B. Stat Major Code

Allied course - II - Statistics and Mathematics - II

Code: 18K2SAS2

UNIT - I

Index numbers - Definition, uses problems in the Constructions of index numbers. Methods of index numbers - Simple aggregate index. weighted index numbers - laspeyer's paasche's and fisher's index numbers. Time reversal and factor reversal tests, Cost of living index numbers - methods of construction. (family budget method) and aggregate expenditure method.

UNIT - II

Statistical decision theory - common elements of decision theory - pay off table - problems, maximin and minimax principles - EMV, EOL, EPPI, EVPI - Problems.

UNIT - III

Differentiation - Definition, formulae Simple problems, Inverse function, Differentiation by transformation, differentiation of implicit function, Higher derivatives, simple problems.

UNIT - IV

Partial derivatives - Definition Homogeneous functions, and Euler's theorem - Statement and simple problems. Rolle's theorem (with proof).

UNIT - V :

Complex numbers - properties Arithmetic Operations, Demoivre's theorem (Statement only), Expansion of $\cos n\theta$, $\sin n\theta$ and expansion of $\cos^n\theta$, $\sin^n\theta$ - Simple problems.

BOOKS FOR STUDY :

1. Dr. S. Arumugam and A. Thangapandi Issac - Calculas volue - I (Differentiation and Applications) (1999)
2. Calculas Vol-I, S. Narayanan, T.K. Manicavasagam pillay (2010)
3. A. Singaravelu - Allied mathematics - I (2002)
4. A. Singaravelu - Allied mathematics - II (1998)
5. S.P. Gupta - Statistical methods (Revised Edition 2001).

UNIT - I.

INDEX NUMBERS.

Definition :

Index numbers are devices for measuring differences in the magnitude of a group of related variables.

- Croxton and cowden.

Characteristics of Index number:

On the basis of the study and analysis of definitions of index numbers, the following points are worth considering;

(1) Index numbers are specialised averages:

Normally, averages can be used to compare only those variables which are expressed in same units. but, index numbers help in comparing the changes in variables, which are in different units.

(2) Index numbers are expressed in percentages:

Index numbers are expressed in terms of percentages so as to show the extent of change. However, percentage sign (%) is never used.

(3) Index numbers measure changes not capable of direct measurement :

Where it is difficult to measure the variation in the effects of a group of variables directly or where the variations are entirely incapable of direct quantitative study, relative variations are measured with the help of index numbers.

(4) Index numbers are for comparison :

The index numbers by their nature are Comparative. They compare changes taking place overtime or between places and like catogaries.

USES :-

(1) They measure the relative change :

Index number are particallarly useful in measuring relative changes. " Index numbers are used to measure the changes to some quantity, which we cannot observe directly".

(2) They are of better comparison :

The index numbers reduce the changes of price level into more useful and understandable form.

(3) They are good guides :

The index numbers are not restricted to the price phenomenon, which is spread over a period to time, is capable of being expressed numerically through index numbers. Thus, various kinds of index numbers serve different uses.

(4) They are Economic barometers :

Various index numbers computed for different purposes, say employment, trade, transport, agriculture, industry, etc., are of immense value in dealing with different economic problems.

(5) They are the pulse of the economy :

The stability of prices or their inflating or deflating conditions can well be observed with the help of indicates

(6) They are the wage adjuster :

In all the fields of economy, the wages adjustments are done with the study of consumer's price index numbers.

(7) They compare the standard of living :

Index numbers may measure the cost of living of different classes of people. Thus, comparison becomes easy in respect of general price index numbers.

(8) They are a special type of averages :

All the basic ideas of average are employed for the construction of index numbers. In average, the data are homogeneous (in the same units): but in index number averages the variables have different units of measurement.

(9) They help in formulating policies :

Formulation of good policies for the future depends upon past trends. For instance, increase or decrease in wages required to study the cost of living index number.

(10) They measure the purchasing power of money :

Index numbers are helpful in finding out the intrinsic worth of money as contrasted with its nominal worth. Generally statements are seen that the purchasing power of Indian rupee in 1997 is only 10 paise as compared to its purchasing power in 1990.

TYPES OF INDEX NUMBERS :

(1) Price Index :

For measuring the value of money, the general price index is used. It is an index number which compares the prices for a group of commodities at a certain time or at a place. with prices of a base period. There are wholesale price index numbers and retail price index numbers. Retail price index reveals the changes in the retail prices of commodities such as consumption goods, bank deposits, bon bonds, etc.

(b) Quantity Index:

Quantity index numbers study the changes in the volume of goods produced or consumed; For instance, industrial production, agricultural production, import, export, etc. They are useful and helpful to study the output in an economy.

(c) Value Index:

These index numbers compare the total value of a certain period. with the total value of the base period. Here the total value is equal to the price of each, multiplied by the quantity for instance, indices of profits, sales, inventories, etc.

Interpretation of Index Numbers.

Index numbers are pure numbers without the units of measurement. Suppose we say the profit for this year is 120% of the profit for the previous year. It means; the previous year's profit was say Rs. 100; this year there is clear that the profit of the current year (this year) is Rs. 120.

Problems in the construction of Index numbers:

(1) purpose or object:

The statistician must clearly determine the purpose for which the index numbers are to be constructed, because there is no all purpose index numbers. Every index number has got its own uses and limitations. For example if we want to study the changes in the value of money, then we have to construct index numbers of wholesale price. Cost of living index numbers of workers in an industrial area and those of the workers of an agricultural area are different in respect of requirements.

(2) Selection of base:

The base period of an index number is very important as it is used for the construction of index numbers. Every index number must have a base. Thus when selecting a base period, the year must be recent and normal. A normal year is one which is free from economic and natural ~~distrib~~ disturbances, widespread failure of rains, earthquakes, war, strikes, production crisis, etc.,

Therefore, the year to be selected as base year must be normal year or a typical year and a recent year.

(a) Fixed base

(b) Average base

(c) Chain base.

(a) Fixed base:

The name reveals that the base year is a fixed one. The prices of a particular year, selected as a base period are treated as equal to 100.

(b) Average base:

Sometimes it is difficult to select an year as base through normality. Under such a critical position, the average of several years is considered better, as abnormalities can be reduced to great extent.

(c) chain base:

In this method, there is no fixed based year. It changes from year to year. when a comparison is desired from year to year, a system of chain base is used.

(3) Selection of commodities: [selection of Regimen]

If we study the price changes of one commodity, we have to include only one item. For instance, if we study the changes in production of cloth, then we may include the production of mill cloth, silk, khadi, etc., and there is no problem Another example, say index of retail price:

(4) source of data:

The price relating to the thing to be measured must be collected. If we want to study the changes in industrial production, we must collect the prices relating to the production of various goods of factories. The collection of data, i.e., price must be representative, comparable and accurate.

(5) Selection Of Averages:

One can use any average. But in practice, the arithmetic average is used, because it is easy for computation; geometric mean and harmonic mean are difficult to calculate. But, geometric mean is preferred because of the following characteristics. (a) geometric mean is the best measure and (b) It gives less harmonic weight to bigger items and more weight to smaller items.

(6) Weighting:

All commodities are not equally important. The main purpose of an index number of prices of an index number of prices, is to ascertain the changes in the price level. In case of simple average, all commodities will have equal importance. Therefore to stress the importance, the system of weighting is adopted.

There are two methods of weighting:

a) Implicit weighting
b) Explicit weighting.

Methods of Construction Of Index numbers.

unweighted Index numbers:

(1) Simple aggregate method.

This is the simplest method of constructing the index numbers. The prices of the different commodities of the current year are added and the total is divided by the sum of the prices of the base year commodity and multiplied by 100. Symbolically,

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

P_{01} = Price index number for the current year with reference to the base year.

ΣP_1 = Aggregate of prices for the current year.
ΣP_0 = Aggregate of prices for the base year.

Steps :-

* Prices of all commodities of current years are added to get ΣP_1.
* Prices of all commodities of base years are added to get ΣP_0
* ΣP_1 is divided by ΣP_0 and multiplied with 100.

Problem - 1 :-

From the following data construct an index for 1999 taking 1998 as base.

Commodity	Price in 1998 [P_0]	Price in 1999 (P_1)
A	50	70
B	40	60
C	80	90
D	110	120
E	20	20
	$\Sigma P_0 = 300$	$\Sigma P_1 = 360$

Price Index $\Big\}$ $P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$

$= \frac{\overset{12}{36\cancel{0}}}{30\cancel{0}} \times 100$

$P_{01} = 120$

2) Simple average of price relative method.

In this method, the price relative of each item is calculated and separately and then averaged. A price relative is the price of the current year expressed as a percentage of the price of the base year, symbolically,

$$P_{01} = \frac{\Sigma\left(P_1/P_0 \times 100\right)}{N}$$

$$P_{01} = \frac{\Sigma P}{N}$$, N = Number of items.

When the geometric mean is employed, in state instead of the arithmetic mean then the formula is,

$$P_{01} = \text{Antilog} \frac{\Sigma \log\left(P_1/P_0 \times 100\right)}{N} = \text{antilog}\left[\frac{\Sigma \log P}{N}\right]$$

where, $P = \frac{P_1 \times 100}{P_0}$.

From the following data construct an index for 2001 taking 2000 as base year by the average of relative methods by using.
a) arithmetic mean b) Geometric mean for averaged relatives.

Commodity	Price in 2000	price in 2001
A	50	70
B	40	60
C	80	90
D	110	120
E	20	20

Index number using arithmetic mean of price relative

Commodity	Price in 2000 (P_0)	Price in 2001 (P_1)	Price relative $P = P_1/P_0 \times 100$
A	50	70	140
B	40	60	150
C	80	90	112.5
D	110	120	109.09
E	20	20	100
Total	$\Sigma P_1 = 300$	$\Sigma P_0 = 360$	$\Sigma P = 611.59$

arithmetic mean, $P_{01} = \frac{\Sigma P}{N}$

$$P_{01} = \frac{\Sigma\left(P_1/P_0 \times 100\right)}{N} = \frac{611.59}{5}$$

$P_{01} = 122.32$

Index numbers using Geometric mean of price relative.

commodity	Price 2000	Price 2001	price relative	log P
A	50	70	140	2.14612
B	40	60	150	2.17609
C	80	90	112.5	2.05115
D	110	120	109.09	2.03778
E	20	20	100	2
				10.41114

Geometric mean, $P_{01} = \text{Antilog}\left(\frac{\Sigma \log P}{N}\right)$

$\Sigma \log P = 10.41$

$P_{01} = \text{Antilog}\left(\frac{10.41}{5}\right) = \text{Antilog}(2.08)$

$= 120.78$

Merits of Simple Average of Price relative method:

* It gives equal importance to all items.
* Extreme items do not unduly affect the index numbers.
* The influence due to different units is completely removed.

Demerits:

* The use of geometric mean involves difficulties of computation.
* It fails to give any consideration to the relative importance of different items.

WEIGHTED INDEX NUMBERS:

(i) Weighted Aggregate Index numbers:

According to this method, prices themselves are weighted by quantities; i.e., $P \times q$. Thus physical quantities are used as weights. Some of the important formulae are given below.

a) Laspeyre's method b) Paasche's method c) Fisher's Ideal method.

A) Laspeyre's Method:

In this method, the base year quantities are taken as weights:

symbolically, $$P_{01}(La) = \frac{\Sigma P_1 q_0 \times 100}{\Sigma P_0 q_0}.$$

STEPS :- * Multiply the current year prices of various commodities with base year weights and obtain $P_1 q_0$.

* Multiply the base year prices of various commodities with base year weights and obtain $P_0 q_0$.

* Divide $P_1 q_0$ by $P_0 q_0$ and multiply the quotient by 100. The above steps give us the price index.

B) Paasche's Method:

In this method, the current year quantities are taken as weights; Symbolically, $$P_{01}(Pa) = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100.$$

STEPS: * Multiply current year prices of various commodities with current year weights and obtain P_1q_1.

* Multiply the base year prices of various commodities with the current year weights and obtain P_0q_1.

* Divide P_1q_1 by P_0q_1 and multiply the quotient by 100.

c) Fisher's Ideal method:

Fisher's Ideal index number is given by the geometric mean of Laspeyres and Paasche's formula: Symbolically

$$P_{01}(F) = \sqrt{L \times P} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}} \times 100$$

2) Weighted average of price relative:

Price relative is not calculated by the weighted aggregate method. If we know the values consumed in the base year, then we can construct the weighted index number according to the weighted averages of relative method.

$$P_{01} = \frac{\frac{P_1}{P_0} \times P_0 q_0}{\Sigma P_0 q_0} \times 100 = \frac{\Sigma PV}{\Sigma V}$$

P = price relative ($\frac{P_1}{P_0} \times 100$)

V = value weight ($P_0 q_0$)

Weighted aggregative method:

Construct index numbers of price from the following data by applying (1) Laspeyre's (2) Paasche's (3) Fisher's Ideal methods.

Commodity	1999		2000	
	price	quantity	price	quantity
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	13

Calculation of various index.

Commodity	1999		2000		P_1q_0	P_0q_0	P_1q_1	P_0q_1
	Price	quant	price	quant				
A	2	8	4	6	32	16	24	12
B	5	10	6	5	60	50	30	25
C	4	14	5	10	70	56	50	40
D	2	19	2	13	38	38	26	26

$\Sigma P_1 q_0 = 200$.

$\Sigma P_0 q_0 = 160$

$\Sigma P_1 q_1 = 130$.

$\Sigma P_0 q_1 = 103$.

Laspeyre's method: $P_{01} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$

$$= \frac{200}{160} \times 100 = 125.$$

Paasche's method ; $P_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100 = \frac{130}{103} \times 100 = 126.2.$

Fisher's Ideal method ;

$$P_{01}(F) = \sqrt{L \times P} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}} \times 100 = \sqrt{\frac{200}{160} \times \frac{130}{103}} \times 100$$

$$= \sqrt{1.25 \times 1.262} \times 100 = \sqrt{1.575} \times 100 = 1.2549 \times 100$$

$$= 125.499 = 125.5.$$

From the following data compute price index by supplying weighted average of price method using (a) A.M (b) G.M.

Commodity	P_0	q_0	P_1
Sugar	3.0	20kg	4.0
Flour	1.5	40kg	1.6
Milk	1.0	10 litre	1.5

Index number using weighted arithmetic mean of price relatives.

commodity	P_0	q_0	P_1	$V = P_0 q_0$	$P = \frac{P_1}{P_0} \times 100$	PV
Sugar	3.0	20	4.0	60	133.3	7998
Flour	1.5	40	1.6	60	106.67	6396
Milk	1.0	10	1.5	10	150	1500

Arithmetic Mean $= P_{01} = \frac{\Sigma PV}{\Sigma V}$

$$= \frac{15900}{130} = 122.31.$$

Index number using G.M of Price relatives :

commodity	P_0	q_0	P_1	$V = P_0 q_0$	P	log P	V log P
Sugar	3.0	20	4.0	60	133.3	2.1248	127.5
Flour	1.5	40	1.6	60	106.6	2.028	121.68
Milk	1.0	10	1.5	10	150	2.176	21.76
							270.94

$$G.M = \text{Antilog}\left[\frac{\Sigma V \log P}{\Sigma V}\right]$$

$$= \text{Antilog}\left[\frac{270.94}{130}\right] = \text{Antilog}(2.084)$$

$$= 121.338$$

Price index by Laspeyre's method.

Commodity	P_0	q_0	P_1	$P_1 q_0$	$P_0 q_0$
Sugar	3.0	20	4.0	80	60
Flour	1.5	40	1.6	64	60
Milk	1.0	10	1.5	15	10
				159	130

$$P_{01} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$$

$$= \frac{159}{130} \times 100$$

$$= 1.2230 \times 100$$

$$= 122.30.$$

Test of adequency of index number of formula:

1) Unit test: The unit test requires that the formula for constructing an index should be independent of the units in which or for which prices and quantities are noted expect for a simple (unweighted) aggregative index are other formula discussed in this chapter satisfy this test.

2) The reversal test: According to professor fisher the formula for Calculating an index number should be such that it gives the same ratio between one point of comparison and the other, no matter which of the two is taken as the base. Symbolically, the fg. relation should be statisfy:

$$P_{01} \times P_{10} = 1.$$

Laspayre's Method :-

$$P_{01} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0}, \quad P_0 = \frac{\Sigma P_0 q_1}{\Sigma P_1 q_1}, \quad P_{01} \times P_{10} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_0 q_1}{\Sigma P_1 q_1}$$

and the test is not statisfy when $P_{01} \times P_{10} \neq 1.$

paasche's method.

$$P_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}, \quad P_{10} = \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}, \quad P_{01} \times P_{10} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}.$$

Marshall worth method let us now, see now fisher's ideal formula statisfy the test.

$$P_{01} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}}$$

changing time (i.e) 0 to 1 and 1 to 0

$$P_{10} = \sqrt{\frac{\Sigma P_0 q_1}{\Sigma P_1 q_0} \times \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}} \times \sqrt{\frac{\Sigma P_0 q_1}{\Sigma P_1 q_0} \times \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}} = \sqrt{1} = 1.$$

3) Factor reversal test:

According to prof. fisher's just as each formula should permit the interchange of the two times without giving inconsistent result, so it ought to permit interchanging the prices and quantities without giving inconsisting result. The total value of a given Commodity in a given year and the product of the quantity and the price per unit.

(TOTAL VALUE = $P \times q$)

The ration of the total value in the one year to the total value in the preciving year is $\frac{P_1 q_1}{P_0 q_0}$. If from one year to the next year both the prices and quantity could double.

$$P_{01} \times Q_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0}.$$

UNIT - II.

Meaning and scope :

Decision making is a process of choosing a best course of action out of several alternative courses for the purpose of achieving a goal or goals. It refer to the selection of an act from amongest various alternative, one which is judged to be the best under given circumstances.

Decisions making is defined as :

"A process which results in the selection from a set of alternative courses, or actions, that course of action which is considered to meet the objectives of the decision problem more statisfactory than others as judged by the decision makes".

Elements of decisions making problem:

I. Decision makes:

The decision makes (who may be an individual or a group of individuals) is interested with the responsibility for making the decision or selection of one act from a set of possible courses of Act.

II. Act :

A decision situation arises only when the decision makes has more than one course of actions open to him. The decision always involves a choice among several alternatives These several alternative are known as acts. Thus the acts are the alternative Course of action or strategies, that are available to a decision makes.

III. States of Nature or events :

These are the occurances, which are beyond the control of the decision makers. The uncontrollable occurance are called outcomes or states of nature and their existence create difficulties as well as interest in decision making under uncertainly.

The status of nature and denoted by $S_1, S_2, S_3, \ldots, S_m$. A common element in most decision making problem is uncertainly.

IV. Pay off:

When the value of consequence is expressed directly in terms of gain expressed in money, it is called a pay off. It can also be termed as cost saving or times saving but the expression of pay off is always in quantitative terms to help precise analysis. Thus a payoff measures the net benefit to the decision maker that occurs from a given combination of decision alternative (i.e., $A_1, A_2, A_3, \ldots$) and events (i.e, $S_1, S_2, \ldots, S_m$).

V. Pay off Table:

A pay off table indicates the relation between all possible states of nature, all possible actions and the value associated with the consequence. Thus there are m, n pay offs in all. The totality of m, n pay offs arranged in a tabular from is called a payoff table.

State of Nature	Acts ($A_1, A_2, A_3, \ldots A_n$)				Conditional Pay offs
	A_1	A_2	A_3	$\ldots\ldots$	A_n
S_1	P_{11}	P_{12}	P_{13}		P_{1n}
S_2	P_{21}	P_{22}	P_{23}		P_{2n}
S_3	P_{31}	P_{32}	P_{33}		P_{3n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$
S_m	P_{m1}	P_{m2}	P_{m3}		P_{mn}.

Pay off = Sales revenue from estimated demand − Total variable cost − fixed cost.

IV. Opportunity loss or regret:

The difference between profit actually derived from a certain decision and that which would have been derived if the decision had been for the event that actually occured is known as the opportunity loss is the loss incurred because of failure to take the best possible action.

$$\left.\begin{array}{c}\text{opportunity}\\ \text{loss}\end{array}\right\} = \begin{array}{c}\text{Highest possible}\\ \text{profit for an}\\ \text{event (state of}\\ \text{nature)}\end{array} (-) \begin{array}{c}\text{Actual profit}\\ \text{obtained from a}\\ \text{particular action taken.}\end{array}$$

VII. Opportunity loss table or regret table:

Table indicates the difference between the payoff due to a certain decision and the best pay off that could have been realised had he known that state of Nature was going to occur. Consider a fixed state of nature S_i the payoffs corresponding to the n strategies are given by $P_{11}, P_{12}, P_{13}, \ldots P_n$. Suppose M_i, is the maximum of these quantities. These if A_1, is used by the decision makes there is loss of opportunity of $M_i = P_{11}$ and so on.

Regret (opportunity loss) table:

Event (states of Nature)	Conditional opportunity loss (Rs) Alternative strategies (Acts)		
	A_1	$A_2 \ldots\ldots$	A_n
S_1	$M_1 - P_{11}$	$M_1 - P_{12} \ldots$	$M_1 - P_n$
S_2	$M_2 - P_{21}$	$M_2 - P_{22} \ldots$	$M_2 - P_{2n}$
S_3	$M_3 - P_{31}$	$M_3 - P_{32} \ldots$	$M_3 - P_{3n}$.
$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$M_m - P_{m1}$	$M_m - P_{m2} \ldots$	$M_m - P_{mn}$.

Working rules:-

Take the highest possible profit of an event.

Take the actual profit obtained from the actual action.

Take the difference between the step 1 and step 2 is the opportunity loss or regret is

$$\left.\begin{array}{c}\text{opportunity}\\ \text{loss}\end{array}\right\} = \begin{array}{c}\text{Value of}\\ \text{Step 1}\end{array} (-) \begin{array}{c}\text{Value of}\\ \text{Step 2.}\end{array}$$

Example:

A factory produces three variables of fountaines pens. The fixed and variables costs and given below.

	Fixed Cost	variable cost (per unit)
Type 1	Rs. 2,00,000	Rs. 10
Type 2	Rs. 3,00,000	Rs. 8
Type 3	Rs. 6,00,000	Rs. 6

The likely demands under the situations are given below:

Demand poor = 25,000 units.

Demand Moderate = 1,00,000 units.

Demand High = 1,50,000 units.

If the price of each type is Rs. 20 prepare the payoff table and regret table after showing the necessary calculation.

Solution:-

Pay off = Sale revenues from the estimated demand − Total variable Cost − fixed cost.

Payoff Table (1000 Rs)

State of Nature (or) Event	Acts T_1	T_2	T_3
D_1	50 (T_1D_1)	−20 (T_2D_1)	−250 (T_3D_1)
D_2	800 (T_1D_2)	880 (T_2D_2)	800 (T_3D_2)
D_3	1300 (T_1D_3)	1480 (T_2D_3)	1500 (T_3D_3)

The maximum payoff in event, D_1 } = Rs. 50.

The maximum payoff in event, D_1 = Rs. 50

The maximum Payoff in event, D_2 = Rs. 880

The maximum Payoff in event, D_3 = Rs. 1500

Regret table or opportunity loss table.

State of Nature (or) Events	Acts T_1	T_2	T_3
D_1	$50-50=0$	$50-(-20)=70$	$50-(-250)=300$
D_2	$880-800=80$	$880-880=0$	$880-800=80$
D_3	$1500-1300=200$	$1500-480=20$	$1500-1500=0$.

Type of decision making situations:

Decision making under certainly:

In case of decision making under certainly. The state of nature is known. In this case that payoff matrix is reduced to on Column only dealing with the state of Nature about which are is certain
* Minimum Criterion: Under this criterion that alternative is selected which maximises the minimum cost data.
* Minimax regrets criterion: Regrets criterion favaces the regret that one might have while arriving at a particular decision. The regret is measured by the difference between the payoff due to certain decision and the best payoff that could have been realized had he known what state of nature was going to occur.

Decision making under condition of risk:

When the state of nature is unknown but on the basis of Objective or empirical data. It is possible to assign probabilities to various States of nature. This method is generally referred to decision making under the condition or risk.

Than decision makes has to choose the Stategy which will give him maximum payoffs in terms of utility. If however the measure is the cost, then the stategy with the lowest Cost is picked up.

Decision making under uncertainly:

In decision making under the condition of uncertainly. We do not have set of Probabilities for known States of nature. In this process the decision makes has to first select a criterion and then decide which decision criterion.

Decision making under uncertainly without use of probability.

(i) Maximum criterion:

Maximum criterion is appropriate if the decision maker is Pessimist in nature. Out of these minimum payoffs he will select the stategy which maximses the minimum payoffs. In other words the strategy which gives the highest minimum pay off's will be Selected. In this way the decision maker try to get best out of worst possible consequenes

(ii) Maximum Criterion:

This criterion is appropriate if the decision maker is optimist in nature the would always think that the state of nature could be best from his point of view i.e., favourable to him.

He believes that, given to the selection of any Strategy, nature will action in a way which provides the greatest reward.

(iii) Laplace criterion:

Under Condition of uncertainly when there is Complete ignorance about the probability of occurence of each State of nature is Same. The decision maker will find out the expected payoffs of all the stategies. It is assumed that the person wishes to invest all of the funds in a plan

Alternative Investment	Economic Condition		
	High Growth (Rs)	Normal Growth (Rs)	Slow Growth (Rs).
Stock	10000	7000	3000
Bonds	8000	6000	1000
Debentures	6000	6000	6000

Determine the best investment plan using each of the following Criteria (i) Maximum (ii) Minimax (iii) Laplace.

Solution:

Payoff table.

States of Nature / acts	S_1	S_2	S_3
A_1	10,000	7000	3000
A_2	8,000	6000	1000
A_3	6,000	6000	6000

Maximam Criterion:

under this criterion we take the minimum payoff of each action out of these actions we select that one which

	Minimum Payoff
A_1	3,000
A_2	1,000
A_3	6,000

Laplace criterion Here the probability of each plan = $1/3$.

$P(A_1) = P_1 = 1/3$ $\quad$ $P(A3) = P_3 = 1/3$

$P(A_2) = P_2 = 1/3$

$$E.M.V(A_1) = 1/3 \times 1000 + 1/3 \times 7000 + 1/3 \times 3000 = \frac{20{,}000}{3}$$

$= Rs.\ 6666.67/.$

$E.M.V(A_2) = \frac{1}{3} \times 8000 + \frac{1}{3} \times 6000 + \frac{1}{3} \times 1000 = \frac{15000}{3}$

$= Rs.\ 5000.$

$E.M.V(A_3) = \frac{1}{3} \times 6000 + \frac{1}{3} \times 6000 + \frac{1}{3} \times 6000 = \frac{18000}{3}$

$= Rs.\ 6000$

Here $E.M.V(A_1)$ is Maximum $E.M.V(A_1) = 6666.6$

EMV - Expected monetary value

EOL - Expected opportunity loss

EVPI - Expected value of perfect information.

Example:

Calculate (a) expected EMV and EP : and (b) EPPI and EVPI (c) EOL from the following payoff table.

Acts → Events or Strategy	A_1	A_2	A_3	probability
S_1	25	-10	-125	0.10
S_2	400	440	400	0.70
S_3	650	740	750	0.20

Solution. (a) :-

Events	probability	Act A_1	A_2	A_3
E_1	0.1	$25 \times 0.1 = 2.5$	$(-10) \times 0.1 = -1.0$	$-125 \times 0.1 = -12.5$
E_2	0.70	$400 \times 0.70 = 280$	$440 \times 0.70 = 308$	$400 \times 0.70 = 280$
E_3	0.20	$650 \times 0.20 = 130$	$740 \times 0.20 = 148$	$750 \times 0.20 = 150$
Total	1.0	$EMV(A_1) = 412.5$	$EMV(A_2) = 455$	$EMV(A_3) = 417.5$

The maximum value of EMV is 455 from act A_2.

Hence the expected payoff EP = 455.

b) EVPI : From the given table, we notice the following

Maximum payoff for the event $E_1 = 25$, $P(E_1) = 0.10$

Maximum payoff for the event $E_2 = 440$, $P(E_2) = 0.70$.

Expected payoff for perfect information (EPPI)

= Sum of (maximum payoff × its probability)

$= (25 \times 0.10) + (440 \times 0.70) + (750 \times 0.20) = 25 + 308 + 150$

$= 460.50$

Hence, EPPI = 460.50 , We know that ,

EVPI = EPPI − EP

$= 460.50 - 455 = 5.5$

EVPI = 5.5

c) EOL :

Maximum payoff for the event , $E_1 = 25$, $P(E_1) = 0.10$
Maximum Payoff for the event , $E_2 = 440$, $P(E_2) = 0.70$
Maximum payoff for the event , $E_3 = 750$, $P(E_3) = 0.20$

Acts / Events	Acts		
	A_1	A_2	A_3
E_1	25−25=0	25−(−10)=35	25−(−125)=150
E_2	440−400=40	440−440=0	440−400=40
E_3	750−650=100	750−740=10	750−750=0

Table expected opportunity loss.

Events	probability	Acts		
		Expected Values of A_1	Expected values of A_2	Expected values of A_3
E_1	0.10	$0 \times 0.10 = 0$	$0.10 \times 35 = 3.5$	$0.10 \times 100 = 15$
E_2	0.70	$0.70 \times 40 = 28$	$0.70 \times 0 = 0$	$0.70 \times 40 = 28$
E_3	0.20	$0.20 \times 100 = 20$	$0.20 \times 10 = 20$	$0.20 \times 0 = 0$
Total	0.1	48	5.5	43

Since, the maximum value of opportunity loss is 5.5 for the Acts A_2, so the optimal Act is A_2.

DIFFERENCIATION.

Definition:

Let y be a continuous function of x, then an increase in the value of x will produce an increase or a decrease in the value of y. Usually the average rate of change of y with respect to x, i.e., the ratio $\Delta y/\Delta x$ tends to a definite limit as Δx tends to zero. This limit, when it exists, is called the differential coefficient of y with respect to x and is denoted by the symbol dy/dx

So, if $y = f(x)$ then $y + \Delta y = f(x + \Delta x)$

$$dy/dx = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

STANDARD FORMS.

Differential Coefficient of x^n.

Let y be equal to x^n when x receives an increament Δx, let Δy denote the corresponding increament in y.

Then $y + \Delta y = (x + \Delta x)^n$.

$\therefore \Delta y = (x + \Delta x)^n - x^n$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{(x+\Delta x)^n - x^n}{\Delta x} \qquad \therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x}$$

$$= \lim_{x+\Delta x \to x} \frac{(x+\Delta x)^n - x^n}{(x+\Delta x) - x}$$

$= n \cdot x^{n-1}$ for all values of n. $\therefore \frac{d}{dx}(x^n) = nx^{n-1}$

Differential co-efficient to e^x.

Let y be e^x corresponding to an increament Δx in x. Let the increament in y be Δy

Then. $y + \Delta y = e^{x+\Delta x}$

$$\therefore \Delta y = e^{x+\Delta x} - e^{x}$$

$$\frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x}.$$

$$= \underset{\Delta x \to 0}{Lt} \frac{e^{x+\Delta x} - e^{x}}{\Delta x} = \underset{\Delta x \to 0}{Lt} \frac{e^{x}(e^{\Delta x} - 1)}{\Delta x}$$

$$= e^{x} \underset{\Delta x \to 0}{Lt} \frac{e^{\Delta x} - 1}{\Delta x} = e^{x}.$$

$$\therefore \frac{d}{dx}(e^{x}) = e^{x}.$$

Differential co-efficient of $\log_e x$.

Let y be $\log_e x$; Let the increament in y corresponding to an increament Δx in x be Δy. Then $y + \Delta y = \log_e (x + \Delta x)$

$$\frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x} = \underset{\Delta x \to 0}{Lt} \frac{\log_e (x+\Delta x) - \log_e x}{\Delta x}$$

$$= \underset{\Delta x \to 0}{Lt} \frac{\log_e \left(1 + \frac{\Delta x}{x}\right)}{\Delta x}$$

Substitute h for $\frac{\Delta x}{x}$. as $\Delta x \to 0, h \to 0$.

$$\therefore \frac{dy}{dx} = \underset{h \to \infty}{Lt} \frac{\log_e (1+h)}{xh} = \frac{1}{x} \underset{h \to 0}{Lt} \log_e (1+h)^{1/h}$$

$$= \frac{1}{x} \log_e e = \frac{1}{x}.$$

Differential coefficient of $\sin x$.

If $y = \sin x$, $y + \Delta y = \sin (x + \Delta x)$

$$\frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{\sin (x + \Delta x) - \sin x}{\Delta x}.$$

$$= \underset{\Delta x \to 0}{Lt} \frac{2 \sin \frac{\Delta x}{2} \cos \left(x + \frac{\Delta x}{2}\right)}{\Delta x} = \underset{\Delta x \to 0}{Lt} \frac{\sin \Delta x/2}{\Delta x/2} \lim_{\Delta x \to 0} \cos\left(x + \frac{\Delta x}{2}\right)$$

$= 1 \cdot \cos x$ (as x is measured in radians)

$$\therefore \frac{d}{dx}(\sin x) = \cos x.$$

Differential co-efficient of cos x.

If $y = \cos x$, then $y + \Delta y = \cos(x + \Delta x)$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos x}{\Delta x} = \lim_{\Delta x \to 0} \frac{-2\sin\frac{\Delta x}{2} \cdot \sin\left(x + \frac{\Delta x}{2}\right)}{\Delta x}$$

$$= -\lim_{\Delta x \to 0} \frac{\sin \Delta x/2}{\frac{\Delta x}{2}} \lim_{\Delta x \to 0} \sin\left(x + \frac{\Delta x}{2}\right)$$

$= -1 \times \sin x$ (x being in circular measure).

$\therefore \frac{d}{dx}(\cos x) = -\sin x.$

Differential co-efficient of tan x.

If $y = \tan x$, then $y + \Delta y = \tan(x + \Delta x)$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\tan(x+\Delta x) - \tan x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x)\cos x - \sin x \cos(x+\Delta x)}{\cos(x+\Delta x)\cos x \cdot \Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} \cancel{\lim_{\Delta x \to 0}} \frac{1}{\cos(x+\Delta x)} \cdot \frac{1}{\cos x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} \cdot \lim_{\Delta x \to 0} \frac{1}{\cos(x+\Delta x)} \cdot \frac{1}{\cos x} = 1 \cdot \frac{1}{\cos x} \cdot \frac{1}{\cos x}.$$

$= \sec^2 x$ (~~cos~~ x is measured in radians) $\therefore \frac{d}{dx}(\tan x) = \sec^2 x.$

Differential co-efficient of a sum or difference:

Let $y = u + v - w$, where u, v, w are functions of x.

Let $\Delta y, \Delta u, \Delta v, \Delta w$ be the increments on y, u, v, w respectively corresponding to an increment Δx in x.

Then $y + \Delta y = u + \Delta u + v + \Delta v - (w + \Delta w)$

$\therefore \Delta y = \Delta u + \Delta v - \Delta w$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta x} - \frac{\Delta w}{\Delta x}.$$ when $\Delta x \to 0$, we have,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}.$$

Hence the differential co-efficient of sum of a finite number of functions is equal to the sum of the differential co-efficients of those functions.

For example: The differential co-efficient of $ax^2 + 2bx + c$ is $2ax + 2b$.

Similarly, $d/_{dx}(\sin x + k\cos x) = \frac{d}{dx}(\sin x) + \frac{d}{dx}(k\cos x)$

$= \cos x + k\, d/_{dx}(\cos x) = \cos x - k\sin x.$

$d/_{dx}(a-bx)^2 = d/_{dx}(a^2 - 2abx + b^2x^2) = -2ab + 2b^2x$

$d/_{dx}(x^{1/2} - 2)^2 = d/_{dx}(x - 4x^{1/2} + 4) = 1 - 4 \times 1/2\, x^{-1/2} = 1 - \frac{2}{x^{1/2}}.$

Product Rule:

Let $y = uv$ where u and v are functions of x. Let $\Delta y, \Delta u, \Delta v$ be the increments in y, u, v respectively corresponding to an increament in x.

Then $y + \Delta u = (u + \Delta u)(v + \Delta v)$

$\therefore \Delta y = (u + \Delta u)(v + \Delta v) - uv = u\Delta v + v\Delta u + \Delta u\Delta v$

$$\therefore \frac{\Delta y}{\Delta x} = u\frac{\Delta v}{\Delta x} + v\frac{\Delta u}{\Delta x} + \frac{\Delta u}{\Delta x}\Delta v$$

when Δx and therefore $\Delta u, \Delta v, \Delta y$ all tend to zero.

$\frac{\Delta y}{\Delta x}, \frac{\Delta u}{\Delta x}, \frac{\Delta v}{\Delta x}$ tend to $\frac{dy}{dx}, \frac{du}{dx}, \frac{dv}{dx}.$

respectively, $\frac{\Delta u}{\Delta x}\Delta v$ will tend to $\frac{\Delta u}{\Delta x} \times 0$ i.e., to zero

$$\therefore \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}.$$

Hence the differential co-efficient of the product of two functions = first function × differential co-efficient of the second function + the second function × differential coefficient of the first function.

Examples: Ex. 1: $\frac{d}{dx}\left\{x(x^2-1)(x^2+4)\right\}$

Let $y = x(x^2-1)(x^2+4)$

By, $\frac{dy}{dx} = (x^2-1)(x^2+4) + 2x^2(x^2+4) + 2x^2(x^2-1)$

Ex. 2:- $\frac{d}{dx}(e^x \sin x \log x)$

Let $y = e^x \sin x \log x$.

By, $dy/dx = e^x \sin x \log x + e^x \cos x \log x + \frac{e^x \sin x}{x}$.

Quotient Rule:

Let $y = u/v$ where u and v are functions of x. when x becomes $x+\Delta x$. let y, u, v become $y+\Delta y$, $u+\Delta u$, $\Delta + \Delta v$ respectively.

$\therefore y+\Delta y = \frac{u+\Delta u}{v+\Delta v}$ $\therefore \Delta y = \frac{u+\Delta u}{v+\Delta v} - \frac{u}{v} = \frac{v\Delta u - u\Delta v}{v(v+\Delta v)}$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{u\frac{\Delta u}{\Delta x} - u\frac{\Delta v}{\Delta x}}{v^2 + v\Delta v}$$

In the limit, when $\Delta x \to 0$ and therefore $\Delta u, \Delta v$ and Δy tend to zero we get,

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

Hence the derivative of a quotient.

denominator × derivative of numerator − numerator × derivative of denominator.