# ALLIED COURSE - III

## STATISTICS AND MATHEMATICS III

### UNIT - I

Analysis of Time series - uses components of time series, Measurement of trend - free hand method, semi-average method, moving average method and method of least squares - problems.

### UNIT - II

Business forecasting : steps in forecasting, Methods of forecasting, choice of forecasting - theories of business forecasting - cautions of business forecasting.

### UNIT - III

Integration - Definition . Important results (simple problem) Integration by the method of substitution (an important formulas). Trigonometric substitution - Simple problems.

### UNIT - IV

Integration of Rational algebraic function.

Type I - $\int P(x) / Q(x)$ - Problems. Integration by the method

method of Partial functions – simple problems.

Type II - Partial functions.

Type III - $\int \frac{dx}{ax+bx+c}$ and simple problems.

## UNIT-V

Reduction formula for $\int \sin^n x\,dx$, $\int \cos^n x\,dx$, $\int \tan^n x\,dx$ - simple problems. Beta function and Gamma function definition and properties simple problems.

# ALLIED COURSE - III

# STATISTICS AND MATHEMATICS - III

## UNIT - I

## TIME SERIES AND FORECASTING

### DEFINITION

Any sequences of measurement taken on a response, that is variable over time is called a time series. It can be written as (t) for t = 1, 2, 3. . . .

A set of data depending on the time is called time series.

- Kenny

A time series is a set of statistical observations arranged in chronological order.

- Morris Humburg.

### ANALYSIS OF A TIME SERIES:

The time series analysis consists of :

(i) Identifying or determining the various fores (or) influences.

(ii) Isolating, studying, analysis and measuring them independently.

The time series analysis is of great importance not only to businessman or to an economics but also to People working in various disciplines in natural, social and physical sciences

## USES OF TIME SERIES:

- It enables us to study the past behaviour of the phenomenon under consideration.
- It helps us to compare the changes in the values of different phenomena at different places etc.,

## MEASUREMENT OF TREND:

The methods which are used to measure the trend.

1. Free hand or graphic method.
2. Method of semi-Averages.
3. Method of moving Averages.
4. Method of least squares.

## 1. FREEHAND OR GRAPHIC METHOD:

This is the simplest and the most flexible method of estimating the secular trend and consists of first obtaining a histogram plotting the time series values on a graph paper and then drawing a freehand smooth curve through these points so that it accurately reflects the long term tendency of the data.

PROBLEM:

From the following data, of the solieris figures determine the trend like by freehand curve method.

| year | 1978 | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 |
|---|---|---|---|---|---|---|---|
| Solarios | 60 | 80 | 70 | 100 | 80 | 120 | 110 |

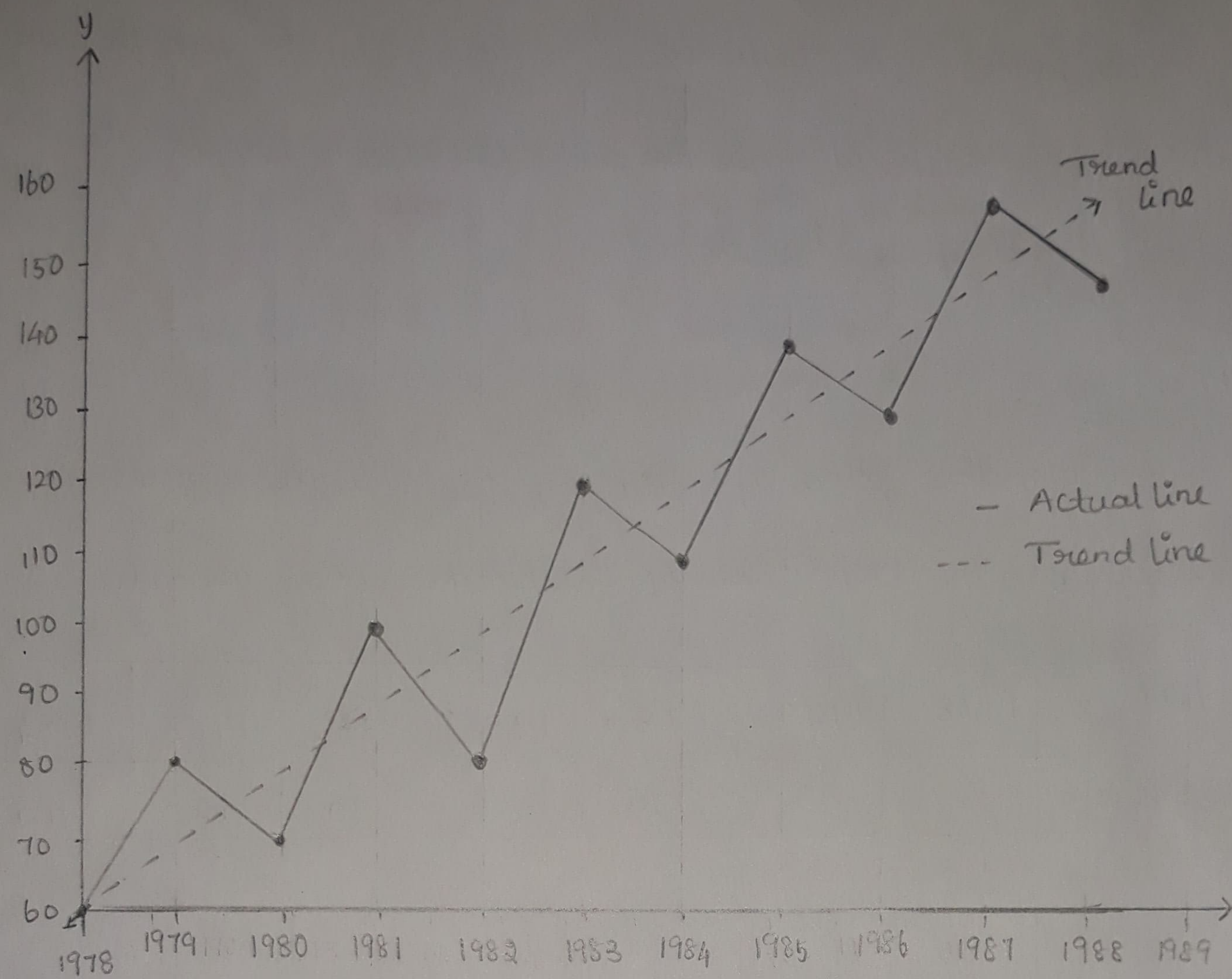
| 1985 | 1986 | 1987 | 1988 |
|---|---|---|---|
| 140 | 130 | 160 | 150 |

Note:

The following points may be considered:

* It should be smooth
* The number of points above the trend curve should be more or less equal to the number of points below it.
* The sum of vertical deviation is equal in both above and below line.
* The sum of squares of vertical deviations of the given points from the trend line should be minimum possible.

Solution:




## 2. SEMI - AVERAGE METHOD:

The average value of those two halves would be calculated. These average values would be plotted against the mid value of each half. The two points are then joined by a straight line which can be extended on either side. This line would be the trend line by the method of semi-averages.

PROBLEM:

Fit a trend line to the following data by the method of semi Average.

| year | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 |
|---|---|---|---|---|---|---|---|
| Output | 600 | 800 | 1000 | 800 | 1200 | 1000 | 1400 |

Soln :-

Since, in this problem the given data have seven years. Therefore the medical year shall be left out and an arrange average of the first three years and the last 3 years shall be uptieded.

$$\text{Average of first 3 year} = \frac{600 + 800 + 1000}{3}$$

$$= 800$$

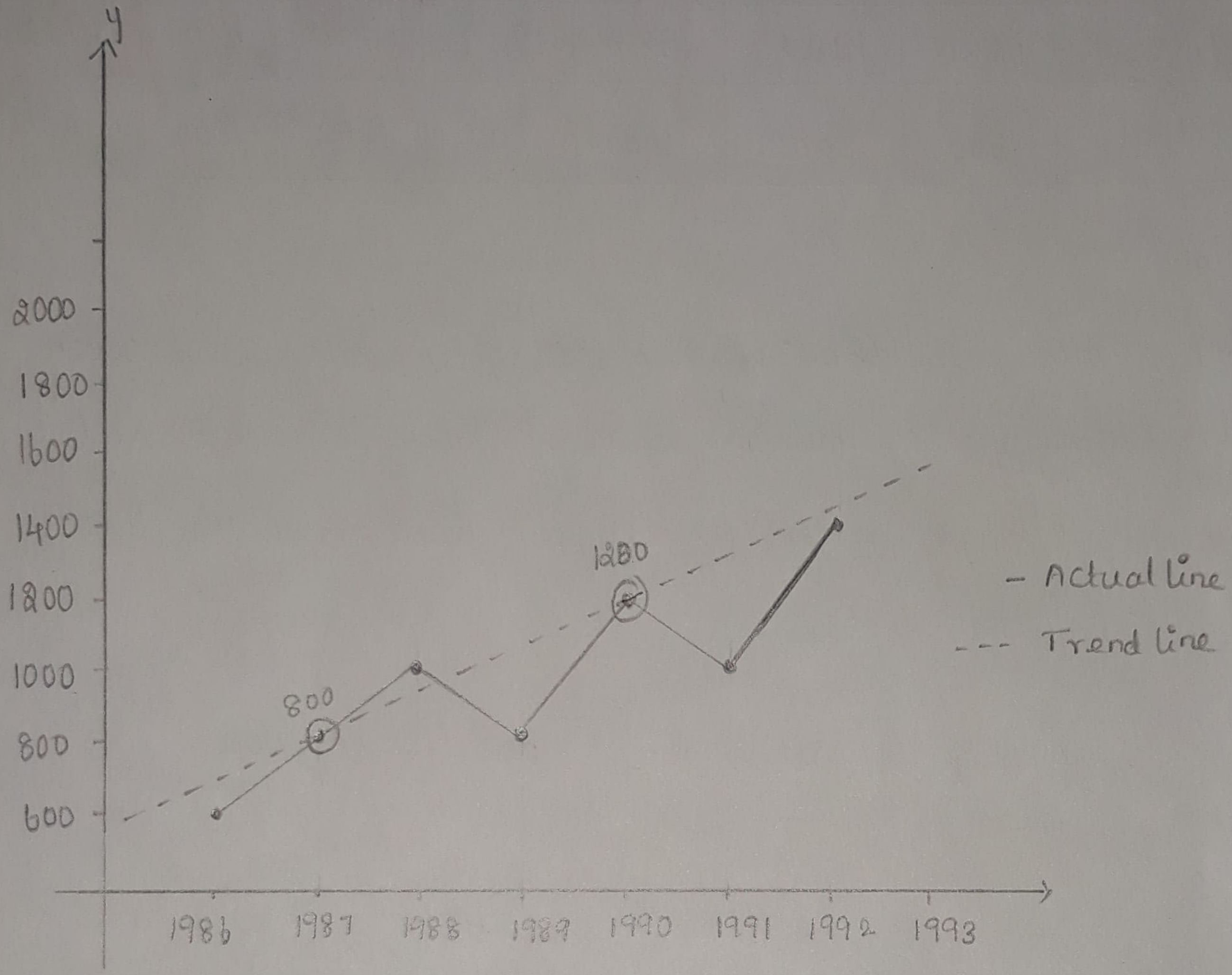
$$\text{last of first 3 year} = \frac{1200 + 1000 + 1400}{3}$$

$$= 1230$$

Average of the first 3 year that is 800 will be plotted the midpoint to the first 3 year. That is the year 1987 and the semi-average.

Average of the last 3 years that 1200 would be

Plotted against the midpoint to the last 3 year that is the year 1990 and the semi-average by joining this Point we shall uptieded the required trend lines.




3. METHOD OF MOVING AVERAGES:

This method is a simple device of reducing fluctuations and obtaining trend values with a fair degree of accuracy. This method consists of taking arithmetic mean of the values of a certain time span and placing it at the centre of the time span. The average value of a year is taken as the trend value for the middle point of the period of moving averages.

## WEIGHTED MOVING AVERAGE:

It is obtained on dividing the weighted moving totals by the sum of weight.

$$\boxed{W.M.A. = \frac{\sum w_1 x_1}{\sum w_1}}$$

## PROBLEM:

Calculate 3 yearly moving Average of the data given below:

| Years | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 |
|---|---|---|---|---|---|---|---|---|
| Sales (M. in RS). | 3 | 4 | 8 | 6 | 7 | 11 | 9 | 10 |

| 1988 | 1989 |
|---|---|
| 14 | 12 |

Soln

Calculating of 3 year moving Average.

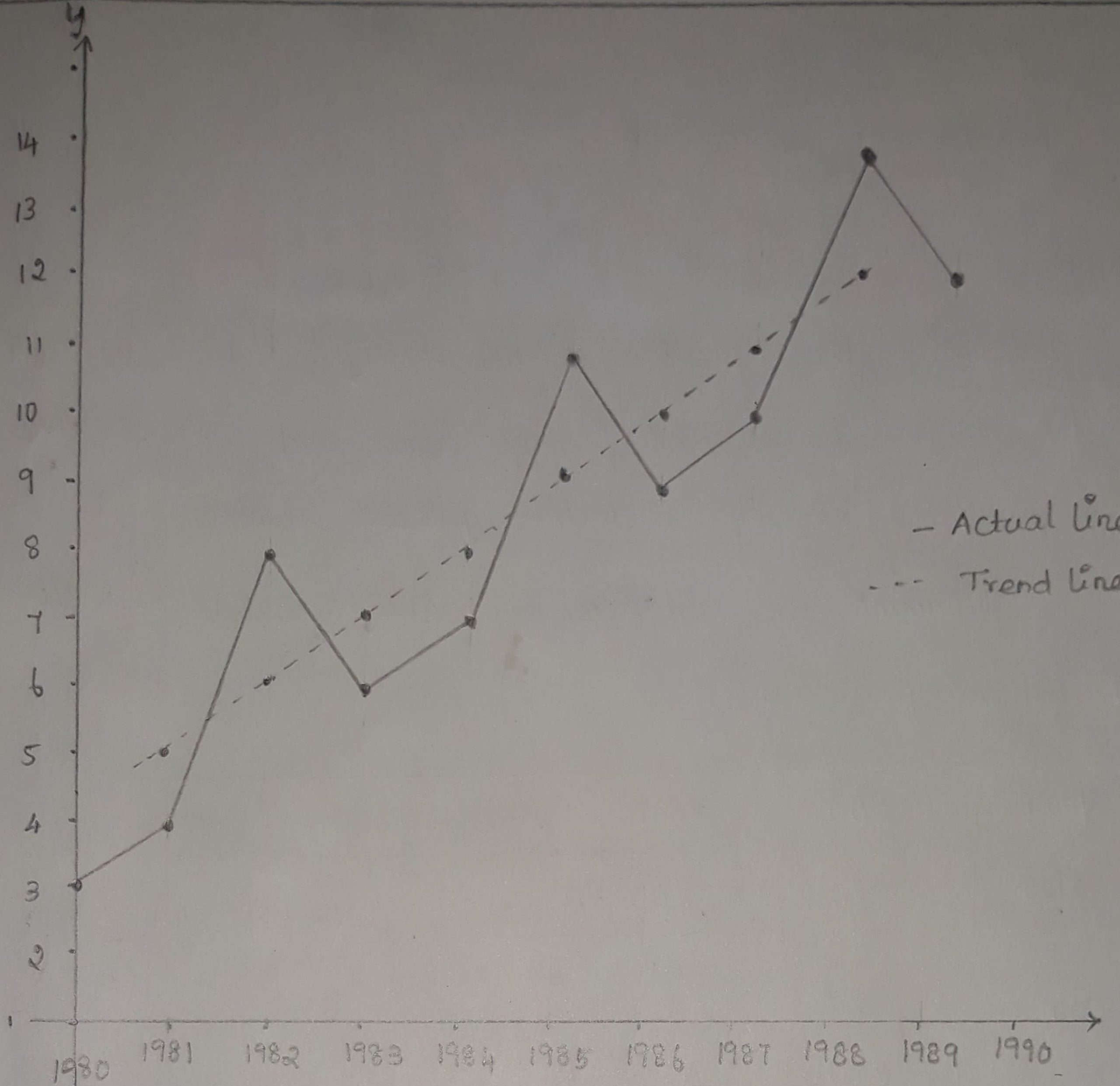
| Year | sales | 3 yearly totals | 3 yearly moving Average (trends) |
|---|---|---|---|
| 1980 | 3 | | |
| 1981 | 4 | 15 | $\frac{15}{3} = 5$ |
| 1982 | 8 | | |

| | | | |
|---|---|---|---|
| 1983 | 6 | 18 | 18/3 = 6 |
| 1984 | 7 | 21 | 21/3 = 7 |
| 1985 | 11 | 24 | 24/3 = 8 |
| 1986 | 9 | 27 | 27/3 = 9 |
| 1987 | 10 | 30 | 30/3 = 10 |
| 1988 | 14 | 33 | 33/3 = 11 |
| 1989 | 12 | 36 | 36/3 = 12 |
| | | | TOTAL = 68 |

Steps: odd the values of the 1st three year (namely 1980, 1981, 1982. that is 3+4+8 = 15) and place of the Total against the medid year 1981.

[Ex: $\frac{a+b+c}{3}$, $\frac{b+c+d}{3}$, $\frac{c+d+e}{3}$, $\frac{d+e+f}{3}$].

Steps: leave the first year and odd up the values of next three 1981, 1982, 1983 that is 4+6+8=18 and the place of total against the medial year 1982.




4. METHOD OF LEAST SQUARES :

Sum of sequare the deviations would be the least as compared to the sum of squares of the deviations obtained by using other lines.

The straight line trend has on equation of the type : $y = a + bx$.

$\Sigma y = Na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

N = number of year for which data.

$$\Sigma y = Na \Rightarrow a = \frac{\Sigma y}{N} = \bar{y}$$

$$\Sigma xy = b\Sigma x^2 \Rightarrow b \frac{\Sigma xy}{\Sigma x^2}$$

PROBLEM:

Determine the equation of the straight line of best fit with the origin at the method year 2002, and unit of $x$ as 1 year be $y = a + bx$ by the method of least squares.

Determine the equation of a straight which best fit the following data.

| year | 2000 | 2001 | 2002 | 2003 | 2004 |
|---|---|---|---|---|---|
| Sales | 35 | 56 | 79 | 80 | 40 |

Soln :-

The values a or b.

$$a = \frac{\Sigma y}{N}$$

$$b = \frac{\Sigma xy}{\Sigma x^2}$$

$$y = a + bx$$

Calculation for the line of best fit:

| Year | Sales (Y) | X 2002 | $X^2$ | XY |
|---|---|---|---|---|
| 2000 | 35 | -2 | 4 | -70 |
| 2001 | 56 | -1 | 1 | -56 |
| 2002 | 79 | 0 | 0 | 0 |
| 2003 | 80 | 1 | 1 | 80 |
| 2004 | 40 | 2 | 4 | 80 |
| | 290 | 0 | 10 | 0 44 |

$$Y = a + bx$$

$$a = \frac{290}{5}$$

$$= 58$$

$$b = \frac{34}{10}$$

$$= 3.4$$

Substituting this value in (1)

$$Y = 58 + 3.4x$$

$$Y = 58 + 3.4(0)$$

$$Y = 58.$$

# UNIT-II

# BUSINESS FORECASTING

When the estimates of future conditions are made on systematic basis, the process is referred to as "forecasting" and the figure or statement obtained is known as a "forecast".

## STEPS IN FORECASTING:

(1) understanding why changes in the past have occurred.

(2) Determining which phases of business activity must be measured.

(3) selecting and compiling data to be used to as measuring devices.

(4) Analysis the data

## METHODS IN FORECASTING:

(1) Business Barometers

(2) Extrapolation

(3) Regression Analysis

(4) Economic Models

(5) Correcting by the use of time series Analysis.

(6) opinion Polling.

(7) Casual Models

(8) Exponential smoothing

(9) survey method.

(1) BUSINESS BAROMETERS:

Of great assistance in practical forecasting is a series that can be used " index" or "indicator" is also ~~als~~ study though loosely, used in business statistics.

The following are some of the important series which aid businessman in forecasting:

Gross national product, Employment, wholesale Prices, Consumer prices, Industrial production, Volume of bank deposits and currently outstanding, Consumer credit, Disposable personal income, Department store sales, stock prices, Bond yields.

(2) EXTRAPOLATION:

Extrapolation is the simplest for yet often a useful method of forecasting in many forecasting situations the most reasonable expectation is that the variable will follow its already established path Extrapolation relies on the relative constancy in the pattern of past movements in some time series.

(3) REGRESSION ANALYSIS:

Regression relationship may involves on Predicted or dependent may and one independent variable - simple regression, or it may involve relationship between the variables to be forecast and several independent variables - multiple regression.

(4) ECONOMETRIC MODELS:

Economic techniques which originated in the eighteenth country, have recently gained in Popularity for forecasting.

## (5) FORECASTING BY THE USE OF TIME SERIES ANALYSIS:

The time series analysis helps to identify and explain :

a) Any regular or systematic variation in the series of data which is due to seasonality the "seasonals".

b) cyclical patterns

c) Trends in the data

d) Growth rates of these trends.

unfortunately, most existing methods identify only the seasonals, the combined effects of trends and cycles and the irregular or chance component. That is, they do not seperate trend from other.

## (6) OPINION POLLING:

opinion poll is the survey of opinion of experts, ie, knowledgable persons in the field whose views carry lot of weight.

## (7) CASUAL MODELS:

A Casual model is the most sophisticated kind of forecasting tool. It expresses mathematically the relevant caused relationships and market survey information, If may also directly incorporate the results of a time series analysis.

(8) EXPONENTIAL SMOOTHING:

This method is an outgrowth of the recent attempts to maintain the smoothing function of moving averages without their corresponding drawbacks and limitations.

THEORIES OF BETWEEN BUSINESS FORECASTING:

Several theories have been developed out of researchers conducted by individual and institutions on business forecasting.

1) SEQUENCE OR TIME-LOG THEORY:

This is by far the most important theory of business forecasting. It is based on the assumption that means most of the business data have the log and lead relationship. There is time-lag between different movements.

## (2) ACTION AND REACTIONS THEORY:

This theory is based on two assumption

- every action has a reaction
- Magnitude of the original action influences the reaction.

we find four phases of a business cycle.

- Prosperity
- Decline
- Depression and
- Improvement.

## 3) ECONOMIC RHYTHM THEORY:

The basic assumption of this theory is that history repeats itself and hence the exponents of this theory believe that economic phenomena behave in a rhythmic order. cycles of nearly the same intensity and duration bend to recur.

## 4) SPECIFIC HISTORICAL ANALYSED:

This theory is based on a more realistic assumption ie, that all business cycles are not uniform in amplitude or duration and as such the use of history is made not by projecting and fancied economic rhythm into the future.

5) CROSS- SECTION ANALYSIS:

This theory is based on the knowledge and interpretain of current forces rather than Projection of past trends. The theory assumes that no two cycles are alike but the like factors bearing upon a given situations are assembled and relying upon the knowledge of economic processes, the forecast concludes the situations is favourable or not.

CAUTIONS WHILE USING FORECASTING TECHNIQUES:

Forecasting business conditions is a complex task which cannot be accomplished with exactness.

The economic, social and political forces which shape the future.

It may be pointed out that forecasting is much more than projecting a series mechanically.

Though future is some sort of extension of the past but it can hardly be expected to be an exact replica.

# UNIT - III

## INTEGRAL CALCULAS

We have so far considered the problem of differentiation, being given $y = f(x)$, find $\frac{dy}{dx}$, Now we Pass on the process called integration which may be regarded as the inverse of differentiation.

Some of them are necessary to commit them to memory,

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ for all values of $n$ expect when $n = -1$.
- In the case when $n=1$, $\frac{dx}{x} = \log x + c$.
- $\int e^x dx = e^x$
- $\int \sin x\, dx = -\cos x$
- $\int \cos x\, dx = \sin x$
- $\int \sec^2 x\, dx = \tan x$
- $\int \operatorname{cosec}^2 x\, dx = -\cot x$
- $\int \sec x \tan x\, dx = -\operatorname{cosec} x$
- $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x$

- $\int \cosh x\, dx = \sinh x$
- $\int \sinh x\, dx = + \cosh x$
- $\int \frac{dx}{1+x^2} = \tan^{-1} x$ (or) $- \cot^{-1}$
- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$, (or) $- \cos^{-1} x$
- $\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x$ (or) $\log (x + \sqrt{x^2+1})$
- $\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1} x$ (or) $\log (x + \sqrt{x^2+1})$
- $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x$ (or) $- \operatorname{cosec}^{-1} x$.

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PROBLEM:

1. Evaluate $\int \left(ax + \frac{b}{x^2}\right) dx$.

Soln:

$$\int \left(ax+\frac{b}{x^2}\right)dx = \int a\, x\, dx + \int \frac{b}{x^2}\, dx.$$

$$= \int \left(ax + \frac{b}{x^2}\right) dx$$

$$= a \int x\, dx + b \int 1/x^2\, dx$$

$$= \frac{ax^2}{2} - \frac{b}{x}$$

PROBLEM:

Evaluate : $\int \left(x+\frac{1}{x}\right)^2 . dx$

Soln :

$$= \int \left(x+\frac{1}{x}\right)^2 dx \qquad \left[\text{use } [a+b]^2 = a^2+b^2+2ab\right]$$

$$= \int \left(x^2+2+\frac{1}{x^2}\right) dx$$

$$= \frac{x^3}{3} + 2x - \frac{1}{x}$$

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PROBLEM:

Evaluate : $\int \frac{3x^2+4x-5}{\sqrt{x}} . dx$

soln :

$$= \int \left(\frac{3x^2+4x-5}{\sqrt{x}}\right) . dx$$

$$= 3 \int \left(\frac{3x^2}{\sqrt{x}} + \frac{4x}{\sqrt{x}} - \frac{5}{\sqrt{x}}\right) dx$$

$$= 3 \int x^{3/2} dx + 4 \int x^{1/2} dx - 5 \int x^{-1/2} dx.$$

$$= 3 \, \frac{x^{3/2+1}}{3/2+1} + 4 \, \frac{x^{1/2+1}}{1/2+1} - 5 \, \frac{x^{-1/2+1}}{-1/2+1}$$

$$= 3\,\frac{x^{5/2}}{5/2} + 4\,\frac{x^{3/2}}{3/2} - 5\,\frac{x^{1/2}}{1/2}$$

$$= \frac{6}{5}\,x^{5/2} + \frac{8}{3}\,x^{3/2} - 10\sqrt{x}$$

---

Evaluate : $\int \frac{(x^2+4x)(2x-3)}{x^3}\,dx$

Soln :

$\therefore (x^2+4x)(2x-3)$

$$= \frac{2x^3 - 3x^2 + 8x^2 - 12x}{x^3}$$

$$= \int \frac{(x^2+4x)(2x-3)}{x^3}\,dx$$

$$= \int \left(2 - \frac{3}{x} + \frac{8}{x} - \frac{12}{x^2}\right) dx$$

$$= 2x + 5\log x - 12\left(-\frac{1}{x}\right)$$

$$= 2x + 5\log x + \frac{12}{x}$$

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PROBLEM :

EVALUATE $\int \tan^2 x\,dx$

Soln :

$\because [1 + \tan^2 x = \sec^2 x]$

$$= \int (\sec^2 x - 1)\,dx$$

$$= \int \sec^2 x\,dx - \int dx$$

$$= \tan x - x$$

## INTEGRATION BY THE METHOD OF SUBSTITUTION:

By putting suitable substitution we can convert the given integrals into standard form.

### IMPORTANT FORMULAS:

(1) $$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)}, \; n \neq -1$$

(2) $$\int \frac{dx}{ax+b} = \frac{\log(ax+b)}{a}$$

(3) $$\int e^{ax+b} \cdot dx = \frac{e^{ax+b}}{a}$$

(4) $$\int \sin(ax+b)\, dx = \frac{-\cos(ax+b)}{a}$$

(5) $$\int \cos(ax+b)\, dx = \frac{\sin(ax+b)}{a}$$

(6) $$\int \sec^2(ax+b)\, dx = \frac{\tan(ax+b)}{a}$$

(7) $$\int \operatorname{cosec}^2(ax+b)\, dx = \frac{-\cot(ax+b)}{a}$$

EVALUATE : $\int \frac{\sin^2 x}{1+\cos x} . dx$

<u>Soln</u>

$$\int \frac{\sin^2 x}{1+\cos x} . dx$$

$$= \int \frac{\sin^2 x \,(1-\cos x)}{(1+\cos x)\,(1-\cos x)} . dx$$

$$= \int \left( \frac{\sin^2 x - \sin^2 x \cos x}{1^2 - \cos^2 x} \right) . dx$$

$$= \int \left( \frac{\sin^2 x - \sin^2 x \cos x}{\sin^2 x} \right) . dx$$

$$= \int \frac{\cancel{\sin^2 x}\,(1-\cos x)}{\cancel{\sin^2 x}} . dx$$

$$= \int (1-\cos x) . dx$$

$$= \int dx - \int \cos x . dx$$

$$= x - \sin x$$

EVALUATE: $\int \frac{1}{(4x+5)^2}\,dx$

Sol:

$$\int \frac{1}{(4x+5)^2}\cdot dx$$

$$= \int (4x+5)^{-2}\cdot dx$$

$$= \frac{(4x+5)^{-2+1}}{(-2+1)\times 4}$$

$$= \frac{(4x+5)^{-1}}{-4}$$

$$= -\frac{1}{4(4x+5)}$$

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EVALUATE:

$$\int \sin^3 4x \cdot dx$$

Soln:

W.K.T. $\sin 3x = 3\sin x - 4\sin^3 x$

$$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$$

$$\therefore \int \sin^3 4x\, dx = \int \left(\frac{3\sin 4x - \sin 12x}{4}\right) dx$$

$$= \frac{1}{4}\left\{\int 3\sin 4x\, dx - \int \sin 12x\, dx\right\}$$

$$= \frac{1}{4}\left[-\frac{3\cos 4x}{4} + \frac{\cos 12x}{12}\right]$$

(8) $\int \sec(ax+b)\tan(ax+b)\,dx = \dfrac{\sec(ax+b)}{a}$

(9) $\int \operatorname{cosec}(ax+b)\cot(ax+b)\,dx = -\dfrac{\operatorname{cosec}(ax+b)}{a}$

PROBLEM:

EVALUATE $\int (2x+1)^3\,dx$.

$$= \frac{(2x+1)^{3+1}}{2(3+1)}$$

$$= \int (2x+1)^3\,dx$$

$$= \frac{(2x+1)^4}{4 \times 2} = \frac{(2x+1)^4}{8}$$

EVALUATE $\int \sqrt{1+3x}\,.dx$

Sol.

$$\int \sqrt{1+3x}\,.dx$$

$$= \int (1+3x)^{1/2}\,dx$$

$$= \frac{(1+3x)^{1/2+1}}{3/2 \times 3} = 2/9\,(1+3x)^{3/2}$$

## INTEGRATION BY USING TRIGONOMETRIC SUBSTITUTION:

By using trigonometric substitution, we can evaluate the integral whose is in the form of $\frac{1}{\sqrt{x^2-a^2}}$, $\frac{1}{\sqrt{a^2-x^2}}$, $\frac{1}{\sqrt{x^2+a^2}}$.

### PROBLEM:

EVALUATE $\int \frac{dx}{\sqrt{a^2+x^2}}$

Soln :-

Put $x = a\tan\theta$

$dx = a\sec^2\theta \, d\theta$

$$\therefore \int \frac{dx}{\sqrt{a^2+x^2}} = \int \frac{a\sec^2\theta \, d\theta}{\sqrt{a^2+a^2\tan^2\theta}}$$

$$= \int \frac{a\sec^2\theta \, d\theta}{a\sqrt{1+\tan^2\theta}}$$

$$= \int \frac{\sec^2\theta}{\sec^2\theta} d\theta = \theta$$

$$= \log\left[\sqrt{\tan\theta+1} + \tan\theta\right]$$

$$= \log\left[\sqrt{\frac{x^2}{a^2}+1} + \frac{x}{a}\right]$$

$$= \log\left[\frac{\sqrt{x^2+a^2}}{a} + \frac{x}{a}\right]$$

$$= \log\left[\frac{\sqrt{x^2+a^2}+x}{a}\right]$$

$$= \log\left(\sqrt{x^2+a^2}+x\right) - \log a \quad \left[\log\frac{m}{n} = \log m - \log n\right]$$

$$= \log\left(\sqrt{x^2+a^2}+x\right) + c \quad [c = \log a].$$