SEMESTER : II
ALLIED COURSE : III - Mathematics

Inst Hour	: 4
Credit	: 3
Code	: 18K2CSAM3

NUMERICAL METHODS AND GRAPH THEORY

(For B. Sc., Computer Science Major)

UNIT1:

Algebraic Equations - Method of false position – Bisection method – Iteration method – Solving by Newton Raphson method. (In all problems Approximation upto 2 decimals only)

(Chapter2:Sec2.1-2.5 of Text Book 1)

UNIT 2:

Numerical integration by Trapezoidal and Simpson's rule -Euler's method of solving an ordinary differential equation numerically; Runge- Kutta's second order method of solving ordinary differential equations.(In all problems Approximation upto 2 decimals only)

(Chapter5:5.4.1-5.4.3 & Chapter7: Sec 7.4, 7.4.1, 7.4.2, 7.5 of Text Book 1)

UNIT 3:

Graphs: Definition and examples – Graph models – Precedence Graphs and concurrent processing - Graph terminology – The hand shaking theorem – Underlying undirected graph - bipartite graphs – Union of two graphs

(Chapter 6: Sec6.1- 6.62 of Text Book2)

UNIT 4:

Representation of : graphs (By using adjacency list) - undirected simple graphs (By using Adjacency matrices) – Any undirected graph (By using adjacency matrix) - directed graphs (By adjacency matrix) – undirected graph (by using incidence matrix) – graph isomorphisms.

(Chapter6: Sec 6.63-6.85 of Text Book2)

UNIT 5:

Connectivity – Path circuits and isomorphisms – Euler & Hamiltonian path – Algorithm for constructing Euler circuits – Hamiltonian paths and circuits.

(Chapter6: Sec 6.86 -6.90, 6.115-6.137 of Text Book2)

Text Books:

[1] S.S. Sastry, An introductory Methods of Numerical Analysis, Prentice Hall of India II edition

[2] G. Ramesh ,C.Ganesamoorthy, Discrete Mathematics ,2003

Reference Books:

[1] S.Arumugam, Graph Theory.

[2] Narsingh Deo,Graph Theory with Applications to Engineering and Computer Science

Question Pattern

Section A : 10 x 2 = 20 Marks, 2 Questions from each Unit.

Section B : 5 x 5 = 25 Marks, EITHER OR (a or b) Pattern, One question from each Unit.

Section C : 3 x 10 = 30 Marks, 3 out of 5, One Question from each Unit.

9.3

9/3/18 9/3/18

UNIT - I

The Solution of Numerical Algebraic Transcendental Equation :-

If $f(x)$ is a polynomial of degree two or three or four, exact formulae are available. But if $f(x)$ is a transcendental function like $a + be^{x} + c\sin x + d\log x$ etc. The solution is not exact and we do not have formulae to get the solutions.

If $f(x)$ is continuous in the interval (a,b) and if $f(a)$ and $f(b)$ are of opposite signs, then the equation $f(x)=0$ will have at least one real root between a and b.

BISECTION METHOD :-

The Bisection Method also called as Balzano's Method (or) Interval Halving Method

We have an equation of the form $f(x)=0$ whose solution in the range (a,b). If $f(x)$ is continuous and it can be algebraic (or) transcendental. If $f(a)$ and $f(b)$ are of opposite signs atleast one real root between a and b exist. A first approximation we assume that root be $x_0 = \frac{a+b}{2}$. We form a sequence of approximate roots $x_0, x_1, x_2 \ldots$ whose limit of convergence is the exact root.

EXAMPLE :-

① Find the positive root of $x^3 - x = 1$ correct to four decimal places by bisection method.

Soln :-

Let, $f(x) = x^3 - x - 1$

$f(0) = -1 = (-ve)$
$f(1) = -1 = (-ve)$
$f(2) = 5 = (+ve)$

Hence a root lies between 1 and 2. We can take the range as (1, 2) and Proceed. We can still shorten the range.

$f(1.5) = 0.8750 \quad (+ve)$
$f(1) = -1 \quad (-ve)$

Hence the root lies between 1 and 1.5

let $x_0 = \frac{1+1.5}{2} = 1.2500$

$f(x_0) = f(1.25) = -0.29688$

Hence the root lies between 1.25 and 1.5

$x_1 = \frac{1.25+1.5}{2} = 1.3750$

$f(1.3750) = (1.3750)^3 - (1.3750) - 1 = 0.22461$

$\therefore$ The root lies between 1.2500 and 1.3750

$x_2 = \frac{1.2500 + 1.3750}{2} = 1.3125$

$f(1.3125) = (1.3125)^3 - (1.3125) - 1 = -0.051514$.

$\therefore$ The root lies between 1.3750 and 1.3125

$x_3 = \frac{1.3125 + 1.3750}{2} = 1.3438$

$f(1.3438) = (1.3438)^3 - (1.3438) - 1 = 0.082832$

$\therefore$ The root lies between 1.3125 and 1.3438

$x_4 = \frac{1.3125 + 1.3438}{2} = 1.3282$

$f(1.3282) = (1.3282)^3 - (1.3282) - 1 = 0.014898$

$\therefore$ The root lies between 1.3125 and 1.3282

$x_5 = \frac{1.3125 + 1.3282}{2} = 1.3204$

$f(1.3204) = (1.3204)^3 - (1.3204) - 1 = -0.018340$

$\therefore$ The root lies between 1.3204 and 1.3282

$$x_6 = \frac{1.3204 + 1.3282}{2} = 1.3243$$

$$f(1.3243) = (1.3243)^3 - (1.3243) - 1 = -0.0018$$

$\therefore$ The root lies between 1.3243 and 1.3282

$$x_7 = \frac{1.3243 + 1.3282}{2} = 1.3263$$

$$f(1.3263) = (1.3263)^3 - (1.3263) - 1 = 0.0065$$

$\therefore$ The root lies between 1.3243 and 1.3263

$$x_8 = \frac{1.3243 + 1.3263}{2} = 1.3253$$

$$f(1.3253) = (1.3253)^3 - (1.3253) - 1 = 0.0024$$

$\therefore$ The root lies between 1.3243 and 1.3253

$$x_9 = \frac{1.3243 + 1.3253}{2} = 1.3248$$

$$f(1.3248) = (1.3248)^3 - (1.3248) - 1 = 0.0003$$

$\therefore$ The root lies between 1.3243 and 1.3248

$$x_{10} = \frac{1.3243 + 1.3248}{2} = 1.3245$$

$$f(1.3245) = (1.3245)^3 - (1.3245) - 1 = -0.001$$

$\therefore$ The root lies between 1.3248 and 1.3245

$$x_{11} = \frac{1.3248 + 1.3245}{2} = 1.3247$$

$$f(1.3247) = (1.3247)^3 - (1.3247) - 1 = -0.0001$$

$\therefore$ The root lies between 1.3247 and 1.3248

$$x_{12} = \frac{1.3247 + 1.3248}{2} = 1.3247$$

$\therefore$ The approximate root is 1.3247

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2. Find the positive root of $x - \cos x = 0$ by bisection method.

Solution :-

$$f(x) = x - \cos x$$

$$f(0) = -1 \quad (-ve)$$

$$f(0.5) = -0.37758 \quad (-ve)$$

$$f(1) = 0.45970 \quad (+ve)$$

Hence the root lies between 0.5 and 1.

$$x_0 = \frac{0.5 + 1}{2} = 0.75$$

$$f(0.75) = 0.75 - \cos(0.75) = 0.18311 \quad (+ve)$$

$\therefore$ The root lies between 0.5 and 0.75

$$x_1 = \frac{0.5 + 0.75}{2} = 0.625$$

$$f(0.625) = 0.625 - \cos(0.625) = -0.18596 \quad (-ve$$

$\therefore$ The root lies between 0.625 and 0.750

$$x_2 = \frac{0.625 + 0.750}{2} = 0.6875$$

$$f(0.6875) = 0.6875 - \cos(0.6875) = -0.0853 \quad (+v$$

$\therefore$ The root lies between 0.6875 and 0.75

$$x_3 = \frac{0.6875 + 0.75}{2} = 0.7187$$

$$f(0.7187) = (0.7187) - \cos(0.7187) = -0.03387 \quad (-$$

$\therefore$ The root lies between 0.71875 and 0.75

$$x_4 = \frac{0.71875 + 0.75}{2} = 0.73438$$

$$f(0.73438) = (0.73438) - \cos(0.73438) =$$

$$= -0.0078661 \quad (-ve)$$

$\therefore$ The root lies between 0.73438 and 0.75

$$x_5 = \frac{0.73438 + 0.75}{2} = 0.742190$$

$$f(0.74219) = (0.74219) - \cos(0.74219)$$

$$= 0.005199 \quad (+ve)$$

∴ The Root lies between 0.7348 and 0.742190

$$x_6 = \frac{0.7348 + 0.742190}{2} = 0.73829$$

$f(0.73829) = 0.73829 - \cos(0.73829)$

$= -0.0013305$ (−ve)

∴ The Root lies between 0.73829 & 0.74219

$$x_7 = \frac{0.73829 + 0.74219}{2} = 0.7402$$

$f(0.7402) = 0.7402 - \cos(0.7402)$

$= 0.0018663$ (+ve)

∴ The Root lies between 0.73829 and 0.7402

$$x_8 = \frac{0.73829 + 0.7402}{2} = 0.73925$$

$f(0.73925) = (0.73925) - \cos(0.73925)$

$= 0.00027593$ (+ve)

∴ The root lies between 0.73829 and 0.73925.

$$x_9 = \frac{0.73829 + 0.73925}{2} = 0.7388.$$

∴ The Root is 0.7388.

ITERATION METHOD :-

This iteration method is also called as method of successive Approximation.

$$x = \phi(x) \rightarrow (1)$$

Assume x_0 to be the starting approximate value to the actual root α of $x = \phi(x)$. setting $x = x_0$ in the right hand side of (1) we get the first approximation.

$$x_1 = \phi(x_0)$$

Again setting $x = x_1$ on the R.H.S of (1) we get successive approximations

$$x_2 = \phi(x_1)$$
$$x_3 = \phi(x_2)$$
$$\vdots$$
$$x_n = \phi(x_{n-1})$$

The sequence of approximate roots $x_1, x_2 \ldots x_n$ if it converges to α is taken as the root of the equation $f(x) = 0$.

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(1) Solve the equation $x^3 + x^2 - 1 = 0$ for the positive root by Iteration method.

Soln :-

$$f(x) = x^3 + x^2 - 1$$
$$f(0) = -1 \quad (-ve)$$
$$f(1) = 1 \quad (+ve)$$

$\therefore$ The root lies between 0 and 1

$$x^3 + x^2 - 1 = 0 \Rightarrow x^3 + x^2 = 1$$

$$x^2(x+1) = 1 \Rightarrow x^2 = \frac{1}{x+1}$$

$$x = \frac{1}{\sqrt{x+1}} \Rightarrow x = \frac{1}{(x+1)^{1/2}} \Rightarrow x = (x+1)^{-1/2}$$

$$\phi(x) = (x+1)^{-1/2}$$
$$\phi'(x) = -1/2\,(x+1)^{-3/2}$$
$$= \frac{-1}{2(x+1)^{3/2}}$$

In the Interval $(0,1)$, $|\phi'(x)| < 1$ select

$x_0 = 0.75$

$$x_1 = \frac{1}{\sqrt{0.75+1}} = \frac{1}{\sqrt{1.75}} = 0.7559$$

$$x_2 = \frac{1}{\sqrt{x_1+1}} = \frac{1}{\sqrt{1.7559}} = 0.7547$$

$$x_3 = \frac{1}{\sqrt{x_2+1}} = \frac{1}{\sqrt{1.7547}} = 0.7549$$

$$x_4 = \frac{1}{\sqrt{x_3+1}} = \frac{1}{\sqrt{1.7549}} = 0.7549$$

$$x_5 = \frac{1}{\sqrt{x_4+1}} = \frac{1}{\sqrt{1.7549}} = 0.7549$$

$$x_6 = \frac{1}{\sqrt{x_5+1}} = \frac{1}{\sqrt{1.7549}} = 0.7549$$

we can take 0.7549 as the correct value of the root of the equation.

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(2) Solve $x^3 = 2x+5$ for the positive root by Iteration method.

soln :-

$$x^3 - 2x - 5 = 0$$

$f(0) = 0 - 2(0) - 5 = -5$ (-ve)

$f(1) = 1 - 2(1) - 5 = -6$ (-ve)

$f(2) = 8 - 4 - 5 = -1$ (-ve)

$f(3) = 27 - 6 - 5 = 16$ (+ve)

$\therefore$ The Root lies between 2 and 3.

$$x^3 - 2x - 5 = 0$$

$$\Rightarrow x^3 - 2x = 5$$

$$x^3 = (2x+5)$$

$$x = (2x+5)^{1/3} = \phi(x)$$

$$\phi'(x) = \frac{1}{3}(2x+5)^{-2/3} \cdot 2 = \frac{2}{3}(2x+5)^{-2/3}$$

$$= \frac{2}{3} \frac{1}{\sqrt{(2x+5)}^{1/3}}$$

$$\phi'(2) = (0.6667) \frac{1}{(4.3267)}$$

$$= (0.6667)(0.2311)$$

$$= 0.1541 < 1.$$

$$\phi'(3) = (0.6667)\frac{1}{4.9461} = 0.1348 < 1$$

In the interval $(2,3)$ $|\phi'(x)| < 1$ select

$x_0 = 2$

$$x_1 = (2x_0+5)^{1/3} = \sqrt[3]{(2(2))+5} = 2.0801$$

$$x_2 = (2x_1+5)^{1/3} = \sqrt[3]{2(2.0801)+5}$$
$$= (4.1602+5)^{1/3} = (9.1602)^{1/3}$$
$$= 2.0924.$$

$$x_3 = (2x_2+5)^{1/3} = (4.1848+5)^{1/3} = (9.1848)^{1/3}$$
$$= 2.0942$$

$$x_4 = (2x_3+5)^{1/3} = (4.1884+5)^{1/3} = (9.1884)^{1/3}$$
$$= 2.0945$$

$$x_5 = (2x_4+5)^{1/3} = (4.1890+5)^{1/3} = (9.1890)^{1/3}$$
$$= 2.0945.$$

We can take 2.0945 as the correct value of the root of the equation.

REGULA FALSI METHOD :-

This method is also called as Method of false position.

$$x_1 = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

This value of x_1 gives an approximate value of the root of $f(x) = 0$.

$$x_2 = \frac{a \cdot f(x_1) - x_1 \cdot f(a)}{f(x_1) - f(a)}$$

In the same way, we get $x_3, x_4 \ldots$

This sequence $x_1, x_2, x_3 \ldots$ will converge to the required root.

1. Find the positive root of $x^3 = 2x+5$ by False position method.

Soln :-

Let, $f(x) = x^3 - 2x - 5 = 0$

$f(0) = 0^3 - 2(0) - 5 = -5$ (-ve)

$f(1) = 1^3 - 2(1) - 5 = -7$ (-ve)

$f(2) = 2^3 - 2(2) - 5 = -1$ (-ve)

$f(3) = 3^3 - 2(3) - 5 = 16$ (+ve)

$\therefore$ The Roots lies between 2 and 3

$a = 2$, $b = 3$, $f(a) = -1$ $f(b) = 16$.

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{2(16) - 3(-1)}{16+1}$$

$= 35/17$

$x_1 = 2.0588$

$f(x_1) = (2.0588)^3 - 2(2.0588) - 5$

$\therefore f(x_1) = -0.39079$ (-ve)

$\therefore$ The root lies between 2.0588 and 3

$a = 2.0588$, $b = 3$, $f(a) = -0.39079$, $f(b) = 16$

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{2.0588(16) - 3(-0.39079)}{16 + 0.39079}$$

$x_2 = 2.08126$

$f(x_2) = (2.08126)^3 - 2(2.08126) - 5$

$f(x_2) = -0.147200$ (-ve)

$\therefore$ The Root lies between 2.081264 and 3

$a = 2.081264$, $b = 3$, $f(a) = -0.14720$, $f(b) = 16$

$$x_3 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{(2.081264)(16) - (3)(-0.147200)}{16 + 0.147200}$$

$x_3 = 2.089639$

$f(x_3) = (2.089639)^3 - 2(2.089639) - 5$

$f(x_3) = -0.54679 \quad (-ve)$

$\therefore$ The root lies between 2.089639 and 3

$a = 2.089639, \quad b = 3, \quad f(a) = -0.054679,$
$f(b) = 16$

$$x_4 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{(2.089639)(16) + (3)(0.054679)}{16 + 0.054679}$$

$x_4 = 2.092740$

$f(x_4) = (2.092740)^3 - 2(2.092740) - 5$

$f(x_4) = -0.020298 \quad (-ve)$

$\therefore$ The root lies between 2.092740 and 3

$a = 2.092740, \quad b = 3, \quad f(a) = -0.020198, \quad f(b) = 16$

$$x_5 = \frac{(2.092740)(16) - (-0.020198)(3)}{16 + 0.020198}$$

$x_5 = 2.093884$

$f(x_5) = (2.093884)^3 - 2(2.093884) - 5$

$f(x_5) = -0.007447 \quad (-ve)$

$\therefore$ The root lies between 2.093884 and 3

$a = 2.093884, \quad b = 3, \quad f(a) = -0.007447,$
$f(b) = 16$

$$x_6 = \frac{(2.093884)(16) - (3)(-0.007447)}{16 + 0.007447}$$

$x_6 = 2.094306$

$f(2.094306) = (2.094306)^3 - 2(2.094306) - 5$

$f(x_6) = -0.002740 \quad (-ve)$

$\therefore$ The Root lies between 2.094306 and 3

$a = 2.094306,\ b = 3,\ f(a) = -0.002740,\ f(b) = 16$

$$x_7 = \frac{(2.0943306)(16) - (3)(-0.002740)}{16 + 0.002740}$$

$x_7 = 2.094461$

$f(x_7) = (2.094461)^3 - 2(2.094461) - 5$

$f(x_7) = -0.001010 (-ve)$

$\therefore$ The Root lies between 2.094461 and 3

$a = 2.094461 \quad b = 3 \quad f(a) = -0.001010 \quad f(b) = 16$

$$x_8 = \frac{(2.094461)(16) - (3)(-0.001010)}{16 + 0.001010}$$

$x_8 = 2.0945$.

$\therefore$ The Root is 2.0945.

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(2) Solve for a positive root of $x - \cos x = 0$ by Regular Falsi method

Soln :-

Let $f(x) = x - \cos x$

$f(0) = 0 - \cos(0) = -1 \ (-ve)$

$f(1) = 1 - \cos(1) = 0.459698 \ (+ve)$

$\therefore$ The Root lies between 0 and 1

$a = 0,\ b = 1,\ f(a) = -1,\ f(b) = 0.459698$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{0(0.459698) - 1(-1)}{0.459698 + 1}$$

$x_1 = 0.685073$

$f(x_1) = 0.685073 - \cos(0.685073)$

$= -0.089300 \ (-ve)$

$\therefore$ The Root lies between 0.685073 and 1.

$$x_2 = \frac{0.685073(0.459698) - 1(-0.0893)}{0.459698 + 0.089300}$$

$$= 0.7363$$

$f(x_2) = (0.7363)^3 - \cos(0.7363)$

$f(x_2) = -0.00466$ (-ve)

$\therefore$ The root lies between 0.7363 and 1

$a = 0.7363$, $b = 1$, $f(a) = -0.00466$, $f(b) = 0.4596$

$$x_3 = \frac{(0.7363)(0.459698) - (-1)(-0.00466)}{0.0459698 + 0.00466}$$

$x_3 = 0.738947$

$f(x_3) = (0.738947) - \cos(0.738947)$

$f(x_3) = -0.00023$ (-ve).

$\therefore$ The Root lies between 0.73895 and 1.

$a = 0.73895$, $b = 1$, $f(a) = -0.00023$, $f(b) = 0.4596$

$$x_4 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$x_4 = \frac{(0.73895)(0.459698) - (1)(-0.00023)}{0.459698 + 0.00023}$$

$x_4 = 0.7391$.

$\therefore$ The Root is 0.7391.

NEWTON RAPHSON METHOD (OR) NEWTON METHOD (OR) METHOD OF TANGENT.

FORMULA :-

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)} \qquad r = 0, 1, 2 \ldots$$

① Find the positive root of $f(x) = 2x^3 - 3x - 6$ by Newton Raphson method.

Soln :-

Let, $f(x) = 2x^3 - 3x - 6$; $f'(x) = 6x^2 - 3$

$f(0) = 0 - 0 - 6 = -6$ (-ve)

$f(1) = 2 - 9 = -7$ (-ve)

$f(2) = 16 - 12 = 4$ (+ve)

∴ The root lies between 1 and 2.

$$x_0 = \frac{1+2}{2} = 1.5$$

$f(x_0) = 2(1.5)^3 - 3(1.5) - 6 = -3.75$

$f'(x_0) = 6(1.5)^2 - 3 = 10.5$

put $r = 0 \Rightarrow x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$$x_1 = 1.5 - \left(\frac{-3.75}{10.5}\right) = 1.85714.$$

put $r = 1 \Rightarrow x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$$x_2 = 1.85714 - \left(\frac{1.23902}{17.69381}\right) = 1.78711.$$

put $r = 2 \Rightarrow x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$$x_3 = 1.78711 - \left(\frac{0.05388}{16.16257}\right) = 1.78378.$$

put $r = 3 \Rightarrow x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)}$

$$x_4 = 1.78378 - \left(\frac{0.00018}{16.09123}\right) = 1.78377.$$

put $r = 4 \Rightarrow x_5 = x_4 - \dfrac{f(x_4)}{f'(x_4)}$

$$x_5 = 1.78377 - \left(\frac{0.00002}{16.09101}\right)$$

$$x_5 = 1.78377.$$

∴ The approximate Root is 1.78377.

UNIT-II

TRAPEZOIDAL RULE :-

The Trapezoidal Rule is the simplest of the formulas for the numerical integration, but it is also the least accurate. The accuracy of the result can be improved by decreasing the interval h.

$$\int_{x_0}^{x_0+nh} y(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

This is called the Trapezoidal Rule.

① Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using Trapezoidal Rule with $h=0.2$, Hence determine the value of π.

Soln :-

Here, $h=0.2$. The values of the function $y=\frac{1}{1+x^2}$ for each point of subdivision are given below,

x	0	0.2	0.4	0.6	0.8	1
$y=\frac{1}{1+x^2}$	1	0.9615	0.8621	0.7353	0.6098	0.5
	y_0	y_1	y_2	y_3	y_4	y_5

By Trapezoidal Rule, we have

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$$

$$= \frac{0.2}{2}\left[1.5+2(3.1687)\right]$$

$$= 0.1(7.8374)$$

$$= 0.78374$$

We know that

$$\int_0^1 \frac{dx}{1+x^2} = (\tan^{-1} x)_0^1 = \pi/4$$

$$= 4\,(0.78374)$$

$$= 3.13496$$

Simpson's 1/3 Rule :-

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3+\cdots) + 2(y_2+y_4+\cdots) + y_n\right]$$

$$= \frac{h}{3}\left[y_0 + y_n + 4(\text{sum of odd ordinates}) + 2(\text{sum of even ordinates})\right]$$

This is called Simpson's 1/3 Rule (or) Simpson's one third Rule.

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① Find the value of $\log 2^{1/3}$ from $\int_0^1 \frac{x^2}{1+x^3}\,dx$ using Simpson's 1/3 Rule with $h = 0.25$

Soln:- Given $h = 0.25$. The values of the function $y = \frac{x^2}{1+x^3}$ for each point of subdivision are given below

x	0	0.25	0.5	0.75	1.0
$y = \frac{x^2}{1+x^3}$	0	0.06154	0.2222	0.39560	0.5000
	y_0	y_1	y_2	y_3	y_4

By Simpson's 1/3 Rule

$$\int_0^1 \frac{x^2}{1+x^3}\,dx = \frac{h}{3}\left[(y_0+y_4) + 2y_2 + 4(y_1+y_3)\right]$$

$$= \frac{0.25}{3}\left[(0+0.5) + 2(0.2222) + 4(0.06154 + 0.39560)\right]$$

$$= \frac{0.25}{3}\left[0.5 + 0.4444 + 1.82856\right]$$

$$= 0.231083$$

We know that,

$$\int_0^1 \frac{x^2}{1+x^3}\,dx = \frac{1}{3}\left[\log(1+x^3)\right]_0^1$$
$$= \frac{1}{3}(\log 2 - \log 1) = \frac{1}{3}\log_e 2$$

$$\therefore \log 2^{1/3} = \int_0^1 \frac{x^2}{1+x^3}\,dx = 0.231083.$$

Simpson's 3/8 Rule :-

$$\int_{x_0}^{x_0+nh} y(x)\,dx = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+\cdots y_{n-1}) + 2(y_3+y_6+\cdots+y_{n-3})\right]$$

① Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using trapezoidal rule, Simpson's 1/3 and Simpson's 3/8 Rule.

Soln :- $a = 0$, $b = 6$, $n = 6$

$$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$$

x	0	1	2	3	4	5	6
y	0	0.5	0.2	0.1	0.0588	0.0384	0.0270

Trapezoidal Rule :-

$$I = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$
$$= \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$
$$= \frac{1}{2}\left[(1+0.0270) + 2(0.5+0.2+0.1+0.0588+0.0384)\right]$$
$$= \frac{1}{2}\left[1.0270 + 1.7944\right] = \frac{1}{2}(2.8214)$$
$$= 1.4107.$$

Simpson's 1/3 Rule :-

$$I = \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+\cdots) + 4(y_1+y_3+\cdots)\right]$$
$$= \frac{1}{3}\left[(1+0.0270) + 2(0.2+0.0588) + 4(0.5+0.1+0.0384)\right].$$

$$= \frac{1}{3}[1.0270 + 0.5176 + 2.5536] = 1.366$$

Simpson's 3/8 Rule :-

$$I = \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3 + y_6 + \cdots + y_{n-2})]$$

$$= \frac{3h}{8}[(y_0 + y_6) + 3(0.5 + 0.2 + 0.0588 + 0.0384) + 2(0.1)]$$

$$= 3/8\,[1.0270 + 2.3916 + 0.2] = 1.856975.$$

Euler's Method :-

$$y_{n+1} = y_n + hf(x_n, y_n) \quad , n = 0, 1, 2 \ldots$$

This formula is called Euler's algorithm.

---

① Using Euler's Method find $y(0.2)$ and $y(0.4)$, $y(0.6)$ from $dy/dx = x + y$, $y(0) = 1$, with $h = 0.2$.

Soln :- $f(x,y) = x + y$; $f(x_0) = y_0$

$x_0 = 0$; $y_0 = 1$; $h = 0.2$.

$$\boxed{y_{n+1} = y_n + hf(x_n, y_n)}$$

put $n = 0 \Rightarrow y_1 = y_0 + hf(x_0, y_0)$

$= 1 + 0.2\, f(0,1) = 1 + 0.2[0+1]$

$y_1 = y(0.2) = 1.2$.

$x_1 = 0.2$; $y_1 = 1.2$; $h = 0.2$

put $n = 1 \Rightarrow y_2 = y_1 + hf(x_1, y_1)$

$y_2 = 1.2 + 0.2\, f(0.2, 1.2)$

$= 1.2 + 0.2[0.2 + 1.2] = 1.2 + 0.28$

$y_2 = y(0.4) = 1.48$.

$x_2 = 0.4$; $y_2 = 1.48$; $h = 0.2$

$y_3 = y_2 + hf(x_2, y_2)$

$= 1.48 + 0.2f[0.4, 1.48]$

$= 1.48 + 0.2[0.4 + 1.48] = 1.48 + 0.376$

$y_3 = y(0.6) = 1.856$.

x	0	0.2	0.4	0.6
Euler y	1	1.2	1.48	1.856

Improved Euler Method :-

$$y_{m+1} = y_m + \frac{h}{2}\left[f(x_m, y_m) + f(x_m + h, y_m + h f(x_m, y_m))\right]$$

① Using Improved Euler's Method to find $y(0.1)$.
Given $y' = \frac{y-x}{y+x}$, $y(0) = 1$.

Soln :-

Given, $x_0 = 0$, $y_0 = 1$, $f(x,y) = \frac{y-x}{y+x}$, $h = 0.1$

By Improved Euler Method, we have

$$y_{m+1} = y_m + \frac{h}{2}\left[f(x_m, y_m) + f(x_m + h, y_m + h f(x_m, y_m))\right] \quad \text{①}$$

Put $m=0$ in ①

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_0 + h, y_0 + h f(x_0, y_0))\right]$$

$$f(x_0, y_0) = \frac{y_0 - x_0}{y_0 + x_0} = \frac{1-0}{1+1} = 1$$

$$h f(x_0, y_0) = 0.1(1) = 0.1$$

$$y_0 + h f(x_0, y_0) = 1 + 0.1 = 1.1$$

$$x_0 + h = 0 + 0.1 = 0.1$$

$$f(x_0 + h, y_0 + h f(x_0, y_0)) = f(0.1, 1.1) = \frac{1.1 - 0.1}{1.1 + 0.1} = 0.8333$$

$$f(x_0, y_0) = 1$$

$$f(x_0, y_0) + f(x_0 + h, y_0 + h f(x_0, y_0)) = 1 + 0.8333 = 1.833$$

$$y_1 = 1 + \frac{0.1}{2}(1.8333) = 1.09166$$

$$y(0.1) = 1.09166$$

MODIFIED EULER METHOD :-

$$y_{n+1} = y_n + h f\left[x_n + h/2,\ y_n + h/2\, f(x_n, y_n)\right]$$

① Using Modified Euler Method find $y(0.2)$ & $y(0.1)$ given $dy/dx = x^2 + y^2$, $y(0) = 1$.

Soln :- Given, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$$f(x, y) = x^2 + y^2$$

By modified Euler's Method

$$y_1 = y_0 + \frac{h}{2}\left[x_0 + h/2,\ y_0 + h/2\, f(x_0, y_0)\right]$$

$$f(x_0, y_0) = f(0.1) = 0^2 + 1^2 = 1$$

$$\frac{h}{2} f(x_0, y_0) = \frac{0.1}{2}(1) = 0.05$$

$$y_0 + h/2\, f(x_0, y_0) = 1 + 0.05 = 1.05$$

$$y_1 = 1 + 0.1(1.05) = 1.1105$$

$$\boxed{y(0.1) = 1.1105}$$

$$y_2 = y(1.2) = y_1 + hf[x_1 + h/2 , y_1 + h/2 \, f(x_1, y_1)]$$

$$f(x_1, y_1) = f(0.1, 1.1105)$$

$$= (0.1)^2 + (1.1105)^2 = 1.24321$$

$$\frac{h}{2} f(x_1, y_1) = \frac{0.1}{2}(1.24321) = 0.06216$$

$$y_1 + h/2 \, f(x_1, y_1) = 1.1105 + 0.06216 = 1.172660$$

$$y_2 = 1.1105 + 0.1[f(0.15, 1.172660)]$$

$$= 1.1105 + 0.1[(1.172660)^2 + (0.15)^2]$$

$$= 1.25026$$

$$\boxed{y(0.2) = 1.25026}$$

RUNGE KUTTA METHODS :-

A set of formulae are given without proof for solving a differential equation of the form $dy/dx = f(x, y)$ under the initial condition $y(x_0) = y_0$. Let h denote the length of the interval between equidistant values of x. The various types of formulae according to their order given below.

SECOND RUNGE KUTTA METHOD :-

If the initial values are x_0, y_0 for the differential equation $dy/dx = f(x, y)$ then the first increment in y viz Δy is computed from the formulae

$$\boxed{\begin{aligned} k_1 &= hf(x_0, y_0) \\ k_2 &= hf(x_0 + h/2 , y_0 + k_1/2) \\ \Delta y &= k_2 . \end{aligned}}$$

---

1. Find $y(0.2)$ given $dy/dx = y - x$ by using second order Runge kutta method given $y(0) = 2$, $h = 0.1$.

Soln :- $dy/dx = y - x$; $f(x, y) = y - x$; $y(0) = 2$

(i.e) $x_0 = 0$, $y_0 = 2$

$$\therefore k_1 = hf(x_0, y_0) = 0.1[y_0 - x_0] = 0.1[2-1] = 0.2$$

$$k_2 = hf(x_0 + h/2 , y_0 + k_1/2)$$

$$= 0.1[(y_0 + k_1/2) - (x_0 + h/2)]$$

$= 0.1[(2+\frac{0.2}{2}) - (0+\frac{0.1}{2})]$

$= 0.1[2.1 - 0.05] = 0.1[2.05) = 0.205$

$\Delta y = k_2 = 0.205$

$\therefore y_1 = y_0 + \Delta y$

$y(1) = y(0.1) = 2 + 0.205 = 2.205.$

To find $y(0.2)$

$K_1 = f(x_1, y_1) = y_1 - x_1$

$= 2.205 - 0.1$

$\boxed{K_1 = 2.105}$

$k_2 = hf[x_1 + h/2, y_1 + K_1/2]$

$= 0.1[(y_1 + K_1/2) - (x_1 + h/2)]$

$= 0.1[(2.205 + \frac{2.105}{2}) - (0.1 + \frac{0.1}{2})]$

$= 0.1[3.2575 - 0.15]$

$\boxed{K_2 = 0.3107}$

Now, $\Delta y = k_2 = 0.3107$

$\therefore y_2 = y(0.2)$

$= y_1 + \Delta y$

$= 2.205 + 0.3107$

$\boxed{y_2 = 2.5157}$.

x	0	0.1	0.2
y	1	2.205	2.5157

Fourth order Runge Kutta Method :-

This Method is widely used in problem

The algorithm is given below,

$K_1 = hf(x, y)$

$K_2 = hf(x + h/2, y + K_1/2)$

$K_3 = hf(x + h/2, y + K_2/2)$

$K_4 = hf(x + h, y + K_3)$

$\Delta y = 1/6 (K_1 + 2K_2 + 2K_3 + K_4)$

$y(x+h) = y(x) + \Delta y.$

① Using Runge kutta Method of fourth order Calculate $y(0.1)$, $y(0.2)$ and $y(0.3)$ given that

$dy/dx = \frac{2xy}{1+x^2} + 1$, $y(0) = 0$.

Soln :-

$$f(x,y) = \frac{2xy}{1+x^2} + 1$$

$$x_0 = 0, \quad y = 0, \quad h = 0.1 - 0 = 0.1$$

By fourth order Runge kutta Method for the first interval.

$$k_1 = hf(x_0, y_0)$$

$$= 0.1\left(\frac{2x_0 y_0}{1+x_0^2} + 1\right)$$

$$= 0.1(0+1)$$

$$\boxed{k_1 = 0.1}$$

$$k_2 = hf(x_0 + h/2, \; y_0 + k_1/2)$$

$$= 0.1 f(0.05, 0.05)$$

$$= 0.1\left[\frac{2(0.05)(0.05)}{1+(0.05)^2} + 1\right]$$

$$\boxed{k_2 = 0.1005}$$

$$k_3 = hf(x_0 + h/2, \; y_0 + k_2/2)$$

$$= 0.1 f(0.05, 0.0502)$$

$$= 0.1\left[\frac{2(0.05)(0.0502)}{1+(0.05)^2} + 1\right]$$

$$\boxed{k_3 = 0.1005}$$

$$k_4 = hf(x_0 + h, \; y_0 + k_3)$$

$$= 0.1 f(0.1, 1.1005)$$

$$= 0.1\left[\frac{2(0.1)(0.1005)}{1+(0.1)^2} + 1\right]$$

$$\boxed{k_4 = 0.1019}$$

$$\therefore \Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1/6\,[0.1 + 2(0.1005) + 2(0.1005) + 0.1019]$$

$$\boxed{\Delta y = 0.10066}$$

$y_1 = y(0.1) = y_0 + \Delta y = 0.10066$

$\boxed{y(0.1) = 0.10066}$

Now starting from (x_1, y_1) we get (x_2, y_2)

Again Applying Runge kutta method replacing

(x_0, y_0) by (x_1, y_1) we get

$$k_1 = h f(x_1, y_1)$$
$$= 0.1 f(0.1, 0.1006)$$
$$= 0.1\left[\frac{2(0.1)(0.1006)}{1+(0.1)^2} + 1\right]$$

$\boxed{k_1 = 0.10199}$

$$k_2 = h f(x_1 + h/2, y_1 + k_1/2)$$
$$= 0.1 f(0.15, 0.1516)$$
$$= 0.1\left[\frac{2(0.15)(0.1516)}{1+(0.15)^2} + 1\right]$$

$\boxed{k_2 = 0.1044}$

$$k_3 = h f(x_1 + h/2, y_1 + k_2/2)$$
$$= 0.1 f(0.15, 0.1528)$$
$$= 0.1\left[\frac{2(0.15)(0.1528)}{1+(0.15)^2} + 1\right]$$

$\boxed{k_3 = 0.1044}$

$$k_4 = h f(x_1 + h, y_1 + k_3)$$
$$= 0.1 f(0.2, 0.2051)$$
$$= 0.1\left[\frac{2(0.2)(0.2051)}{1+(0.2)^2} + 1\right]$$

$\boxed{k_4 = 0.10789}$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= \frac{1}{6}[0.10199 + 2(0.1044) + 2(0.1044) + 0.10789]$$
$$= 0.104579$$

$$y_2 = y(0.2) = y_1 + \Delta y$$

$\boxed{y(0.2) = 0.20524}$

Again apply Runge-kutta Method and replacing (x_1, y_1) by (x_2, y_2).

$$k_1 = hf(x_2, y_2)$$

$$= 0.1 f(0.2, 0.20524)$$

$$= 0.1\left[\frac{2\times 0.2\times 0.20524}{1+(0.2)^2}+1\right]$$

$$\boxed{k_1 = 0.107893}$$

$$k_2 = hf\left(x_2+\frac{h}{2}, y_2+\frac{k_1}{2}\right)$$

$$= 0.1 f(0.25, 0.259186)$$

$$= 0.1\left[\frac{2\times 0.2\times 0.259186}{1+(0.2)^2}+1\right]$$

$$\boxed{k_2 = 0.10996}$$

$$k_3 = hf\left(x_2+\frac{h}{2}, y_2+\frac{k_2}{2}\right)$$

$$= 0.1 f(0.25, 0.26022)$$

$$= 0.1\left[\frac{2\times 0.2\times 0.26022}{1+(0.2)^2}+1\right]$$

$$\boxed{k_3 = 0.1100}$$

$$k_4 = hf(x_2+h, y_2+k_3)$$

$$= 0.1 f(0.3, 0.31525)$$

$$= 0.1\left[\frac{2\times 0.3\times 0.31525}{1+(0.3)^2}+1\right]$$

$$\boxed{k_4 = 0.117353}.$$

$$\Delta y = \frac{1}{6}[k_1+2k_2+2k_3+k_4]$$

$$= \frac{1}{6}[0.107893+2(0.1966)+2(0.11004)+0.117353]$$

$$= 0.11076$$

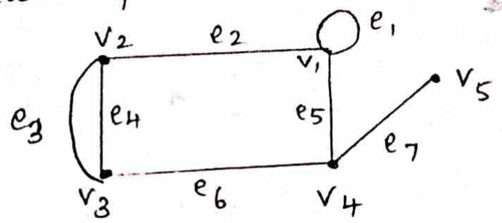
$$y_3 = y_2+\Delta y = 0.2052+0.11076$$

$$\boxed{y(0.3) = y_3 = 0.316001}$$

UNIT - III

GRAPH :-

A graph $G = (V, E)$ consist of two objects called V and E such that $V = \{v_1, v_2, \dots\}$ called vertices and $E = \{e_1, e_2 \dots\}$ called edges and each edge e_k is associated with an unordered pair (v_i, v_j) of vertices.



Graph with 5 vertices and 7 edges.

Loop :-

In a graph G, if an edge e_k has both of its ends as v_i, (i.e) e_k is associated with (v_i, v_i) then e_k is called a Loop.

PARALLEL EDGES (OR) MULTIPLE EDGES :-

In a graph G, if there are more than one edges associated with the same pair of vertices then they are called parallel edges or Multiple edges.

TYPES OF GRAPHS :-

(i) SIMPLE GRAPH :-

If a Graph G, has neither loops nor parallel edges is called a simple Graph.

(ii) MULTIGRAPH :-

If a Graph G has parallel edges and has no loops then it is called a multigraph

(iii) PSEUDOGRAPH :-

A pseudograph is a graph which has both parallel edges and Loops.

(iv) DIRECTED GRAPH :-

In a Graph the edge directions are defined for all edges then it is a directed Graph.

(v) DIRECTED MULTIGRAPH :-

A directed Multigraph is a directed graph with parallel directed edges.

---

① Construct the intersection graph of the following collections of sets.

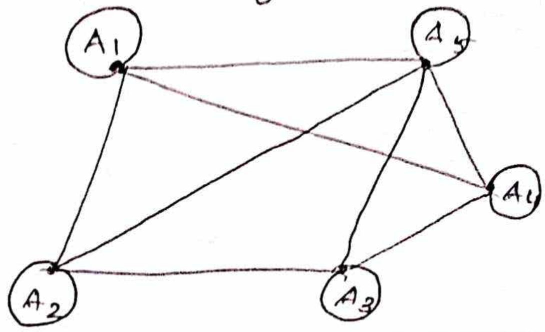
(a) $A_1 = \{0, 2, 4, 6, 8\}$; $A_2 = \{0, 1, 2, 3, 4\}$

$A_3 = \{1, 3, 5, 7, 9\}$; $A_4 = \{5, 6, 7, 8, 9\}$

$A_5 = \{0, 1, 8, 9\}$.

Soln :-

The intersection graph of A_1, A_2, A_3, A_4 & A_5 is.



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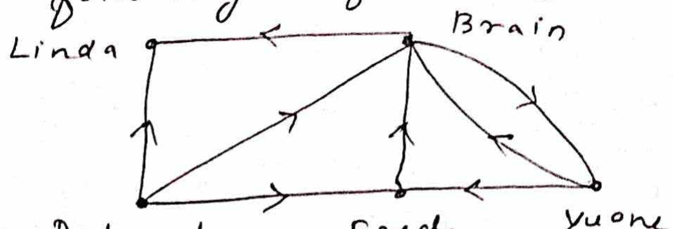
② Who can influence fred and whom can fred in the following influence graph.



Soln :-

Deborah and yuonne can influence fred

Fred can influence Brain.

---

3 INCIDENCE (between vertices and edges)

Let G be a graph, if $e = \{u, v\}$, then the edge e is said to be incident with the vertices u and v.

ADJACENCY (between two vertices) :-

Let G be an undirected graph, two vertices u and v are said to be adjacent in G if there is an edge between u & v.

DEGREE :-

Let G be an undirected graph. The degree of a vertex v in G is the number of edges incident on V, the loop at v contributes twice to the degree of that vertex.

The degree of a vertex is denoted by deg(v).

ISOLATED VERTEX :-

A vertex of degree 0 (zero) is called an isolated vertex.

PENDENT VERTEX :-

A vertex of degree 1 (one) is called an pendent vertex.

---

(1) State and prove Hand shaking Theorem.

Let $G = (V, E)$ be an undirected graph with e edges then,

$$\sum_{v \in V} d(v) = 2e$$

Proof :-

Each Edge in a graph G contributes two degree, Since it is incident on two vertices. This is true for loops also because the degree is counted twice even it is incident with a single vertex.

The degree of a vertex is zero, if there is no edge incident on it.

$\therefore$ The sum of all degrees is equal to twice the number of edges.

$$\sum_{v \in V} d(v) = 2e.$$

(1) How many edges are there in a graph with 5 vertices each of degree with 4?

soln :-

Sum of all degrees is $5 \cdot 4 = 20$

$2e = 20 \Rightarrow e = 20/2 = 10$

$\therefore$ There are 10 edges in the graph.

Theorem :-

The number of vertices of odd degree vertices in an undirected graph is always.

proof :-

Let V_1 & V_2 be the set of vertices of even degree and the set of vertices of odd degree in an undirected graph $G = (V, E)$

$$2e = \sum_{v \in V} \deg(v)$$

$$= \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v) \rightarrow (1)$$

Since $\deg(v)$ is even for all $v \in V_1$, the first term in the R.H.S of equality (1), being the sum of even integers is even.

L.H.S is an even number, the second term in the R.H.S sum is also even.

Since all the terms in this sum is are odd, there must be an even number of sum terms.

$\therefore$ There are an even number of odd degree.

IN-DEGREE & OUT-DEGREE of a Vertex in a directed graph :-

Let $G = (V, E)$ be a directed graph. The in-degree of a vertex v, denoted by $\deg^-(v)$ is the number of edges with v as their terminal vertex.

The out-degree of a vertex v, denoted by $deg^+(v)$, is the number of edges with v as their initial vertex.

Theorem :-

Let $G=(V,E)$ be a graph with directed edges, then

$$\sum_{v \in V} deg^-(v) = \sum_{v \in V} deg^+(v) = |E|.$$

Proof :-

In a directed graph, each edge has an initial vertex and terminal vertex, the sum of the indegrees and the sum of the outdegrees of all the vertices are the same.

Both of these sums are the number of edges in the graph.

UNDERLYING UNDIRECTED GRAPH :-

The undirected graph that results from ignoring directions of all edges of a directed graph is called the underlying undirected graph.

COMPLETE GRAPH :-

The complete graph on n vertices, denoted by K_n, is the simple graph that contains exactly one edge between each pair of distinct vertices.

K_1 K_2 K_3 K_4

CYCLE :-

The cycle C_n, $n \geq 3$, consist of n vertices $v_1, v_2 \ldots v_n$ and edges $(v_1, v_2), (v_2, v_3), (v_3, v_4) \ldots (v_{n-1}, v_n)$ and (v_n, v_1).

Wheel :-

The wheel W_n can be obtained by adding an additional vertex to the cycle C_n for $n \geq 3$, and connecting this new vertex to each of the n vertices in C_n, by 'n' new edges.

BIPARTITE GRAPH :-

A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint non empty sets V_1 & V_2 such that every edge in the graph has one end V_1 & another end V_2.

① Show that C_6 is bipartite ?

Soln :-

The vertex set C_6 can be partitioned in to two disjoint non empty sets $V_1 = \{v_1, v_3, v_5\}$ and $V_2 = \{v_2, v_4, v_6\}$ and every edge of C_6 connects a vertex in V_1 and a vertex in V_2.

② Show that K_3 is not bipartite ?

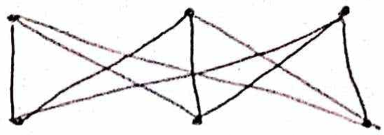
The vertex set of K_3 contain 3 vertices. If we divide this vertex set into two disjoint sets, then one of the two sets must contain two vertices.

If the graph were bipartite, these two vertices cannot be connected by an edge, but in K_3 each vertex is connected to every other vertex by an edge.

$\therefore$ K_3 is not bipartite.

COMPLETE BIPARTITE GRAPH :-

The complete bipartite graph $K_{m,n}$ is the graph that has its vertex set partitioned into two subsets V_1 and V_2 with number of vertices m and n respectively and each vertex of V_1 is connected to every vertex of V_2.



$K_{3,3}$.

SUBGRAPH :-

A subgraph of a graph $G = (V, E)$ is a graph $H = (W, F)$ where $W \subseteq V$ & $F \subseteq E$.

G can also be called as super graph H.

UNION OF TWO GRAPH :-

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs then their union denoted by $G_1 \cup G_2$ is defined to be $G_1 \cup G_2 = (V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$.

---

① Can a simple graph exist with 15 vertices each of degree 5?

Soln :-

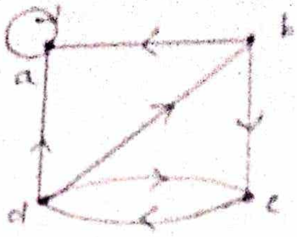
No, a simple graph cannot exist with 15 vertices each of degree 5. Since the degree of vertices is equal to the twice the number of edges, which is even.

But, here each vertex is of degree 5, which is odd, & $15 \times 5 = 75$ sum of degrees also odd

∴ It is not possible, to have a graph with 15 vertices each of degree 5.

---

② In the following graph determine the number of vertices and edges and find the indegree and outdegree of a vertex.



Soln :-

The number of vertices = 4

The number of edges = 7

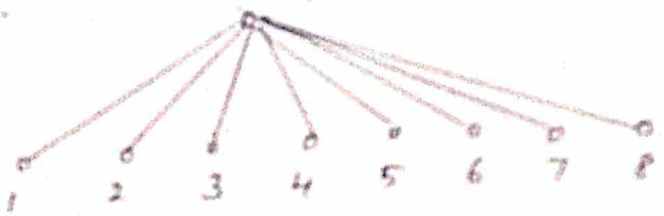
$deg^-(a) = 3$; $deg^-(b) = 1$; $deg^-(c) = 2$, $deg^-(d) = 1$

$deg^+(a) = 0$; $deg^+(b) = 2$; $deg^+(c) = 1$; $deg^+(d) = 3$

③ Draw the following Graphs

(1) $K_{1,8}$

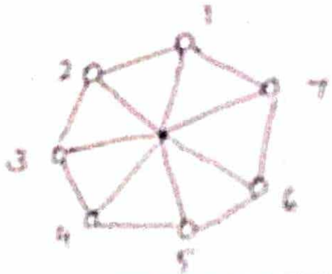
Soln :-



$(K_{1,8})$

(2) W_7

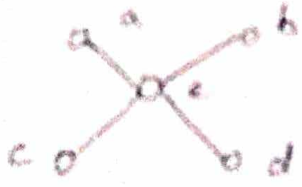
Soln :-



W_7

④ Find whether the graphs are bipartite

(1)



Soln :-

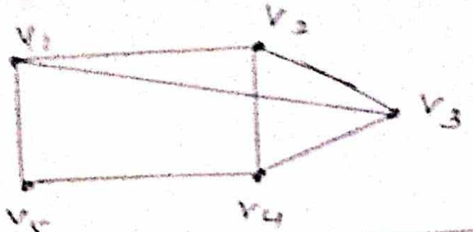
The graph is bipartite with bipartition

$V_1 = \{e\}$; $V_2 = \{a, b, c, d\}$

⑤ Does there exist a simple graph with 5 vertices of degrees 3, 3, 3, 3, 2?

Soln :-

yes, the graph with required property is given below



⑥ Does there exit a simple graph with 5 vertices of the degrees 1, 2, 3, 4, 5?

Soln :-

It is not possible for a simple graph with these vertex degrees. Since in a simple graph with n vertices can have almost $n-1$ degrees.

⑦ Does there exit a simple graph with 5 vertices of the degrees 1, 1, 1, 1, 1?

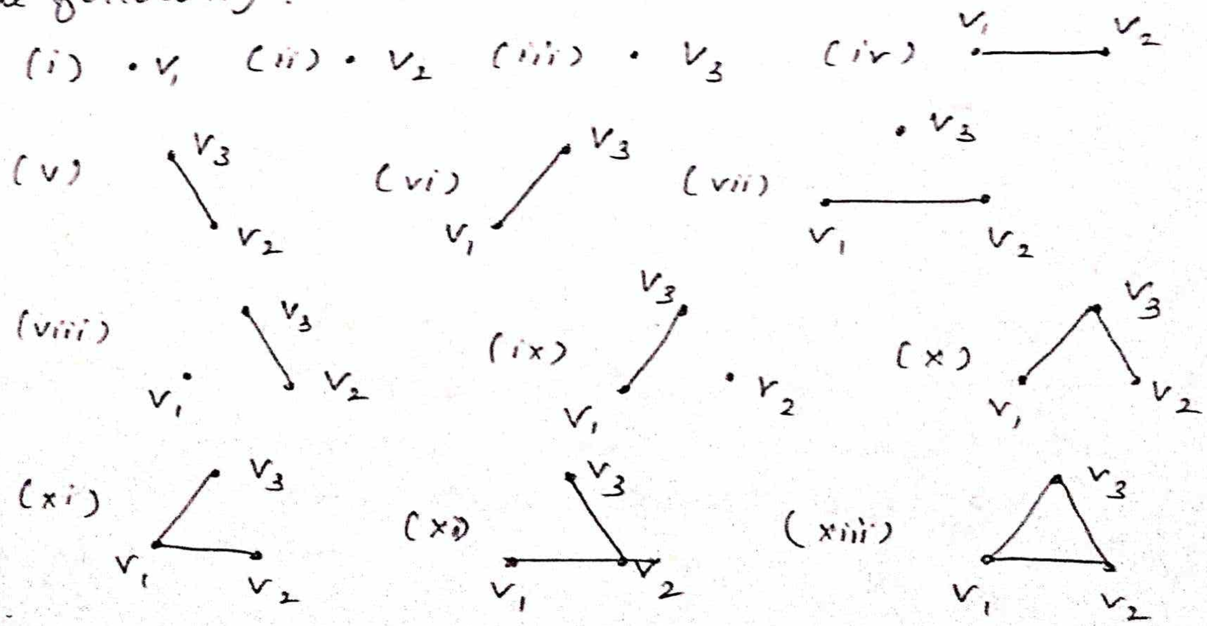
Soln :-

Since the sum of all degrees is equal to 5 which is odd. It is not possible for simple graph with these vertex degree.

⑧ How many subgraph with atleast one vertex does K_3 have?

Soln :-

There are 13 subgraph. The subgraph are following.



(9) For which values of n are the following graphs regular?

(a) K_n (b) C_n (c) W_n (d) Q_n

Soln :-

(a) For all values of n

(b) For all values of n

(c) only for $n = 3$

(d) for all values of n

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(10) For which values of m and n is $K_{m,n}$ regular

Soln :-

The graph $K_{m,n}$ is regular only if $m = n$

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(11) How many vertices does a regular graph of degree 4 with 10 edges have?

Soln :-

Since there are 10 edges $2e = 2 \times 10 = 4v$

(i.e) $20 = 4v$

$v = 20/4 = 5$

$\therefore$ There are 5 vertices.

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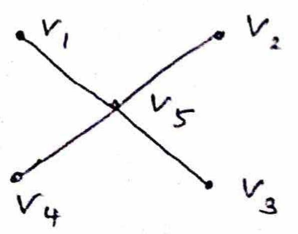
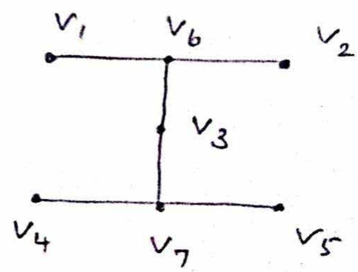
(12) Find the union of the given pair of simple graph?





Soln :-

The union is given below.