/2)$

- ②  $y^2 = 4ax$  എന്ന തിരുത്തൽ  $t$  എന്ന ഹിനിയുടെ  $p = -2a(1+t^2)^{3/2}$ ,  $x = 2a + 3at^2$ ,  $y = -2at^3$  എന്നിവയുടെ തിരുത്തൽ സമവാക്യം കണ്ടെത്തുക.

കിഴിവ് :-

$$x = at^2 \Rightarrow dx/dt = 2at$$

$$y = 2at \Rightarrow dy/dt = 2a$$

$$y_1 = dy/dx = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

$$y_2 = d^2y/dx^2 = \frac{d}{dt} \left( \frac{dy}{dx} \right) \frac{dt}{dx}$$



$$= \frac{d}{dt} \left( \frac{1}{t} \right) \left( \frac{1}{2at} \right) = (-1)t^{-2} \cdot \frac{1}{2at} = -\frac{1}{t^2} \cdot \frac{1}{2at}$$

$$\frac{dy}{dx} = -\frac{1}{2at^3}$$

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2} = \frac{[1 + (1/4t^4)]^{3/2}}{-1/2at^3}$$

$$= -2at^3 [1 + 1/4t^4]^{3/2} = \frac{-2at^3(1+t^2)^{3/2}}{t^3}$$

$$= -2a(1+t^2)^{3/2}$$

$$x = \frac{x - y_1(1+y_1^2)}{y_2} = \frac{at^2 + 1/t [1 + (1/t)^2]}{1/2at^3}$$

$$x = at^2 + (t^2+1)2a = at^2 + 2at^2 + 2a$$

$$y = \frac{y + x(1+y^2)}{y_2} = 2at - \frac{(1+1/t^2)}{1/2at^3}$$

$$= 2at - (t^2+1)2at = 2at - 2at^3 + 2at$$

$$y = -2at^3$$

∴ මහජනගහණයේ මෙම අගයනය (2a + 3at<sup>2</sup>, -2at<sup>3</sup>)  
 සලකමු.

$$x = 2at + 3at^2$$

$$y = -2at^3$$

$$3at^2 = x - 2a$$

$$y = -2a(t^2)^{3/2}$$

$$t^2 = \frac{x-2a}{3a}$$

$$y = -2a \left( \frac{x-2a}{3a} \right)^{3/2}$$

$$y = -2a \left( \frac{x-2a}{3a} \right)^{3/2} \Rightarrow y^2 = \frac{4}{27a} (x-2a)^3$$

$$\Rightarrow (27a)y^2 = 4(x-2a)^3$$

එහෙයින් මෙම අගයනය සලකමු.

③  $x = a(\theta - \sin \theta)$ ;  $y = a(1 - \cos \theta)$  යන ආකාරයේ  
 - නම් එහෙයින් මෙම අගයනය සලකමු.  
 අනුමාන කිරීමේදී මෙම අගයනය සලකමු.



Ex 12 :-

$$x = a(\theta - \sin \theta)$$

$$y = a(1 - \cos \theta)$$

$$dx/d\theta = a(1 - \cos \theta)$$

$$dy/d\theta = a \sin \theta$$

$$dy/dx = \frac{dy/d\theta}{dx/d\theta} = \frac{a \sin \theta}{a(1 - \cos \theta)}$$

$$= \frac{a \cdot 2 \sin \theta/2 \cos \theta/2}{a \cdot 2 \sin^2 \theta/2} = \frac{\cos \theta/2}{\sin \theta/2}$$

$$y_1 = dy/dx = \cot \theta/2$$

$$y_2 = d^2y/dx^2 = \frac{d}{d\theta} (\cot \theta/2) \cdot \frac{1}{a(1 - \cos \theta)}$$

$$= -\operatorname{cosec}^2 \theta/2 \cdot \frac{1}{2} \cdot \frac{1}{a(2 \sin^2 \theta/2)}$$

$$y_2 = -\frac{1}{\sin^2 \theta/2} \cdot \frac{1}{2} \cdot \frac{1}{a(2 \sin^2 \theta/2)} = -\frac{1}{4a \sin^4 \theta/2}$$

$$x = \frac{x - y_1(1 + y_1^2)}{y_2} = \frac{a(\theta - \sin \theta) - (\cot \theta/2)(1 + \cot^2 \theta/2)^2}{-1/4a \sin^4 \theta/2}$$

$$= a(\theta - \sin \theta) + \left(\frac{\cos \theta/2}{\sin \theta/2}\right) \left(1 + \frac{\cos^2 \theta/2}{\sin^2 \theta/2}\right) (4a \sin^4 \theta/2)$$

$$= a(\theta - \sin \theta) + 4a \sin \theta/2 \cdot \cos \theta/2$$

$$= a(\theta - \sin \theta) + 2a \sin \theta$$

$$x = a[\theta + \sin \theta]$$

$$y = y + \frac{(1 + y_1^2)}{y_2} = a(1 - \cos \theta) + \frac{1 + \cot^2 \theta/2}{-1/4a \sin^4 \theta/2}$$

$$= a(1 - \cos \theta) - 4a \sin^4 \theta/2 \cdot \frac{1}{\sin^2 \theta/2}$$

$$= a(1 - \cos \theta) - 4a \sin^2 \theta/2$$

$$= a(1 - \cos \theta) - 2a(1 - \cos \theta)$$

$$y = a(1 - \cos \theta)$$

$$x = a[\theta + \sin \theta] \quad ; \quad y = a[1 - \cos \theta]$$



අනෙක් වක්‍රයක් ලබාදීමේදී දැන් අනෙක් වක්‍රයක් :-

ඒවූ වක්‍රයක් ලෙසින් ඒකාස්‍රීය වක්‍රයක්  
 අනිත්වයක් වක්‍රයක් ලෙසින් ක්‍රියා කරන  
 අනෙක් වක්‍රයක් ලෙසින් ලබාදීමේදී අනෙක් වක්‍රයක්  
 ලෙසින් අනෙක් වක්‍රයක් දැන් ලබාදීමක්  
 ලෙසින්

①  $r = a(1 - \cos \theta)$  - න් වක්‍රයේ ඒකාස්‍රීය වක්‍රයක් ලෙසින්  
 අනිත්වයක්  $\int r$  ලෙසින් ලබා දීම.

දී ඇත :-  $r = a(1 - \cos \theta) \rightarrow$  ①

' $\theta$ ' වෙනස් වීමේදී වක්‍රයේ අනිත්වය

$$\frac{dr}{d\theta} = a \sin \theta \quad \frac{d^2r}{d\theta^2} = a \cos \theta$$

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1r_2 - r_1r_2}$$

$$= \frac{[a^2(1 - \cos \theta)^2 + a^2 \sin^2 \theta]^{3/2}}{a^2(1 - \cos \theta)^2 + 2a^2 \sin^2 \theta - a(1 - \cos \theta) \cdot a \cos \theta}$$

$$= \frac{a^3 [1 - 2\cos \theta + \cos^2 \theta + \sin^2 \theta]^{3/2}}{a^2 [1 - 2\cos \theta + \cos^2 \theta + 2\sin^2 \theta - \cos \theta + \cos^2 \theta]}$$

$$= \frac{a(2 - 2\cos \theta)^{3/2}}{(3 - 3\cos \theta)} = \frac{a^2 \int 2(1 - \cos \theta)^{3/2}}{3(1 - \cos \theta)}$$

$$= \frac{2\sqrt{2}}{3} a \sqrt{\frac{r}{a}}$$

$$= \frac{2\sqrt{2} \int a}{3} \int r$$

②  $r^n = a^n \cos^n \theta$  වක්‍රයේ වක්‍රයේ වක්‍රයේ  
 $\frac{a^n r^{-n+1}}{n+1}$  ලෙසින් ලබා දීම.

දී ඇත :-

$$r^n = a^n \cos^n \theta$$



প্রমাণ করুন,  $\log a^x \cos \theta$

$$\log a^n = \log a^n \cos \theta$$

$$n \log a = \log a^n + \log \cos \theta$$

' $\theta$ '-এর সাপেক্ষে ডিফারেন্সিয়েশন করুন

$$\frac{n}{a} \cdot \frac{dr}{d\theta} = - \frac{n \sin \theta}{\cos^2 \theta}$$

$$\frac{dr}{d\theta} = -r \tan \theta$$

$$\frac{d^2 r}{d\theta^2} = - \frac{dr}{d\theta} \tan \theta - n r \sec^2 \theta$$

$$= r \tan^2 \theta - n r \sec^2 \theta$$

$$\rho = \frac{\{ r^2 + (dr/d\theta)^2 \}^{3/2}}{r^2 + 2(dr/d\theta) - r \frac{d^2 r}{d\theta^2}}$$

$$= \frac{r^2 + r^2 \tan^2 \theta}{r^2 + 2r^2 \tan^2 \theta - r^2 \tan^2 \theta + n r^2 \sec^2 \theta}$$

$$= \frac{r^3 \sec^3 \theta}{r^2 + r^2 \tan^2 \theta + n r^2 \sec^2 \theta}$$

$$= \frac{r \sec^3 \theta}{\sec^2 \theta + n \sec^2 \theta}$$

$$= \frac{r a^n}{(1+n)r^n}$$

$$= \frac{a^n r^{-n+1}}{(n+1)}$$



திரவேகாணம்

திரவேகாணம் திரவேகாணத்தின் பக்கவாக்கம்  
 மெய்யும் கோணவகம்ம கணிதவியக்க மட்டுமே  
 பயன்படுத்தப்படுகிறது. இப்போது எவ்வாறு திரவேகாணம்  
 பயன்படுத்தப்படுகிறது. எவ்வாறு உயர்மட்ட கல்வியில்  
 திரவேகாணம் பயன்படுத்தி விவரிக்கப்படுகிறது  
 அதன் விவரம் கீழ்க்கண்டவாறு விவரிக்கப்படுகிறது.

Sinnθ, cosnθ, tan nθ காரணம் :-

தேர்வு :-

- (i)  $\cos n\theta = \cos^n \theta - n c_2 \cos^{n-2} \theta \sin^2 \theta + \dots$
- (ii)  $\sin n\theta = n \cos^{n-1} \theta \sin \theta - n c_3 \cos^{n-3} \theta \sin^3 \theta + \dots$
- (iii)  $\tan n\theta = \frac{n c_1 \tan \theta - n c_3 \tan^3 \theta + \dots}{1 - n c_2 \tan^2 \theta + n c_4 \tan^4 \theta - \dots}$

எடுத்துக்காட்டு :-

①  $\frac{\sin 7\theta}{\sin \theta} = 7 - 56 \sin^2 \theta + 112 \sin^4 \theta - 64 \sin^6 \theta$ .

என நிரூபிக்க.

தீர்வு :-

$$\begin{aligned}
 (\cos 7\theta + i \sin 7\theta) &= (\cos \theta + i \sin \theta)^7 \\
 &= \cos^7 \theta + 7c_1 \cos^6 \theta (i \sin \theta) + 7c_2 \cos^5 \theta (i \sin \theta)^2 \\
 &\quad + 7c_3 \cos^4 \theta (i \sin \theta)^3 + 7c_4 \cos^3 \theta (i \sin \theta)^4 + \\
 &\quad + 7c_5 \cos^2 \theta (i \sin \theta)^5 + 7c_6 \cos \theta (i \sin \theta)^6 + \\
 &\quad + 7c_7 (i \sin \theta)^7 \\
 &= \cos^7 \theta + 7i \cos^6 \theta \sin \theta - 21 \cos^5 \theta \sin^2 \theta - \\
 &\quad - 35i \cos^4 \theta \sin^3 \theta - 35 \cos^4 \theta \sin^3 \theta + \\
 &\quad + 35 \cos^3 \theta \sin^4 \theta + 21i \cos^2 \theta \sin^5 \theta - \\
 &\quad - 7 \cos \theta \sin^6 \theta - i \sin^7 \theta
 \end{aligned}$$



ഉപയോഗിച്ച് തന്നെ ഉപയോഗിക്കുക.

$$\sin 7\theta = 7 \cos^6 \theta \sin \theta - 35 \cos^4 \theta \sin^3 \theta + 21 \cos^2 \theta \sin^5 \theta - \sin^7 \theta.$$

$$\frac{\sin 7\theta}{\sin \theta} = 7 \cos^6 \theta - 35 \cos^4 \theta \sin^2 \theta + 21 \cos^2 \theta \sin^4 \theta - \sin^6 \theta.$$

$$= 7(1 - \sin^2 \theta)^3 - 35(1 - \sin^2 \theta)^2 \sin^2 \theta + 21(1 - \sin^2 \theta) \sin^4 \theta - \sin^6 \theta.$$

$$= 7(1 - \sin^6 \theta - 3 \sin^2 \theta + 3 \sin^4 \theta) - 35(1 - 2 \sin^2 \theta + \sin^4 \theta) \sin^2 \theta + 21(1 - \sin^2 \theta) \sin^4 \theta - \sin^6 \theta.$$

$$= 7 - 7 \sin^6 \theta - 21 \sin^2 \theta + 21 \sin^4 \theta - 35 \sin^2 \theta + 70 \sin^4 \theta - 35 \sin^6 \theta + 21 \sin^4 \theta - 21 \sin^6 \theta - \sin^6 \theta.$$

$$= 7 - 64 \sin^6 \theta - 56 \sin^2 \theta + 112 \sin^4 \theta.$$

②  $\cos 8\theta = 128 \cos^8 \theta - 256 \cos^6 \theta + 16 \cos^4 \theta - 32 \cos^2 \theta + 1$   
 ന്നതെന്ന് കാണുക.

കണ്ടുക :-

$$\begin{aligned} (\cos 8\theta + i \sin 8\theta) &= (\cos \theta + i \sin \theta)^8 \\ &= \cos^8 \theta + 8C_1 \cos^7 \theta (i \sin \theta) + 8C_2 \cos^6 \theta (i \sin \theta)^2 \\ &+ 8C_3 \cos^5 \theta (i \sin \theta)^3 + 8C_4 \cos^4 \theta (i \sin \theta)^4 + \\ &8C_5 \cos^3 \theta (i \sin \theta)^5 + 8C_6 \cos^2 \theta (i \sin \theta)^6 + \\ &8C_7 \cos \theta (i \sin \theta)^7 + 8C_8 (i \sin \theta)^8 \end{aligned}$$

ഉപയോഗിച്ച് നമ്മെ ഉപയോഗിക്കുക.

$$\cos 8\theta = \cos^8 \theta - 28 \cos^6 \theta \sin^2 \theta + 70 \cos^4 \theta \sin^4 \theta - 28 \cos^2 \theta \sin^6 \theta + \sin^8 \theta.$$

$$= \cos^8 \theta - 28 \cos^6 \theta (1 - \cos^2 \theta) + 70 \cos^4 \theta (1 - \sin^2 \theta)^2 - 28 \cos^2 \theta (1 - \cos^2 \theta)^3 + (1 - \cos^2 \theta)^4.$$



$$\begin{aligned}
 &= \cos^8 \theta - 28 \cos^6 \theta + 28 \cos^4 \theta + 70 \cos^4 \theta - \\
 &\quad 140 \cos^6 \theta + 70 \cos^2 \theta - 28 \cos^2 \theta + \\
 &\quad 84 \cos^4 \theta - 84 \cos^6 \theta + 28 \cos^8 \theta + 1 - 4 \cos^2 \theta + \\
 &\quad 6 \cos^4 \theta - 4 \cos^6 \theta + \cos^8 \theta \\
 &= 128 \cos^8 \theta - 256 \cos^6 \theta + 160 \cos^4 \theta - 32 \cos^2 \theta + 1
 \end{aligned}$$

③  $\tan 7\theta = \frac{7 \tan \theta - 35 \tan^3 \theta + 21 \tan^5 \theta}{1 - 21 \tan^2 \theta + 35 \tan^4 \theta - 7 \tan^6 \theta}$

ଅଟେ ହେଉଛି .

ହିନ୍ଦ :-

$$\begin{aligned}
 \tan 7\theta &= \frac{7c_1 \tan \theta - 7c_3 \tan^3 \theta + 7c_5 \tan^5 \theta -}{1 - 7c_2 \tan^2 \theta + 7c_4 \tan^4 \theta - 7c_6 \tan^6 \theta} \\
 &= \frac{7 \tan \theta - 35 \tan^3 \theta + 21 \tan^5 \theta - \tan^7 \theta}{1 - 21 \tan^2 \theta + 35 \tan^4 \theta - 7 \tan^6 \theta}
 \end{aligned}$$

④ ଶିକ୍ଷକ ଚାହନ୍ତି କିପରି  $\sin^2(2\pi/7), \sin^2(4\pi/7), \sin^2(8\pi/7)$  କରନ୍ତୁ.

$$\cos(2\pi/7) + \cos(4\pi/7) + \cos(8\pi/7) = 1/2 \text{ ଅଟେ ହେଉଛି}$$

ହିନ୍ଦ :-

ଠିକ୍ ଅଟେ ଯେ ଶିକ୍ଷକ ଚାହନ୍ତି କିପରି  $\sin^2$  ର ମୂଲ୍ୟ ଖୋଜିବା ପାଇଁ  $2\pi/7$  (or)  $4\pi/7$  (or)  $8\pi/7$ .

$$\theta = 2\pi/7 \text{ (or) } 4\pi/7 \text{ (or) } 8\pi/7$$

$$7\theta = 2\pi \text{ (or) } 4\pi \text{ (or) } 8\pi$$

$$\therefore \sin 7\theta = 0$$

$$\Rightarrow 7 \cos^6 \theta \sin \theta - 35 \cos^4 \theta \sin^3 \theta + 21 \cos^2 \theta \sin^5 \theta - \sin^7 \theta = 0$$

$$7(1 - \sin^2 \theta)^3 - 35(1 - \sin^2 \theta)^2 \sin^2 \theta +$$

$$21(1 - \sin^2 \theta) \sin^4 \theta - \sin^6 \theta = 0$$

$\sin \theta = t$  ଅଟେ ଚାହନ୍ତି କିପରି



$$7(1-t^2)^3 - 35(1-t^2)t^2 + 21(1-t^2)t^4 - t^6 = 0$$

$t^2 = y$  என  $\sin$  மதிப்புகளை கொண்டு

$$7(1-y)^3 - 35(1-y^2)y + 21(1-y)y^2 - y^3 = 0$$

$$7(1-3y+3y^2-y^3) - 35(1-2y+y^2)y + 21(1-y)y^2 - y^3 = 0$$

$$7 - 21y + 21y^2 - 7y^3 - 35y + 70y^2 - 35y^3 + 21y^2 - 21y^3 - y^3 = 0$$

$$64y^3 - 112y^2 + 56y - 7 = 0$$

மூலங்களைக் கண்டுபிடிக்க.

$$\sin^2(2\pi/7) + \sin^2(4\pi/7) + \sin^2(8\pi/7) = \frac{112}{64}$$

$$\frac{1}{2}(1 - \cos 4\pi/7) + \frac{1}{2}(1 - \cos 8\pi/7) + \frac{1}{2}(1 - \cos 16\pi/7) = \frac{112}{64}$$

$$3/2 - \frac{1}{2}(\cos 4\pi/7 + \cos 8\pi/7 + \cos 16\pi/7) = 7/4$$

$$3 - (\cos 4\pi/7 + \cos 8\pi/7 + \cos 16\pi/7) = 7/2$$

$$3 - 7/2 = \cos 4\pi/7 + \cos 8\pi/7 + \cos 16\pi/7$$

$$\cos 4\pi/7 + \cos 8\pi/7 + \cos 16\pi/7 = -1/2$$

5)  $\tan 2\pi/7, \tan 4\pi/7, \tan 6\pi/7 = \sqrt{7}$  என நிரூபிக்க.

நிரூபிக்க :-

$\theta$  எனில்  $\tan \theta = \sqrt{7}$  எனக் கொள்வோம்.

$2\pi/7$  (அல்லது)  $4\pi/7$  (அல்லது)  $6\pi/7$ .

$$7\theta = 2\pi \text{ (அல்லது) } 4\pi \text{ (அல்லது) } 6\pi$$

$$\tan 7\theta = 0$$

$$\frac{7c_1 \tan \theta - 7c_3 \tan^3 \theta + 7c_5 \tan^5 \theta - 7c_7 \tan^7 \theta}{1 - 7c_2 \tan^2 \theta + 7c_4 \tan^4 \theta - 7c_6 \tan^6 \theta} = 0$$

$$7 \tan \theta - 35 \tan^3 \theta + 21 \tan^5 \theta - \tan^7 \theta = 0$$



$$7(1-t^2)^3 - 35(1-t^2)t^2 + 21(1-t^2)t^4 - t^6 = 0$$

$$t^2 = y \text{ ආකාරයට ධාරණය කිරීම}$$

$$7(1-y)^3 - 35(1-y)^2y + 21(1-y)y^2 - y^3 = 0$$

$$7(1-3y+3y^2-y^3) - 35(1-2y+y^2)y + 21(1-y)y^2 - y^3 = 0$$

$$\tan(2\pi/7) = \tan(\pi - 5\pi/7) = -\tan 5\pi/7$$

$$\tan(4\pi/7) = \tan(\pi - 3\pi/7) = -\tan 3\pi/7$$

$$\tan(6\pi/7) = \tan(\pi - \pi/7) = -\tan \pi/7$$

$$\therefore \text{ධාරණය කිරීම } x^6 - 21x^4 + 35x^2 - 7 = 0$$

ධාරණය කිරීමේ 6 මූලයන්  $\pm \tan 2\pi/7, \pm \tan 4\pi/7, \pm \tan 6\pi/7$ .

$x^2 = y$  ආකාරයට ධාරණය කිරීමේ මූලයන් ධාරණය කිරීමේ මූලයන්,

$$y^3 - 21y^2 + 35y - 7 = 0$$

ධාරණය කිරීමේ මූලයන්  $\tan^2 2\pi/7, \tan^2 4\pi/7, \tan^2 6\pi/7$

$$\therefore \tan^2 2\pi/7 \cdot \tan^2 4\pi/7 \cdot \tan^2 6\pi/7 = 7$$

$$\tan 2\pi/7 \cdot \tan 4\pi/7 \cdot \tan 6\pi/7 = \sqrt{7}$$

$\sin^n \theta$  හා  $\cos^n \theta$  හි අර්ථ දැක්වීම :-

ආකාරය :-

$$\cos^n \theta = \frac{1}{2^{n-1}} \left[ \cos n\theta + n c_1 \cos(n-2)\theta + n c_2 \cos(n-4)\theta + \dots \right]$$

ආකාරය :-

①  $2^5 \cos^6 \theta = \cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10$  ආකාරයට

ආකාරය :-  $x = \cos \theta + i \sin \theta$

$$x + 1/x = 2 \cos \theta \quad ; \quad x^n + 1/x^n = 2 \cos n\theta$$

$$(2 \cos \theta)^6 = (x + 1/x)^6$$



$$= 2x^6 + 6x^4 + 15x^2 + 20 + (15/x^2) + (6/x^4) + (1/x^6)$$

$$= (x^6 + 1/x^6) + 6(x^4 + 1/x^4) + 15(x^2 + 1/x^2) + 20$$

2 കൂടാൻ ഇങ്ങനെ പകർപ്പ് ചേർക്കുക.

$$2^5 \cos 6\theta = \cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10$$

②  $\sin^5 \theta = (1/2^4) [\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta]$

സമാന്തര്യം.

കൃത്യ :-

$$x = \cos \theta + i \sin \theta$$

$$(2i \sin \theta)^5 = (x - 1/x)^5$$

$$= x^5 - 5C_1 x^4 \cdot 1/x + 5C_2 x^3 \cdot 1/x^2 - 5C_3 x^2 \cdot 1/x^3 +$$

$$5C_4 x - 1/x^4 - 5C_5 x/x^5$$

$$i^5 2^5 \sin^5 \theta = x^5 - 5x^3 + 10x - 10/x + 5/x^3 - 1/x^5$$

$$i^5 2^5 \sin^5 \theta = (x^5 - 1/x^5) - 5(x^3 - 1/x^3) + 10(x - 1/x)$$

ഇങ്ങനെ പകർപ്പ് 2i കൂടാൻ ചേർക്കുക, കൂടാൻ (2) ന്

$$i^2 2^5 \sin^5 \theta = 2i \sin^5 \theta - 5 \times 2i \sin 3\theta + 10 \times 2i \sin \theta \rightarrow \textcircled{2}$$

$$\sin^5 \theta = \frac{1}{2^4} [\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta]$$

③  $\sin^3 \theta \cos^5 \theta = x$ ,  $\theta$ -സി ക്രമീകരിക്കുന്ന സാധനം  
 തിരിച്ചറിയാൻ ചർച്ച ചെയ്യേണ്ട.

കൃത്യ :-

$$x = \cos \theta + i \sin \theta$$

$$x + 1/x = 2 \cos \theta ; x - 1/x = 2i \sin \theta$$

$$(2 \cos \theta)^5 (2i \sin \theta)^3 = (x + 1/x)^5 (x - 1/x)^3$$

$$- 2^8 i \cos^5 \theta \sin^3 \theta = (x + 1/x)^5 (x - 1/x)^3$$

$$= [(x + 1/x)^2] [(x + 1/x)(x - 1/x)]^3$$

$$= (x + 1/x)^2 [x^2 - 1/x^2]^3$$



$$= x^2 + 2 + \frac{1}{x^2} \left[ x^6 - \frac{1}{x^6} - 3x^2 + \frac{3}{x^2} \right]$$

$$= x^8 - \frac{1}{x^4} - 3x^4 + 3 + 2x^6 - \frac{2}{x^6} - 6x^2 +$$

$$6/x^2 + x^4 - 1/x^8 - 3 + 3/x^4$$

$$= (x^8 - 1/x^8) + 2(x^6 - 1/x^6) - 2(x^4 - 1/x^4) - 6(x^2 - 1/x^2)$$

$$= 2i \sin 8\theta + 2 \times 2i \sin 6\theta - 2 \times 2i \sin 4\theta - 6 \times 2i \sin 2\theta$$

ഇങ്ങനെ പദകളും  $2i$  കൊണ്ട്  $x$  കൊണ്ട് ചുറ്റിക്കഴിയും.

$$\cos^5 \theta \sin^3 \theta = \frac{-1}{27} \left[ \sin 8\theta + 2 \sin 6\theta - 2 \sin 4\theta - 6 \sin 2\theta \right]$$

cos  $\theta$ , sin  $\theta$  ശമ്പുൾ tan  $\theta$  ന്റെ തിരിച്ചറിയലും ഉ-ന്

എടുക്കുന്നതിന് ശ്രമം കണ്ടു.

ഭേദനം :-

$$(i) \sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$(ii) \cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$$

$$(iii) \tan \theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \dots$$

എടുക്കുന്നതിന് :-

①  $\theta$ -ന് ഇരുപത്തു ഭാഗമായി  $\theta$  എടുക്കാൻ കണ്ടു,  $\frac{\sin \theta}{\theta} = \frac{863}{864}$ .

ശ്രമം :-

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$\frac{\sin \theta}{\theta} = 1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} - \dots$$

$$\frac{863}{864} = 1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} - \dots$$



$$1 - \frac{1}{864} = 1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!}$$

$$\frac{1}{864} = \frac{\theta^2}{3!} - \frac{\theta^4}{5!} + \dots$$

$$\frac{\theta^2}{3!} = \frac{1}{864} \Rightarrow \theta^2 \approx \frac{6}{864} = \frac{1}{144}$$

$\therefore \theta = 1/12$  రెడ్యుషన్.

②  $\cos(\alpha + \alpha) = \cos \alpha - \alpha \sin \alpha - \frac{\alpha^2}{2} \cos \alpha + \frac{\alpha^3}{6} \sin \alpha$   
 కనుగొనండి :-

$$\cos(\alpha + \alpha) = \cos \alpha \cos \alpha - \sin \alpha \sin \alpha$$

$$= \cos \alpha \left[ 1 - \frac{\alpha^2}{2!} + \dots \right] - \sin \alpha \left[ \alpha - \frac{\alpha^3}{3!} + \dots \right]$$

$$= \cos \alpha - \alpha \sin \alpha - \frac{\alpha^2}{2} \cos \alpha + \frac{\alpha^3}{6} \sin \alpha$$

③ కనుగొని  $\cos(\pi/3 + \theta) = 0.49$ .

కనుగొనండి :-

$$\cos(\pi/3 + \theta) = 0.49$$

$$\cos \pi/3 \cos \theta - \sin \pi/3 \sin \theta = 0.49$$

$$\frac{1}{2} (1 - \theta^2/2! + \dots) - \frac{\sqrt{3}}{2} (\theta - \theta^3/3! + \dots) = 0.49$$

$$\frac{1}{2} - \frac{\sqrt{3}}{2} \theta = 0.49$$

$$\frac{\sqrt{3}}{2} \theta = \frac{1}{2} - 0.49 = 1/100$$

$$\theta = \frac{1}{50\sqrt{3}} = \frac{\sqrt{3}}{150} \approx \frac{1.732}{150} = 0.115 \text{ రెడ్యుషన్}$$

$\therefore \theta = 40$  నిమిషాలు.

④  $\sin 3^\circ$  విలువను గా

కనుగొనండి :-

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$180^\circ = \pi \text{ రెడ్యుషన్} \Rightarrow 1^\circ = \pi/180 \text{ రెడ్యుషన్}$$

$$3^\circ = 3\pi/180 \text{ రెడ్యుషన్} \Rightarrow \pi/60 \text{ రెడ్యుషన్}$$



$$\begin{aligned}\sin 3^\circ &= \sin(\pi/60) = \pi/60 - \frac{1}{3!}(\pi/60)^3 + \dots \\ &= \pi/60 \\ &= \frac{22}{7} \times \frac{1}{60} = 0.052.\end{aligned}$$

⑤  $x = \frac{2}{1!} - \frac{4}{3!} + \frac{6}{5!} - \frac{8}{7!}$  மற்றும்  $y = 1 + \frac{2}{1!} - \frac{2^3}{3!} + \frac{2^5}{5!}$

எனவே இருந்தால்  $x^2 = y$  என நிரூபிக்க.

தீர்வு :-

$$x = \frac{1+1}{1!} - \frac{3+1}{3!} + \frac{5+1}{5!} - \frac{7+1}{7!} + \dots$$

$$= \left( \frac{1}{1!} - \frac{3}{3!} + \frac{5}{5!} - \frac{7}{7!} \right) + \left( \frac{1}{1!} - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \dots \right)$$

$$= \left( \frac{1}{1!} - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} \right) + \left( \frac{1}{1!} - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \dots \right)$$

$$= \cos 1 + \sin 1$$

$$x^2 = (\cos 1 + \sin 1)^2$$

$$x^2 = \cos^2 1 + \sin^2 1 + 2 \sin 1 \cos 1$$

$$x^2 = 1 + \sin 2$$

$$x^2 = 1 + \left( \frac{2}{1!} - \frac{2^3}{3!} + \frac{2^5}{5!} - \dots \right)$$

$$x^2 = y$$

⑥  $\theta$ -யின் மதிப்பு சிந்தனையில்,  $\theta \cot \theta = 1 - \frac{\theta^2}{3} - \frac{\theta^4}{45}$

எனவே நிரூபிக்க.

தீர்வு :-

$$\theta \cot \theta = \frac{\theta}{\tan \theta} = \frac{\theta}{\theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15}}$$

$$= \frac{\theta(1)}{\theta \left( 1 + \frac{\theta^2}{3} + \frac{2\theta^4}{15} \right)} = \frac{1}{1 + \frac{\theta^2}{3} + \frac{2\theta^4}{15}}$$

$$= \left( 1 + \frac{\theta^2}{3} + \frac{2\theta^4}{15} \right)^{-1}$$



$$\begin{aligned}
 &= 1 - \left( \frac{0^2}{3} + \frac{2 \cdot 0^4}{15} \right) + \left( \frac{0^2}{3} + \frac{2 \cdot 0^4}{15} \right)^2 - \dots \\
 &= 1 - \frac{0^2}{3} - \left( \frac{2}{15} - \frac{1}{9} \right) 0^4 \\
 &= 1 - \frac{0^2}{3} - \frac{0^4}{45}
 \end{aligned}$$

7)  $\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x - \sin x} = 24$  का मान ज्ञात करें।

हल :-

$$\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x - \sin x}$$

$$= \lim_{x \rightarrow 0} \left[ \frac{3 \left( \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} \dots \right) - \left( \frac{3x}{1!} - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} \dots \right)}{x - \left( x/1! - x^3/3! + x^5/5! \dots \right)} \right]$$

$$= \lim_{x \rightarrow 0} \left[ \frac{\left( -x^3/2 + 27x^3/6 \right) - \left( \frac{3x^5}{5!} - \frac{(3x)^5}{5!} + \dots \right)}{x^3/3! - x^5/5! + \dots} \right]$$

$$= \lim_{x \rightarrow 0} \left[ \frac{\left( -1/2 + 9/2 \right) - \left( \frac{3}{5!} - \frac{3^5 x^2}{5!} \right) + \dots}{1/3! - x^2/5!} \right]$$

$$= 24.$$

8)  $\lim_{x \rightarrow 0} \left( \frac{\cos^2 ax - \cos^2 bx}{1 - \cos cx} \right) = 2 \left( \frac{b^2 - a^2}{c^2} \right)$

हल :-

$$\lim_{x \rightarrow 0} \left( \frac{\cos^2 ax - \cos^2 bx}{1 - \cos cx} \right) = \lim_{x \rightarrow 0} \left( \frac{(\cos ax)^2 - (\cos bx)^2}{1 - \cos cx} \right)$$

$$= \lim_{x \rightarrow 0} \left[ \frac{\left( 1 - \frac{a^2 x^2}{2} + \dots \right)^2 - \left( 1 - \frac{b^2 x^2}{2} + \dots \right)^2}{1 - \left( 1 - \frac{c^2 x^2}{2!} + \dots \right)} \right]$$



$$= \lim_{x \rightarrow 0} \left[ \frac{\left(1 - \frac{2a^2x^2}{2!} + \frac{a^4x^4}{(2!)^2} + \dots\right) - \left(1 - \frac{2b^2x^2}{2!} + \frac{b^4x^4}{2!} + \dots\right)}{c^2x^2/2!} \right]$$

$$= \lim_{x \rightarrow 0} \left[ \frac{0 + \frac{(-2a^2 + 2b^2)x^2}{2!}}{c^2x^2/2!} \right]$$

$$= \lim_{x \rightarrow 0} \left[ \frac{\frac{2(b^2 - a^2)x^2}{2!}}{c^2x^2/2!} \right]$$

$$= \lim_{x \rightarrow 0} \left[ \frac{x^2(b^2 - a^2)}{x^2 \left(\frac{c^2}{2!}\right)} \right]$$

$$= \frac{b^2 - a^2}{c^2/2}$$

$$= \frac{2(b^2 - a^2)}{c^2}$$



වසන්ත වර්තමාන

වසන්ත වර්තමාන අගයන් වනුයේ

- වර්තමාන

- ①  $\sinh x = \frac{e^x - e^{-x}}{2}$
- ②  $\cosh x = \frac{e^x + e^{-x}}{2}$
- ③  $\tanh x = \frac{\sinh x}{\cosh x}$
- ④  $\coth x = \frac{\cosh x}{\sinh x}$
- ⑤  $\operatorname{cosech} x = \frac{1}{\sinh x}$
- ⑥  $\operatorname{sech} x = \frac{1}{\cosh x}$

පරීක්ෂණ :-

①  $\cosh^2 x - \sinh^2 x = 1$

අනුමාන :-

$$\begin{aligned} \cosh^2 x - \sinh^2 x &= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 \\ &= \frac{e^{2x} + e^{-2x} + 2 - e^{2x} - e^{-2x} + 2}{4} \\ &= \frac{4}{4} = 1 \end{aligned}$$

②  $2 \sinh x \cosh x = \sinh 2x$

අනුමාන :-

$$\begin{aligned} 2 \sinh x \cosh x &= 2 \left[ \left(\frac{e^x - e^{-x}}{2}\right) \left(\frac{e^x + e^{-x}}{2}\right) \right] \\ &= \frac{2}{4} [e^x e^x + e^x e^{-x} - e^{-x} e^x - e^{-x} e^{-x}] \\ &= \frac{1}{2} [e^{2x} - e^{-2x}] = \sinh 2x. \end{aligned}$$

③  $\cosh^2 x + \sinh^2 x = \cosh 2x$

අනුමාන :-

$$\begin{aligned} \cosh^2 x + \sinh^2 x &= \left(\frac{e^x + e^{-x}}{2}\right)^2 + \left(\frac{e^x - e^{-x}}{2}\right)^2 \\ &= \frac{1}{4} [(e^{2x} + e^{-2x} + 2e^x e^{-x}) + (e^{2x} + e^{-2x} + 2e^x e^{-x})] \\ &= \frac{1}{4} [2e^{2x} + 2e^{-2x}] = \frac{2}{4} [e^{2x} + e^{-2x}] \\ &= \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x. \end{aligned}$$



(7)

(6) 1 ⇒ cosh<sup>2</sup>x - sinh<sup>2</sup>x = 1

cosh<sup>2</sup>x = 1 + sinh<sup>2</sup>x → (1)

(6) 3 ⇒ cosh<sup>2</sup>x + sinh<sup>2</sup>x = cosh 2x → (2)

(7) - x 2 - ñ ungn'g'g'g'g'

1 + sinh<sup>2</sup>x + sinh<sup>2</sup>x = cosh 2x  
2 sinh<sup>2</sup>x = cosh 2x - 1  
sinh<sup>2</sup>x =  $\frac{\cosh 2x - 1}{2}$

(6) 4 ⇒ sinh<sup>2</sup>x = cosh<sup>2</sup>x - 1 → (3)

(8) - x 2 - ñ ungn'g'g'g'g'

cosh<sup>2</sup>x + cosh<sup>2</sup>x - 1 = cosh 2x  
2 cosh<sup>2</sup>x = cosh 2x + 1  
cosh<sup>2</sup>x =  $\frac{\cosh 2x + 1}{2}$

16ma 2mmw d'nniyam 16w 2uc d'nniyam 6w  
2mm unniyam :-

Gejym :-

- (i) sin ix = i sinh x
- (ii) cos ix = cosh x
- (iii) tan ix = i tanh x

Praty :-

(i) sin(ix) = (ix) -  $\frac{(ix)^3}{3!}$  +  $\frac{(ix)^5}{5!}$   
 = i  $\left[ x - \frac{i^2 x^3}{3!} + \frac{i^4 x^5}{5!} \right]$   
 = i  $\left[ x + \frac{x^3}{3!} + \frac{x^5}{5!} \right]$   
 = i sinh x

(ii) cos(ix) = 1 -  $\frac{(ix)^2}{2!}$  +  $\frac{(ix)^4}{4!}$  -  $\frac{(ix)^6}{6!}$   
 = 1 -  $\frac{i^2 x^2}{2!}$  +  $\frac{i^4 x^4}{4!}$  -  $\frac{i^6 x^6}{6!}$



$$(ii) \tan(ix) = \frac{\sin(ix)}{\cos(ix)} = \frac{i \sinh x}{\cosh x} = i \tanh x$$

සබවුම් :-

- ①  $\sinh x = (1/i) (\sin(ix)) = -i \sin(ix)$
- ②  $\cosh x = \cos ix$
- ③  $\tanh x = -i \tan(ix)$

සුඛවිඳි ලබන අනුරෝධන :-

$$① \sinh^{-1} x = \log_e (x + \sqrt{x^2 + 1})$$

ප්‍රිය :-

$$y = \sinh^{-1} x \Rightarrow x = \sinh y$$

$$x = \frac{e^y - e^{-y}}{2} \Rightarrow 2x = e^y - e^{-y} \Rightarrow 2x = e^y - \frac{1}{e^y}$$

$$\Rightarrow 2x e^y = e^{2y} - 1$$

$$\Rightarrow e^{2y} - 2x e^y + 1 = 0$$

$$e^y = \frac{2x \pm \sqrt{(2x)^2 - 4(1)(-1)}}{2(1)} \Rightarrow e^y = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$$

$$= \frac{2x \pm 2(\sqrt{x^2 + 1})}{2} = \frac{2(x \pm \sqrt{x^2 + 1})}{2}$$

$$e^y = x \pm \sqrt{x^2 + 1}$$

සූඛවිඳි ලබන අනුරෝධන

$$\log e^y = \log_e (x \pm \sqrt{x^2 + 1})$$

$$y = \log_e (x \pm \sqrt{x^2 + 1})$$

$$② \text{අනුරෝධන,} \\ \cosh^{-1} x = \log_e (x + \sqrt{x^2 - 1})$$

$$③ \tanh^{-1} x = \frac{1}{2} \log \left( \frac{1+x}{1-x} \right)$$

ප්‍රිය :-

$$y = \tanh^{-1} x$$

$$x = \tanh y \Rightarrow x = \frac{\sinh y}{\cosh y}$$

$$x = \frac{e^y - e^{-y}}{e^y + e^{-y}} \Rightarrow x = \frac{e^y - e^{-y}}{e^y + e^{-y}}$$

$$x(e^y + e^{-y}) = e^y - e^{-y}$$

$$xe^y + xe^{-y} = e^y - e^{-y}$$

$$e^{-y}(1+x) = e^y - xe^y$$

$$1+x = e^{2y}(1-x) \Rightarrow e^{2y} = \frac{1+x}{1-x}$$

ଉତ୍ତର ଦେବା ପାଇଁ ଉଭୟ ପାର୍ଶ୍ଵକୁ ଲଗ୍ ନେବା,

$$\log e^{2y} = \log_e \left( \frac{1+x}{1-x} \right) \Rightarrow 2y = \log_e \left( \frac{1+x}{1-x} \right)$$

$$\Rightarrow y = \frac{1}{2} \log_e \left( \frac{1+x}{1-x} \right)$$

ଉଦାହରଣ ୧ :-

$$(1) (\cosh x + \sinh x)^n = \cosh nx + \sinh nx$$

ପ୍ରମାଣ :-

$$\begin{aligned} (\cosh x + \sinh x)^n &= (\cos ix + \sin ix)^n \\ &= \cos(ix) - i \sin(ix) \\ &= \cos i(nx) - i \sin i(nx) \\ &= \cosh nx + \sinh nx \end{aligned}$$

$$(2) \frac{1 + \tanh x}{1 - \tanh x} = \cosh 2x + \sinh 2x \text{ ଉପର ପ୍ରମାଣ ଦେବା.}$$

ପ୍ରମାଣ :-

$$\frac{1 + \tanh x}{1 - \tanh x} = \frac{1 + \left( \frac{\sinh x}{\cosh x} \right)}{1 - \left( \frac{\sinh x}{\cosh x} \right)} = \frac{\cosh x + \sinh x}{\cosh x - \sinh x}$$

$$= \left( \frac{\cosh x + \sinh x}{\cosh x - \sinh x} \right) \times \left( \frac{\cosh x + \sinh x}{\cosh x + \sinh x} \right)$$

$$= \frac{(\cosh x + \sinh x)^2}{\cosh^2 x - \sinh^2 x}$$



$$= \frac{\cosh^2 x + \sinh^2 x + 2 \sinh x \cosh x}{\cosh^2 x - \sinh^2 x}$$

$$= \cosh^2 x + \sinh^2 x$$

③  $\tan(a+ib) = x+iy$ ,  $\therefore \frac{x}{y} = \frac{\sin 2a}{\sinh 2b}$

எனவே கிடைக்கிறது.

கிடைக்கிறது :-

$$x+iy = \tan(a+ib) = \frac{\sin(a+ib)}{\cos(a+ib)}$$

$$= \frac{\sin(a+ib)}{\cos(a+ib)} \times \frac{\cos(a-ib)}{\cos(a-ib)} = \frac{\sin 2a + \sin(i2b)}{\cos 2a + \cos(i2b)}$$

$$= \frac{\sin 2a + i \sinh 2b}{\cos 2a + \cosh 2b}$$

இதிலிருந்து  $x$  மற்றும்  $y$  கிடைக்கிறது.

$$x = \frac{\sin 2a}{\cos 2a + \cosh 2b} \quad ; \quad y = \frac{\sinh 2b}{\cos 2a + \cosh 2b}$$

$$\frac{x}{y} = \frac{\sin 2a}{\sinh 2b}$$

④  $x+iy = \tan(A+iB)$  எனில்,  $x^2+y^2+2xy \cot 2A = 1$   
எனவே கிடைக்கிறது.

கிடைக்கிறது :-

$$x+iy = \tan(A+iB) \quad ; \quad x-iy = \tan(A-iB)$$

$$\cot 2A = \frac{1}{\tan 2A} = \frac{1}{\tan[(A+iB)+(A-iB)]}$$

$$= \frac{1 - \tan(A+iB) \tan(A-iB)}{\tan(A+iB) + \tan(A-iB)}$$

$$= \frac{1 - (x+iy)(x-iy)}{(x+iy) + (x-iy)} = \frac{1 - (x^2+y^2)}{2x}$$

$$2x \cot 2A = 1 - x^2 - y^2$$

$$\Rightarrow x^2 + y^2 + 2x \cot 2A = 1.$$

5)  $x + iy = \sin(A + iB)$  எனில்  $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

என நிரூபிக்க.

நீர்ச்சு :-

$$x + iy = \sin(A + iB)$$

$$= \sin A \cosh B + i \cos A \sinh B$$

$$= \sin A \cosh B + i \cos A \sinh B.$$

இருபுறமும் எய் பகுதியை கருவியாக எடுத்துக் கொள்ள

$$x = \sin A \cosh B \Rightarrow \cosh B = \frac{x}{\sin A} \rightarrow \textcircled{1}$$

$$y = \cos A \sinh B \Rightarrow \sinh B = \frac{y}{\cos A} \rightarrow \textcircled{2}$$

இருபுறமும் சதுரம் எடுத்துக்கொள்ள

$$\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = \cosh^2 B - \sinh^2 B.$$

$$\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1.$$

6)  $\cos(x + iy) = \cos x \cosh y - i \sin x \sinh y$ , எனில்  
 $\cos 2x + \cosh 2y = 2$  என நிரூபிக்க.

நீர்ச்சு :-

$$\cos \theta + i \sin \theta = \cos(x + iy)$$

$$= \cos x \cosh y - i \sin x \sinh y$$

$$= \cos x \cosh y - i \sin x \sinh y.$$

இருபுறமும் எய் பகுதியை கருவியாக எடுத்துக் கொள்ள

$$\cos \theta = \cos x \cosh y \rightarrow \textcircled{1}; \sin \theta = -\sin x \sinh y$$



⑧ ⑨ ⑩ ⑪ ⑫ ⑬ ⑭ ⑮ ⑯ ⑰ ⑱ ⑲ ⑳

$$\begin{aligned} \cos^2 \theta + \sin^2 \theta &= \cos^2 x \cosh^2 y + \sin^2 x \sinh^2 y \\ &= \cos^2 x \cosh^2 y + (1 - \cos^2 x) \sinh^2 y \\ &= \cos^2 x \cosh^2 y + \sinh^2 y - \cos^2 x \sinh^2 y \\ &= \cos^2 x (\cosh^2 y - \sinh^2 y) + \sinh^2 y \end{aligned}$$

$$\cos^2 \theta + \sin^2 \theta = \cos^2 x + \sinh^2 y$$

$$1 = \frac{1 + \cos 2x}{2} + \frac{\cosh 2y - 1}{2}$$

$$2 = 1 + \cos 2x + \cosh 2y - 1$$

$$2 = \cos 2x + \cosh 2y$$

⑦  $\cos \alpha + i \sin \alpha = \cos(\theta + i\phi)$  or  $\sin^2 \theta = \pm \sin \alpha$   
 or  $\sin^2 \theta = \pm \sin \alpha$   
 Find :-

$$\begin{aligned} \cos \alpha + i \sin \alpha &= \cos(\theta + i\phi) \\ &= \cos \theta \cosh \phi - i \sin \theta \sinh \phi \end{aligned}$$

$$\cos \alpha = \cos \theta \cosh \phi \quad ; \quad \sin \alpha = -\sin \theta \sinh \phi$$

$$\Rightarrow \cosh \phi = \frac{\cos \alpha}{\cos \theta} \quad ; \quad \sinh \phi = -\frac{\sin \alpha}{\sin \theta}$$

$$\Rightarrow \cosh^2 \phi = \frac{\cos^2 \alpha}{\cos^2 \theta} \quad ; \quad \sinh^2 \phi = \frac{\sin^2 \alpha}{\sin^2 \theta} \rightarrow \textcircled{2}$$

① - ②

$$\cosh^2 \phi - \sinh^2 \phi = \frac{\cos^2 \alpha}{\cos^2 \theta} - \frac{\sin^2 \alpha}{\sin^2 \theta}$$

$$1 = \frac{\cos^2 \alpha \sin^2 \theta - \sin^2 \alpha \cos^2 \theta}{\cos^2 \theta \sin^2 \theta}$$

$$\cos^2 \alpha \sin^2 \theta - \sin^2 \alpha \cos^2 \theta = \cos^2 \theta \sin^2 \theta$$

$$(1 - \sin^2 \alpha) \sin^2 \theta - \sin^2 \alpha (1 - \sin^2 \theta) = (1 - \sin^2 \theta) \sin^2 \theta$$

$$\begin{aligned} \sin^2 \theta - \sin^2 \theta \sin^2 \alpha - \sin^2 \alpha + \sin^2 \alpha \sin^2 \theta &= \sin^2 \theta - \sin^4 \theta \\ &= \sin^2 \theta - \sin^4 \theta \end{aligned}$$

$$\sin^2 \theta - \sin^2 \alpha - \sin^2 \theta = -\sin^4 \theta$$

$$-\sin^2 \alpha = -\sin^4 \theta$$

$$\therefore \sin^2 \theta = \pm \sin \alpha$$

$$\textcircled{8} \quad \cosh^7 x = \frac{1}{2^6} [\cosh 7x + 7 \cosh 5x + 21 \cosh 3x + 35 \cosh x]$$

Soln :-

$$\begin{aligned} \cosh^7 x &= \left( \frac{e^x + e^{-x}}{2} \right)^7 \\ &= \frac{1}{2^7} [e^{7x} + (7C_1 e^{6x} e^{-x}) + (7C_2 e^{5x} e^{-2x}) + \\ &\quad (7C_3 e^{4x} e^{-3x}) + (7C_4 e^{3x} e^{-4x}) + \\ &\quad (7C_5 e^{2x} e^{-5x}) + (7C_6 e^x e^{-6x}) + (7C_7 e^{-7x})] \\ &= \frac{1}{2^7} [ (e^{7x} + e^{-7x}) + 7(e^{5x} + e^{-5x}) + \\ &\quad 21(e^{3x} + e^{-3x}) + 35(e^x + e^{-x}) ] \\ &= \frac{1}{2^6} \left[ \frac{e^{7x} + e^{-7x}}{2} + \frac{7(e^{5x} + e^{-5x})}{2} + \frac{21(e^{3x} + e^{-3x})}{2} \right. \\ &\quad \left. + 35 \frac{(e^x + e^{-x})}{2} \right] \\ &= \frac{1}{2^6} [\cosh 7x + 7 \cosh 5x + 21 \cosh 3x + 35 \cosh x] \end{aligned}$$

$\textcircled{9}$  Show by using binomial expansion method that

(i)  $\sinh(\alpha + i\beta)$

Proof :-

$$x + iy = \sinh(\alpha + i\beta) = -i \sin i(\alpha + i\beta)$$

$$x + iy = -i \sin(i\alpha - \beta)$$

$$= -i (\sin i\alpha \cos \beta - \cos i\alpha \sin \beta)$$

$$= -i (i \sinh \alpha \cos \beta - \cosh \alpha \sin \beta)$$

$$x + iy = \sinh \alpha \cos \beta + i \cosh \alpha \sin \beta$$

$$x = \sinh \alpha \cos \beta ; y = \cosh \alpha \sin \beta$$

$$\text{Show} = \sinh \alpha \cos \beta ; \text{Argument} = \cosh \alpha \sin \beta.$$



(ii)  $\tan h(1+i)$

தீர்வு :-

$$x+iy = \frac{\sinh(1+i)}{\cosh(1+i)} = \frac{-i \sin(1+i)}{\cos i(1+i)} = \frac{-i \sin(i-1)}{\cos(i-1)}$$

$$= \frac{-i \cdot 2 \sin(i-1) \cos(i+1)}{2 \cos(i-1) \cos(i+1)} = \frac{-i [\sin 2i - \sin 2]}{-\cos 2i + \cos 2}$$

$$= \frac{-i [i \sinh 2 - \sin 2]}{\cosh 2 + \cos 2} = \frac{\sinh 2 + i \sin 2}{\cosh 2 + \cos 2}$$

$$\text{Real part} = \frac{\sinh 2}{\cosh 2 + \cos 2} \quad ; \quad \text{Imaginary part} = \frac{\sin 2}{\cosh 2 + \cos 2}$$

(iii)  $\tan^{-1}(x+iy)$

தீர்வு :-

$$\tan^{-1}(x+iy) = A+iB$$

$$\tan(A+iB) = x+iy \quad ; \quad \tan(A-iB) = x-iy$$

$$\tan 2A = \tan(A+iB) + \tan(A-iB)$$

$$= \frac{\tan(A+iB) + \tan(A-iB)}{1 - \tan(A+iB)\tan(A-iB)} = \frac{(x+iy) + (x-iy)}{1 - (x+iy)(x-iy)}$$

$$= \frac{2x}{1 - (x^2 + y^2)}$$

$$\therefore 2A = \tan^{-1}\left(\frac{2x}{1 - (x^2 + y^2)}\right)$$

$$\therefore \text{Real part} = \frac{1}{2} \tan^{-1}\left(\frac{2x}{1 - (x^2 + y^2)}\right)$$

உண்மை,

Imaginary part

$$\frac{1}{2} \tan^{-1}\left[\frac{2y}{1 + x^2 + y^2}\right]$$

$$(70) \quad \tan(\theta + i\phi) = \cos \alpha + i \sin \alpha \text{ or } e^{i\alpha}$$

$$(i) \quad \theta = \frac{1}{2} n\pi + \frac{1}{4} \pi \quad (ii) \quad \phi = \frac{1}{2} \log \tan \left( \frac{\pi}{4} + \frac{\alpha}{2} \right)$$

Priz :-

$$\tan(\theta + i\phi) = \cos \alpha + i \sin \alpha$$

$$\tan(\theta - i\phi) = \cos \alpha - i \sin \alpha$$

$$(i) \quad \text{Now } 2\theta = (\theta + i\phi) + (\theta - i\phi)$$

$$\tan 2\theta = \tan [(\theta + i\phi) + (\theta - i\phi)]$$

$$= \frac{\tan(\theta + i\phi) + \tan(\theta - i\phi)}{1 - \tan(\theta + i\phi) \tan(\theta - i\phi)}$$

$$= \frac{\cos \alpha + i \sin \alpha + \cos \alpha - i \sin \alpha}{1 - (\cos \alpha + i \sin \alpha)(\cos \alpha - i \sin \alpha)}$$

$$= \frac{2 \cos \alpha}{1 - (\cos^2 \alpha - i^2 \sin^2 \alpha)} = \frac{2 \cos \alpha}{1 - 1} = \frac{2 \cos \alpha}{0}$$

$$= \infty = \tan \frac{\pi}{2}$$

$$2\theta = n\pi + \frac{\pi}{2} \Rightarrow \theta = \frac{1}{2} \left( n\pi + \frac{\pi}{2} \right)$$

$$(ii) \quad \tan(2i\phi) = \tan [(\theta + i\phi) - (\theta - i\phi)]$$

$$= \frac{\tan(\theta + i\phi) - \tan(\theta - i\phi)}{1 - \tan(\theta + i\phi) \tan(\theta - i\phi)}$$

$$= \frac{(\cos \alpha + i \sin \alpha) - (\cos \alpha - i \sin \alpha)}{1 - (\cos \alpha + i \sin \alpha)(\cos \alpha - i \sin \alpha)}$$

$$= \frac{\cos \alpha + i \sin \alpha - \cos \alpha - i \sin \alpha}{1 - (\cos^2 \alpha - i^2 \sin^2 \alpha)}$$

$$= \frac{2i \sin \alpha}{1 + 1} = i \sin \alpha$$

$$i \tanh 2\phi = i \sin \alpha$$

$$\Rightarrow \tanh 2\phi = \sin \alpha$$

$$2\phi = \tanh^{-1}(\sin \alpha) \Rightarrow 2\phi = \frac{1}{2} \log \left( \frac{1 + \sin \alpha}{1 - \sin \alpha} \right)$$



$$= \frac{1}{2} \log \left[ \frac{1 + \frac{2 \tan \alpha/2}{1 + \tan^2 \alpha/2}}{1 - \frac{2 \tan \alpha/2}{1 + \tan^2 \alpha/2}} \right] = \frac{1}{2} \log \left[ \frac{(1 + \tan \alpha/2)^2}{(1 - \tan \alpha/2)^2} \right]$$

$$= 2 \cdot \frac{1}{2} \log \left[ \frac{1 + \tan \alpha/2}{1 - \tan \alpha/2} \right] = \log \left[ \frac{\tan \pi/4 + \tan \alpha/2}{1 - \tan \pi/4 \tan \alpha/2} \right]$$

$$2\phi = \log \left[ \tan \pi/4 + \alpha/2 \right]$$

$$\phi = \frac{1}{2} \log \left[ \tan \pi/4 + \alpha/2 \right]$$

(11)  $\tan \alpha/2 = \tanh \alpha/2$  . अतः  $\cos x \cosh x = 1$   
 समीकरण .

हल :-

$$\cosh x = \frac{1 + \tanh^2 x/2}{1 - \tanh^2 x/2} = \frac{1 + \tan^2 x/2}{1 - \tan^2 x/2}$$

$$= \frac{1 + \frac{\sin^2 x/2}{\cos^2 x/2}}{1 - \frac{\sin^2 x/2}{\cos^2 x/2}} = \frac{\frac{\cos^2 x/2 + \sin^2 x/2}{\cos^2 x/2}}{\frac{\cos^2 x/2 - \sin^2 x/2}{\cos^2 x/2}}$$

$$\therefore \cosh x = \frac{1}{\cos^2 x/2 - \sin^2 x/2}$$

$$= \frac{1}{\cos 2 \cdot x/2} = \frac{1}{\cos x}$$

$$\cosh x \cdot \cos x = 1$$

(12)  $u = \log_e \tan(\pi/4 + \theta/2)$  iff  $\cosh u = \sec \theta$

हल :-

$$\cosh u = \sec \theta \Rightarrow u = \cosh^{-1}(\sec \theta)$$

$$u = \log_e (\sec \theta + \sqrt{\sec^2 \theta - 1}) = \log_e (\sec \theta + \sqrt{\tan^2 \theta})$$

$$= \log_e (\sec \theta + \tan \theta) = \log_e \left( \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \right)$$

$$= \log_e \left( \frac{1 + \sin \theta}{\cos \theta} \right) = \log_e \left( \frac{1 + 2 \tan \theta/2 / 1 + \tan^2 \theta/2}{1 - \tan^2 \theta/2 / 1 + \tan^2 \theta/2} \right)$$

$$= \log_e \left( \frac{(1 + \tan \theta/2)^2}{1 - \tan^2 \theta/2} \right) = \log_e \left( \frac{1 + \tan \theta/2}{1 - \tan \theta/2} \right)$$

$$= \log_e \left[ \frac{\tan \pi/4 + \tan \theta/2}{1 - \tan \pi/4 \tan \theta/2} \right]$$

$$= \log_e [\tan (\pi/4 + \theta/2)]$$

अतः निम्नानुसारं,

$$u = \log_e \tan (\pi/4 + \theta/2)$$

$$e^u = \tan (\pi/4 + \theta/2)$$

$$e^{u/2} \cdot e^{u/2} = \tan (\pi/4 + \theta/2)$$

$$\frac{e^{u/2}}{e^{-u/2}} = \frac{\tan \pi/4 + \tan \theta/2}{1 - \tan \pi/4 \tan \theta/2} = \frac{1 + \tan \theta/2}{1 - \tan \theta/2}$$

$$\frac{e^{u/2} - e^{-u/2}}{e^{u/2} + e^{-u/2}} = \frac{1 + \tan \theta/2 - (1 - \tan \theta/2)}{1 + \tan \theta/2 + (1 - \tan \theta/2)}$$

$$= \frac{2 \tan \theta/2}{2} = \tan \theta/2$$

$$\frac{e^{u/2} - e^{-u/2}}{e^{u/2} + e^{-u/2}} = \tan \theta/2$$

$$\tanh (u/2) = \tan \theta/2$$

$$\cosh u = \frac{1 + \tanh^2 u/2}{1 - \tanh^2 u/2} = \frac{1 + \tan^2 \theta/2}{1 - \tan^2 \theta/2} = \frac{1 + \frac{\sin^2 \theta/2}{\cos^2 \theta/2}}{1 - \frac{\sin^2 \theta/2}{\cos^2 \theta/2}}$$

$$= \frac{\cos^2 \theta/2 + \sin^2 \theta/2}{\cos^2 \theta/2 - \sin^2 \theta/2} = \frac{1}{\cos^2 \theta/2 - \sin^2 \theta/2}$$

$$= \frac{1}{\cos 2 \cdot \theta/2} = \frac{1}{\cos \theta}$$

$$\cosh u = \sec \theta$$

(13)  $\cos (\alpha + i\gamma) = x (\cos \alpha + i \sin \alpha)$  माना है

$$y = 1/2 \log \left[ \frac{\sin (\alpha - \alpha)}{\sin (\alpha + \alpha)} \right] \text{ मान लिया है .}$$



Priz :-

$$\begin{aligned}\cos(x+iy) &= \cos x \cos iy - \sin x \sin(iy) \\ &= \cos x \cosh y - \sin x i \sinh y\end{aligned}$$

$$\cos x \cosh y - \sin x i \sinh y = r(\cos \alpha + i \sin \alpha)$$

ඉහත දෙකට වෙන් වෙන්, මෙහිදී අනුමාන වශයෙන්ම  
සමාන කර

$$r \cos \alpha = \cos x \cosh y \rightarrow (1)$$

$$r \sin \alpha = -\sin x \sinh y \rightarrow (2)$$

(2) ආ (1) ආවේ වෙනස් කරමු.

$$\frac{r \sin \alpha}{r \cos \alpha} = -\frac{\sin x \sinh y}{\cos x \cosh y} \Rightarrow \tan \alpha = -\tan x \tanh y$$

$$\tanh y = -\frac{\tan \alpha}{\tan x} \Rightarrow y = \tanh^{-1} \left( -\frac{\tan \alpha}{\tan x} \right)$$

$$y = \frac{1}{2} \log \left[ \frac{1 - \tan \alpha / \tan x}{1 + \tan \alpha / \tan x} \right] = \frac{1}{2} \log \left[ \frac{\tan x - \tan \alpha}{\tan x + \tan \alpha} \right]$$

$$= \frac{1}{2} \log \left[ \frac{\frac{\sin x}{\cos x} - \frac{\sin \alpha}{\cos \alpha}}{\frac{\sin x}{\cos x} + \frac{\sin \alpha}{\cos \alpha}} \right]$$

$$= \frac{1}{2} \log \left[ \frac{\sin x \cos \alpha - \sin \alpha \cos x}{\sin x \cos \alpha + \sin \alpha \cos x} \right]$$

$$y = \frac{1}{2} \log \left[ \frac{\sin(x-\alpha)}{\sin(x+\alpha)} \right]$$

ਸਰਲੀਕਰਣ :-

①  $\text{Log } i = i(4n+1)(\pi/2)$  ਸਮਝਾਓ .

ਪ੍ਰਮਾਣ :-

$$i = \cos(\pi/2) + i \sin(\pi/2)$$

$$\begin{aligned} \text{Log } i &= \log 1 + i[(\pi/2) + 2n\pi] \\ &= i[(\pi/2) + 2n\pi] \\ &= i(4n+1)(\pi/2) . \end{aligned}$$

② (i)  $\text{Log}(1+i)$  (ii)  $\text{Log}(-e)$  ਲੱਭੋ .

ਪ੍ਰਮਾਣ :-

(i)  $1+i = \sqrt{2} [\cos(\pi/4) + i \sin(\pi/4)]$

$$\begin{aligned} \therefore \text{Log}(1+i) &= \log \sqrt{2} + i(\pi/4) + 2n\pi i \\ &= \frac{1}{2} \log 2 + i(2n+1)(\pi/4) . \end{aligned}$$

(ii)  $-e = e(\cos \pi + i \sin \pi)$

$$\begin{aligned} \text{Log}(-e) &= \log e + i(\pi + 2n\pi) \\ &= 1 + i(2n+1)\pi . \end{aligned}$$

③  $i^i = e^{-(4n+1)\pi/2}$  ਸਮਝਾਓ .

ਪ੍ਰਮਾਣ :-

$$\begin{aligned} i^i &= e^{i \text{Log } i} = e^{i(4n+1)\pi/2} \\ &= e^{-(4n+1)\pi/2} . \end{aligned}$$

④  $i^{a+ib} = a+ib$  ਜੇਕਰ  $a^2+b^2 = e^{-(4n+1)\pi}$  .

ਪ੍ਰਮਾਣ :-

$$i^{a+ib} = a+ib \rightarrow \text{①}$$

$$\begin{aligned} i^{a+ib} &= e^{(a+ib) \text{Log } i} \\ &= e^{(a+ib)i(4n+1)(\pi/2)} \\ &= e^{(ai-b)(4n+1)(\pi/2)} \\ &= e^{ia(4n+1)\pi/2} e^{-b(4n+1)\pi/2} \end{aligned}$$



$$= e^{-b(4n+1)(\pi/2)} [\cos \theta + i \sin \theta]$$

$$a + ib = e^{-b(4n+1)(\pi/2)} [\cos \theta + i \sin \theta]$$

ഇതുകൊണ്ട്  $a$  ന്റെ  $b$  ന്റെ കണക്കുകൾ ഉണ്ടാക്കാം.

$$a = e^{-b(4n+1)(\pi/2)} \cdot \cos \theta$$

$$b = e^{-b(4n+1)(\pi/2)} \cdot \sin \theta$$

$$\therefore a^2 + b^2 = e^{-b(4n+1)\pi} = e^{-(4n+1)\pi b}$$

മുൻകരുതലുകൾ :-

1.  $\sin \theta \neq 0$  ആകണം :-

$$\textcircled{1} \frac{1}{\sin \theta \sin 2\theta} + \frac{1}{\sin 2\theta \sin 3\theta} + \dots \text{ (ന  $n$  വരെ)}$$

$$\operatorname{cosec} \theta \operatorname{cosec} 2\theta + \operatorname{cosec} 2\theta \operatorname{cosec} 3\theta + \dots$$

$n$ -ആം പദം വരെ കൂട്ടി കണക്കാക്കണം.

മുൻകരുതലുകൾ :-

$$S_n = \frac{1}{\sin \theta \sin 2\theta} + \frac{1}{\sin 2\theta \sin 3\theta} + \dots$$

$$T_n = \frac{1}{\sin n\theta \sin (n+1)\theta} = \frac{1}{\sin \theta} \left[ \frac{\sin \theta}{\sin n\theta \sin (n+1)\theta} \right]$$

$$= \frac{1}{\sin \theta} \left[ \frac{\sin (n+1)\theta - n\theta}{\sin n\theta \sin (n+1)\theta} \right]$$

$$= \frac{1}{\sin \theta} \left[ \frac{\sin (n+1)\theta \cos n\theta - \cos (n+1)\theta \sin n\theta}{\sin n\theta \sin (n+1)\theta} \right]$$

$$= \operatorname{cosec} \theta [\cot (n\theta) - \cot (n+1)\theta]$$

$$T_1 = \operatorname{cosec} \theta [\cot \theta - \cot 2\theta]$$

$$T_2 = \operatorname{cosec} \theta [\cot 2\theta - \cot 3\theta]$$

$$T_3 = \operatorname{cosec} \theta [\cot 3\theta - \cot 4\theta]$$

$$\vdots$$

$$T_n = \operatorname{cosec} \theta [\cot n\theta - \cot (n+1)\theta]$$

$$\therefore S_n = \operatorname{cosec} \theta [\cot \theta - \cot (n+1)\theta]$$

② लघु अंश

$$S_n = \frac{\sin \theta}{\cos \theta + \cos 2\theta} + \frac{\sin 2\theta}{\cos \theta + \cos 4\theta} + \frac{\sin 3\theta}{\cos \theta + \cos 6\theta} + \dots$$

पहिले :-

$$T_n = \frac{\sin n\theta}{\cos \theta + \cos 2n\theta}$$

$$= \frac{1}{2\sin(\theta/2)} \left[ \frac{2\sin(\theta/2)\sin n\theta}{2\cos\left(\frac{(2n+1)\theta}{2}\right)\cos\left(\frac{(2n-1)\theta}{2}\right)} \right]$$

$$= \frac{1}{2\sin(\theta/2)} \left[ \frac{\cos(n-1/2)\theta - \cos(n+1/2)\theta}{2\cos\left(\frac{(2n+1)\theta}{2}\right)\cos\left(\frac{(2n-1)\theta}{2}\right)} \right]$$

$$= \frac{1}{4} \operatorname{cosec}(\theta/2) \left[ \frac{\cos\left(\frac{(2n-1)\theta}{2}\right) - \cos\left(\frac{(2n+1)\theta}{2}\right)}{\cos\left(\frac{(2n+1)\theta}{2}\right)\cos\left(\frac{(2n-1)\theta}{2}\right)} \right]$$

$$= \frac{1}{4} \operatorname{cosec}(\theta/2) \left[ \sec\left(\frac{(2n+1)\theta}{2}\right) - \sec\left(\frac{(2n-1)\theta}{2}\right) \right]$$

$$T_1 = \frac{1}{4} \operatorname{cosec}(\theta/2) \left[ \sec(3\theta/2) - \sec(\theta/2) \right]$$

$$T_2 = \frac{1}{4} \operatorname{cosec}(\theta/2) \left[ \sec(5\theta/2) - \sec(3\theta/2) \right]$$

$$\vdots$$

$$T_n = \frac{1}{4} \operatorname{cosec}(\theta/2) \left[ \sec\left(\frac{(2n+1)\theta}{2}\right) - \sec\left(\frac{(2n-1)\theta}{2}\right) \right]$$

$$\therefore S_n = \frac{1}{4} \operatorname{cosec}(\theta/2) \left[ \sec\left(\frac{(2n+1)\theta}{2}\right) - \sec(\theta/2) \right]$$

② लघु अंश

$$S_n = \frac{\sin 2\theta}{\cos \theta \cos 3\theta} + \frac{\sin 4\theta}{\cos 3\theta \cos 5\theta} + \frac{\sin 6\theta}{\cos 5\theta \cos 7\theta} + \dots$$

पहिले :-

$$T_n = \frac{\sin 2n\theta}{\cos(2n-1)\theta \cos(2n+1)\theta}$$



$$= \frac{1}{2 \sin \theta} \left[ \frac{2 \sin \theta \sin 2n \theta}{\cos (2n-1) \theta \cos (2n+1) \theta} \right]$$

$$= \frac{1}{2 \sin \theta} \left[ \frac{\cos (2n-1) \theta - \cos (2n+1) \theta}{\cos (2n-1) \theta \cos (2n+1) \theta} \right]$$

$$= \frac{1}{2} \operatorname{cosec} \theta [\sec (2n+1) \theta - \sec (2n-1) \theta]$$

$$T_1 = \frac{1}{2} \operatorname{cosec} \theta [\sec 3\theta - \sec \theta]$$

$$T_2 = \frac{1}{2} \operatorname{cosec} \theta [\sec 5\theta - \sec 3\theta]$$

$$\vdots$$

$$T_n = \frac{1}{2} \operatorname{cosec} \theta [\sec (2n+1) \theta - \sec (2n-1) \theta]$$

$$\therefore S_n = \frac{1}{2} \operatorname{cosec} \theta [\sec (2n+1) \theta - \sec \theta]$$

(4) લઘુકોણ અનુક્રમ

$$S_n = \frac{\sin 2\theta}{\sin \theta \sin 3\theta} - \frac{\sin 4\theta}{\sin 3\theta \sin 5\theta} + \frac{\sin 6\theta}{\sin 5\theta \sin 7\theta} + \dots$$

ક્રમિક :-

$$T_n = (-1)^{n+1} \left[ \frac{\sin 2n\theta}{\sin (2n-1)\theta \sin (2n+1)\theta} \right]$$

$$= \frac{(-1)^{n+1}}{2 \cos \theta} \left[ \frac{2 \cos \theta \sin 2n\theta}{\sin (2n-1)\theta \sin (2n+1)\theta} \right]$$

$$= \frac{(-1)^{n+1}}{2 \cos \theta} \left[ \frac{\sin (2n+1)\theta + \sin (2n-1)\theta}{\sin (2n-1)\theta \sin (2n+1)\theta} \right]$$

$$= (-1)^{n+1} \left( \frac{1}{2} \sec \theta \right) [\operatorname{cosec} (2n-1)\theta + \operatorname{cosec} (2n+1)\theta]$$

$$T_1 = \frac{1}{2} \sec \theta [\operatorname{cosec} \theta + \operatorname{cosec} 3\theta]$$

$$T_2 = -\frac{1}{2} \sec \theta [\operatorname{cosec} 3\theta + \operatorname{cosec} 5\theta]$$

$\vdots$

$$T_n = (-1)^{n+1} \left( \frac{1}{2} \sec \theta \right) [\operatorname{cosec} (2n-1)\theta + \operatorname{cosec} (2n+1)\theta]$$

$$S_n = \frac{1}{2} \sec \theta [\operatorname{cosec} \theta + (-1)^{n+1} \operatorname{cosec} (2n+1)\theta]$$

⑤ लघुय क्रमिक

$$S_n = \tan \theta \sec 2\theta + \tan 2\theta \sec 4\theta + \tan 4\theta \sec 8\theta + \dots$$

ह्रिय :-

$$T_n = \tan (2^{n-1} \theta) \sec (2^n \theta)$$

$$= \frac{\sin (2^{n-1} \theta)}{\cos (2^{n-1} \theta) \cos (2^n \theta)}$$

$$= \frac{\sin (2^n \theta - 2^{n-1} \theta)}{\cos (2^{n-1} \theta) \cos (2^n \theta)}$$

$$= \frac{\sin (2^n \theta) \cos (2^{n-1} \theta) - \cos (2^n \theta) \sin (2^{n-1} \theta)}{\cos (2^{n-1} \theta) \cos (2^n \theta)}$$

$$T_n = \tan (2^n \theta) - \tan (2^{n-1} \theta)$$

$$T_1 = \tan 2\theta - \tan \theta$$

$$T_2 = \tan 4\theta - \tan 2\theta$$

$$\vdots$$

$$T_n = \tan (2^n \theta) - \tan (2^{n-1} \theta)$$

$$S_n = \tan (2^n \theta) - \tan \theta$$

⑥  $S_n = \tan x \tan (x+y) + \tan (x+y) \tan (x+2y) + \dots$   
 $\tan (x+2y) \tan (x+3y) + \dots$  लघुय क्रमिक

ह्रिय :-

$$T_n = \tan [x + (n-1)y] \tan (x+ny)$$

$$\tan y = \tan [x+ny - (x+(n-1)y)]$$

$$= \frac{\tan (x+ny) - \tan (x+(n-1)y)}{1 + \tan (x+ny) \tan (x+(n-1)y)}$$

$$(1 + \tan (x+ny) \tan (x+(n-1)y)) \cdot \frac{1}{\tan y} [\tan (x+ny) - \tan (x+(n-1)y)]$$

$$(1 + \tan (x+ny) \tan (x+(n-1)y)) = \cot y [\tan (x+ny) - \tan (x+(n-1)y)]$$

$$1 + T_n = \cot y [\tan (x+ny) - \tan (x+(n-1)y)]$$

$$T_n = \cot y [\tan (x+ny) - \tan (x+(n-1)y)]$$



$$T_n = \cot y [\tan(x+ny) - \tan\{x+(n-1)y\}]$$

$$T_1 = \cot y [\tan(x+y) - \tan x] - 1$$

$$T_2 = \cot y [\tan(x+2y) - \tan(x+y)] - 1$$

⋮

$$T_n = \cot y [\tan(x+ny) - \tan(x+(n-1)y)] - 1$$

$$S_n = \cot y [\tan(x+ny) - \tan x] - n.$$

⑦  $S_n = \tan^{-1}(1/3) + \tan^{-1}(1/7) + \tan^{-1}(1/13) + \dots$

लक्ष्य बनाना.

धिया :-

$$S_n = \tan^{-1}\left(\frac{1}{1+1 \cdot 2}\right) + \tan^{-1}\left(\frac{1}{1+2 \cdot 3}\right) + \tan^{-1}\left(\frac{1}{1+3 \cdot 4}\right) + \dots$$

$$\therefore T_n = \tan^{-1}\left[\frac{1}{1+n(n+1)}\right] = \tan^{-1}\left[\frac{n+1-n}{1+n(n+1)}\right]$$

$$T_n = \tan^{-1}(n+1) - \tan^{-1}(n)$$

$$T_1 = \tan^{-1}(2) - \tan^{-1}(1)$$

$$T_2 = \tan^{-1}(3) - \tan^{-1}(2)$$

⋮

$$T_n = \tan^{-1}(n+1) - \tan^{-1}(n)$$

$$\therefore S_n = \tan^{-1}(n+1) - \tan^{-1}(1) = \tan^{-1}\left[\frac{(n+1)-1}{1+(n+1)}\right]$$

$$= \tan^{-1}\left[\frac{n}{n+2}\right].$$

⑧  $S_n = \tan^{-1}\left(\frac{1}{1+1+1^2}\right) + \tan^{-1}\left(\frac{1}{1+2+2^2}\right) + \tan^{-1}\left(\frac{1}{1+3+3^2}\right) + \dots$

लक्ष्य बनाना.

+ ...

धिया :-

$$T_n = \tan^{-1}\left(\frac{1}{1+n+n^2}\right) = \tan^{-1}\left(\frac{1}{1+n(n+1)}\right)$$

$$= \tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)$$

$$T_n = \tan^{-1}(n+1) - \tan^{-1}(n)$$

$$\therefore S_n = \tan^{-1}\left(\frac{n}{n+2}\right)$$

⑨  $\operatorname{cosec} \alpha + \operatorname{cosec} 2\alpha + \operatorname{cosec} 2^2\alpha + \dots$  को गुणक श्रृंखला.  
 सिद्ध :-

$$T_1 = \frac{1}{\sin \alpha} = \frac{\sin \alpha/2}{\sin \alpha \sin \alpha/2} = \frac{\sin(\alpha - \alpha/2)}{\sin \alpha \sin \alpha/2}$$

$$= \frac{\sin \alpha \cos \alpha/2 - \cos \alpha \sin \alpha/2}{\sin \alpha \sin \alpha/2}$$

$$T_1 = \cot \alpha/2 - \cot \alpha$$

$$T_2 = \cot \alpha - \cot 2\alpha$$

$$T_3 = \cot 2\alpha - \cot 2^2\alpha$$

$$\vdots$$

$$T_n = \cot(2^{n-2}\alpha) - \cot(2^{n-1}\alpha)$$

$$\therefore S_n = \cot \alpha/2 - \cot(2^{n-1}\alpha)$$

सिद्ध करने के लिए :-

①  $S_n = \sin \alpha + \sin 2\alpha + \sin 3\alpha + \dots$  को गुणक श्रृंखला.

सिद्ध :-

$$2 \sin(\alpha/2) S_n = 2 \sin \alpha \sin(\alpha/2) + 2 \sin 2\alpha \sin(\alpha/2) + \dots + 2 \sin n\alpha \sin(\alpha/2)$$

$$= [\cos(\alpha/2) - \cos(3\alpha/2)] + [\cos(3\alpha/2) - \cos(5\alpha/2)]$$

$$+ \dots + \left[ \cos \frac{(2n-1)\alpha}{2} - \cos \frac{(2n+1)\alpha}{2} \right]$$

$$= \cos(\alpha/2) - \cos[(2n+1)\alpha/2]$$

$$= 2 \sin \left[ \frac{(n+1)\alpha}{2} \right] \sin \left( \frac{n\alpha}{2} \right)$$

$$S_n = \frac{\sin \left( \frac{(n+1)\alpha}{2} \right) \sin \left( \frac{n\alpha}{2} \right)}{\sin(\alpha/2)}$$



②  $\tan n\alpha = \frac{\sin \alpha + \sin 3\alpha + \sin 5\alpha + \dots}{\cos \alpha + \cos 3\alpha + \cos 5\alpha + \dots}$  *cosu anna*

Pris :-

$$\begin{aligned} \text{R.H.S} &= \frac{\sin \left[ \alpha + \frac{(n-1)2\alpha}{2} \right] \sin \left( \frac{n2\alpha}{2} \right) \sin \alpha}{\cos \left[ \alpha + \frac{(n-1)2\alpha}{2} \right] \sin \left( \frac{n2\alpha}{2} \right) \sin \alpha} \\ &= \frac{\sin [\alpha + (n-1)\alpha] \sin(n\alpha)}{\cos [\alpha + (n-1)\alpha] \sin(n\alpha)} \\ &= \frac{\sin^2 n\alpha}{\cos n\alpha \sin n\alpha} \\ &= \tan n\alpha \end{aligned}$$

③ Find  $S_n = \sin \alpha - \sin 2\alpha + \sin 3\alpha - \sin 4\alpha + \dots$  *cosu anna*

Pris :-

$$\sin(\pi + 2\alpha) = -\sin 2\alpha$$

$$\sin(2\pi + 3\alpha) = \sin 3\alpha ; \sin(3\pi + 4\alpha) = -\sin 4\alpha \dots \text{etc.}$$

$$\therefore S_n = \sin \alpha + \sin(\pi + 2\alpha) + \sin(2\pi + 3\alpha) + \sin(3\pi + 4\alpha) + \dots$$

$$= \sin \alpha + \sin[\alpha + (\alpha + \pi)] + \sin[\alpha + 2(\alpha + \pi)] + \dots$$

$$\therefore S_n = \frac{\sin \left[ \alpha + \frac{(n-1)(\pi + \alpha)}{2} \right] \sin \left[ \frac{n(\pi + \alpha)}{2} \right]}{\sin \left( \frac{\pi + \alpha}{2} \right)}$$

$$= \frac{\sin \left[ \alpha + \frac{(n-1)(\pi + \alpha)}{2} \right] \sin \left[ \frac{n(\pi + \alpha)}{2} \right]}{\cos(\alpha/2)}$$

④  $S_n = \cos \alpha \cos 3\alpha + \cos 3\alpha \cos 5\alpha + \cos 5\alpha \cos 7\alpha + \dots$  *cosu anna*

Pris :-

$$S_n = \left(\frac{1}{2}\right) (\cos 4\alpha + \cos 2\alpha) + \left(\frac{1}{2}\right) (\cos 8\alpha + \cos 2\alpha) + \dots$$

$$\begin{aligned}
&= (1/2) [n \cos 2\alpha + (\cos 4\alpha + \cos 8\alpha + \cos 12\alpha + \dots)] \\
&= 1/2 \left[ n \cos 2\alpha + \frac{\cos(4\alpha + \frac{(n-1)4\alpha}{2}) \sin(\frac{4n\alpha}{2})}{\sin 2\alpha} \right] \\
&= 1/2 \left[ n \cos 2\alpha + \frac{\cos[2(n+1)\alpha] \sin[2n\alpha]}{\sin 2\alpha} \right]
\end{aligned}$$

5) 6504 Ananth,  $S_n = \cos^3 \alpha + \cos^3 2\alpha + \cos^3 3\alpha + \dots$   
 தீர்வு :-

$$S_n = 1/4 [\cos 3\alpha + 3 \cos \alpha] + 1/4 [\cos 6\alpha + 3 \cos 2\alpha] + \dots$$

$$= (1/4) [\cos 3\alpha + \cos 6\alpha + \dots] + (3/4) (\cos \alpha + \cos 2\alpha + \dots)$$

$$= 1/4 \left[ \frac{\cos(3\alpha + \frac{(n-1)3\alpha}{2}) \sin(\frac{n3\alpha}{2})}{\sin(3\alpha/2)} \right] +$$

$$3/4 \left[ \frac{\cos(\alpha + \frac{(n-1)\alpha}{2}) \sin(\frac{n\alpha}{2})}{\sin(\alpha/2)} \right]$$

$$= \frac{1}{4} \left[ \frac{\cos[(3/2)(n+1)\alpha] \sin[(3/2)n\alpha]}{\sin(\alpha/2)} \right] +$$

$$\frac{3}{4} \left[ \frac{\cos[(n+1)\alpha/2] \sin(n\alpha/2)}{\sin(\alpha/2)} \right]$$

தீர்வுகளின் தொகுப்பு :-

எடுத்துக்காட்டு :-

$-1/4 \pi \leq \theta \leq \pi/4$ , காண்க  $\tan^3 \theta + \tan^5 \theta + \dots$

$$0 = \tan \theta - 1/3 \tan^3 \theta + 1/5 \tan^5 \theta + \dots$$

தீர்வு :-



$$1 + i \tan \theta = 1 + i \left( \frac{\sin \theta}{\cos \theta} \right) = \frac{\cos \theta + i \sin \theta}{\cos \theta}$$

$$1 + i \tan \theta = (\sec \theta) e^{i\theta}$$

$$\begin{aligned} \log(1 + i \tan \theta) &= \log(\sec \theta) + \log(e^{i\theta}) \\ &= \log \sec \theta + i\theta \rightarrow \textcircled{1} \end{aligned}$$

$$-\frac{1}{4}\pi \leq \theta \leq \frac{\pi}{4}, \quad |\tan \theta| \leq 1,$$

$$|i \tan \theta| = |\tan \theta| \leq 1$$

$\log(1 + i \tan \theta)$  2nd 2nd 4th 5th 7th 9th 11th

$$(i \tan \theta) - \frac{1}{2} (i \tan \theta)^2 + \frac{1}{3} (i \tan \theta)^3 -$$

$$\frac{1}{4} (i \tan \theta)^4 + \dots = \log(\sec \theta) + i\theta$$

$$\begin{aligned} i \tan \theta + \frac{1}{2} \tan^2 \theta + \frac{1}{3} i \tan^3 \theta - \frac{1}{4} \tan^4 \theta + \dots \\ = \log(\sec \theta) + i\theta \end{aligned}$$

Equating real & imaginary parts.

$$0 = \tan \theta - \frac{1}{3} \tan^3 \theta + \frac{1}{5} \tan^5 \theta - \dots$$

Problem 2

①  $x$  varies from  $-\pi/4$  to  $\pi/4$  then  $\tanh x$  varies from  $-\frac{1}{\sqrt{2}}$  to  $\frac{1}{\sqrt{2}}$ .

$$\tan x - \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} - \dots = \tanh x + \frac{\tanh^3 x}{3} + \frac{\tanh^5 x}{5} - \dots$$

Let  $\tanh x = y$ .

$$\text{Given: } \tanh x = y$$

$$\text{R.H.S} = y + \frac{y^3}{3} + \frac{y^5}{5} - \dots = \frac{1}{2} \log \left( \frac{1+y}{1-y} \right) = \tanh^{-1}(y)$$

$$= \tanh^{-1}(\tanh x) = x \rightarrow \textcircled{1}$$

$$\text{put } \tan x = z$$

$$\begin{aligned} \text{L.H.S} = z - \frac{z^3}{3} + \frac{z^5}{5} - \dots = \tan^{-1}(z) = \tan^{-1}(\tan x) \\ = x \rightarrow \textcircled{2} \end{aligned}$$

Both 1 & 2 are correct

$$\tan x - \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} - \dots = \tanh x + \frac{\tanh^3 x}{3} + \frac{\tanh^5 x}{5} - \dots$$

$$(2) \quad \pi = 2\sqrt{3} \left[ 1 - \frac{1}{8^2} + \frac{1}{5} \cdot \frac{1}{3^2} - \frac{1}{7} \cdot \frac{1}{3^2} + \dots \right]$$

ਸਮਾਨ ਕਰੋ।

ਕੀਮਤ :-

$$R.H.S = 2\sqrt{3} \left[ 1 - \frac{1}{3^2} + \frac{1}{5} \cdot \frac{1}{3^2} - \frac{1}{7} \cdot \frac{1}{3^2} + \dots \right]$$

$$= 2\sqrt{3} \sqrt{3} \left[ \frac{1}{\sqrt{3}} - \frac{1}{3^{5/2}} + \frac{1}{5} \cdot \frac{1}{3^{5/2}} - \dots \right]$$

$$= 6 \left[ \frac{1}{\sqrt{3}} - \frac{1}{3} \left( \frac{1}{\sqrt{3}} \right)^3 + \frac{1}{5} \left( \frac{1}{\sqrt{3}} \right)^5 - \dots \right]$$

$$= 6 \tan^{-1} \left( \frac{1}{\sqrt{3}} \right)$$

$$= 6 \left( \frac{\pi}{6} \right)$$

$$= \pi$$

(3)  $\theta$  ਸਮਾਂ  $\frac{1}{4}\pi$  ਤੋਂ  $\frac{3}{4}\pi$  ਤੱਕ  $\frac{1}{4}\pi$  ਦੇ ਅੰਤਰਾਂ ਵਿੱਚ ਲਿਆਂਦਾ ਹੈ,

$$\theta = \frac{1}{2}\pi - \cot \theta + \frac{1}{3} \cot^3 \theta - \frac{1}{5} \cot^5 \theta + \dots \text{ ਸਮਾਨ ਕਰੋ।}$$

ਕੀਮਤ :-

$$\frac{1}{4}\pi \leq \theta \leq \frac{3}{4}\pi$$

$$\therefore \frac{1}{4}\pi - \frac{1}{2}\pi \leq \theta - \frac{1}{2}\pi \leq \frac{3}{4}\pi - \frac{1}{2}\pi$$

$$-\frac{1}{4}\pi \leq \theta - \frac{1}{2}\pi \leq \frac{1}{4}\pi$$

ਕਿਉਂਕਿ  $\theta - \frac{1}{2}\pi$  ਸਮਾਂ  $\frac{1}{4}\pi$  ਦੇ ਅੰਤਰਾਂ ਵਿੱਚ ਲਿਆਂਦਾ ਹੈ,  $\theta - \frac{1}{2}\pi$  ਸਮਾਂ  $\frac{1}{4}\pi$  ਦੇ ਅੰਤਰਾਂ ਵਿੱਚ ਲਿਆਂਦਾ ਹੈ।

$$\theta - \frac{1}{2}\pi = \tan \left( \theta - \frac{1}{2}\pi \right) - \frac{1}{3} \tan^3 \left( \theta - \frac{1}{2}\pi \right) + \frac{1}{5} \tan^5 \left( \theta - \frac{1}{2}\pi \right) + \dots$$

$$= -\cot \theta + \frac{1}{3} \cot^3 \theta - \frac{1}{5} \cot^5 \theta + \dots$$

$$\theta = \frac{1}{2}\pi - \cot \theta + \frac{1}{3} \cot^3 \theta - \frac{1}{5} \cot^5 \theta + \dots$$

(4)  $(1 - 3^{-1/2}) - \frac{1}{3} (1 - 3^{-3/2}) + \frac{1}{5} (1 - 3^{-5/2}) - \dots = \frac{\pi}{12}$

ਸਮਾਨ ਕਰੋ।



प्रश्न :-

$$\begin{aligned} \text{LHS} &= (1 - 3^{-1/2}) - \frac{1}{3} (1 - 3^{-3/2}) + \frac{1}{5} (1 - 3^{-5/2}) - \dots \\ &= \left(1 - \frac{1}{\sqrt{3}}\right) - \frac{1}{3} \left(1 - \frac{1}{3\sqrt{3}}\right) + \frac{1}{5} \left(1 - \frac{1}{3^2\sqrt{3}}\right) - \dots \\ &= \left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\right] - \left[\frac{1}{\sqrt{3}} - \frac{1}{3} \left(\frac{1}{\sqrt{3}}\right)^3 + \frac{1}{5} \left(\frac{1}{\sqrt{3}}\right)^5 - \dots\right] \end{aligned}$$

$$= \tan^{-1}(1) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \pi/4 - \pi/6$$

$$= \pi/12$$

$$\therefore S = \pi/12.$$