Sub. Code: 18K3MLE01

(1)

Non - Major Elective Course : I

Mathematics

UNIT – I

1.1. Introduction:

Operation Research is the study of Optimisation techniques. The existence of optimisation techniques can be traced at least to the days of Newton and Lagrange. Rapid development and invention of new techniques occured since the world war II essentially because of the necessary to win the war with the limited resources available. Different teams had to do research on military operations in order to invent techniques to manage with available resources so as to obtain the desired objective. Hence the nomenclature Operation Research or Resource Management Techniques.

Scope or Uses or Applications of O.R (Some O.R. Models):

O.R. is useful for solving :-

(i) Resource allocation problems

(ii) Inventory control problems

(iii) Maintenance and Replacement Problems

(iv) Sequencing and Scheduling Problems

(v) Assignment of jobs to applicants to maximise total Profit or minimize total cost.

vi) Transportation Problems

(vii) Shortest route problems like travelling sales person problems.

(viii) Marketing Mangement problems

(ix) Finance Management Problems.

(x) Production, Planning and control Problems

(xi) Design problems.

(xii) Queuing problems, etc to mention a few.

1.2: Role of Operation research in Business and Management

1. Marketing Management Operations research techniques have definitely a role to play in.

a) Product selection

b) Competitive strategies

c) Advertising strategy etc.

2. Production Management

The O.R techniques are very useful in the following areas of production management.

(a) Production Scheduling

(b) Project scheduling

(c) Allocation of resources

(d) Location of factories and their sizes

(e) Equipment replacement and Maintenance

(f) Inventory policy etc.

3. Fenance Management

The techniques O.R are applied to Budgeting and investment areas and especially to

(a) Cash flow analysis

(b) Capital requirement

(c) Credit policies

(d) Credit risks etc.

4. Personal Management

(a) Recruitment policies and

(b) Assignment of jobs are some of the areas of personnel management where O.R. techniques are useful.

5. Purchasing and Procurement

(a) Rules for purchasing

(b) Determining the quantity

(c) Determine the time of purchases are some of the areas where O.R. techniques can be applied.

6. Distrubution:

In determining,

(a) location of ware houses

(b) size of the ware houses

(c) rental outlets

(d) Transportation strategies O.R. techniques are useful.

1.3. Role of O.R in Engineering:

(i) Optimal design of water resources systems
(ii) Optimal design of structures
(iii) Production, planning, Scheduling and control.
(iv) Optimal design of electrical networks.
(v) Inventory control.
(vi) Planning of maintenance and replacements of equipment.
(vii) Allocation of resources of services to maximize the benefit
(viii) Design of material handling
(ix) Optimal design of machines
(x) Optimum design of control systems
(xi) Optimal selection of sites for an industry to mention a few.

1.4 Classification of Models:-

Iconic Model:-

This is a physical, or pictorial representation of various aspects of a system.
Example: Toy, Miniature model of a building, scaled up model of a cell in biology etc.

Example: A linear programming problem, an assignment problem, transportation problem etc.

Dynamic model:-

Dynamic model is a model which considers time as one of the important variables.

Example: A Dynamic programming problem. A replacement problem.

Deterministic model: This is a model which does not take uncertainity into account.

Example: A linear programming problem, an assignment problem etc.

Stochastic model: This is a model which considers uncertaininty as an important aspect of the problem.

Example: Any stochastic programming problem, Stochastic inventory models etc.

Descriptive model:- This is one which just describes a situation or system.

Example: An opinion poll, any survey.

Predictive model: is one which predicts something based on some data. Predicting election results before actually the counting is completed.

Analogue or Schematic Model:

This uses one set of properties to represent another set of properties which a system under study has.

Example: A network of water pipes to represent the flow of current in an electrical network or graphs, organisational charts etc.

Mathematical model (or) Symbolic model:-

This uses a set of mathematical symbols to represent the decision variables of a system under consideration. These variables related by mathematical equations or inequalities which describes the properties of the system.

Example:

A linear programming model, A system of equations representing an electrical network or differential equations representing dynamic systems etc.

Static Model:

This is a model which does not take time into account. It assumes that the values of the variables do not change with time during a certain period of time horizon.

<u>Prescriptive model</u>: is one which prescribes or suggests a course of action for a problem.

<u>Example</u>: Any programming (linear, nonlinear, dynamic, geometric etc) problem.

<u>Analytical model</u>: is a model in which exact solution is obtained by mathematical methods in closed form.

<u>Simulation model</u> is representing the reality through the use of a model or device which will react in the same manner as reality under a given set of conditions. Once a simulation model is designed, it takes only a little time, in general, to run a simulation on a computer.

It is usually less mathematical and less time consuming and generally least expensive as well, in many situations.

<u>Example</u>: queuing problems, inventory problems.

1.5 <u>Some characteristics of a good Model</u>:

(i) It shoul be reasonably simple.

(ii) A good model should be capable of taking into account new changes in the situation affecting its frame significantly with ease.

ie updating the models should be as simple and easy as possible.

(iii) Assumptions made to simplify the model should be as small as possible.

(iv) Number of variables used should be as small as possible.

(v) The model should be open to parametric treatment

1.6. Principles of Modelling :-

(i) Do not build up a complicated model while a simple one will suffice.

(ii) Beware of moulding the problems to fit a techniques

(iii) Deduction must be made carefully.

(iv) Models should be validated prior to emplementation

(v) A model should neither be pressed to do not criticised for failing to do that for which it was never entended.

(vi) Beware of overselling the model in cases where assumption made for the construction of the model can be challenged.

(vii) The solution of a model cannot be more accurate than the accuracy of the information that goes into the construction of the model.

(viii) Models are only aids in decision making

(ix) Models should be as accurate possible.

1.7 : General Methods for Solving O.R. Models.

(i) Analytic Procedure : Solving models by classical Mathematical techniques like differential calculus, finite differences etc. to obtain analytic solutions.

(ii) Iterative Procedure : Starts with a trial solution and a set of rules for improving it by repeating the procedure until further improvement is not possible.

(iii) Monte-Carlo technique : Taking sample observations, computing probability distributions for the variable using random numbers and constructing some functions to determine values of the decisions variables.

1.8 Main Phases of O.R :

(i) Formulation of the problems : Identifying the objectives, the decision variables involved and the constraints that arise involving the decision variables.

(ii) Construction of a Mathematical model:

Expressing the measure of effectiveness which may be total profit, total cost, utility etc. to be optimised by a mathematical function called objective function. Representing the constraints like budget constraints, raw materials constraints resources constraints, quality constraints etc, by means of mathematical equations or inequalities.

(iii) Solving the model constructed: Determining the solution by analytic or iterative or monte-carlo method depending upon the structure of the mathematical model.

(iv) Controlling and updating: A solution which is optimum today may not be so tomorrow. The values of the variables may change, new variables may also emerge. The structural relationship between the variables may also undergo a change. All these are determined in updating.

(V) Testing the model and its solution (ie) -, Validating the model: checking as far as possible either from the past available data or by expertise and experience whether the model gives a solution which can be used in practice.

(VI) Implementation : Implement using the solution to achieve the describe goal.

UNIT – II

2.1 Linear Programming Problems.

Linear Programming problems deals with the optimization (Maximization or Minimization) of a function of decisions variables (The variables whose values determine the solution of a problem are called decision variables of the problem) known as objective function, subject to a set of simultaneous linear equations known as constraints.

The term linear means that all the first degree in the problems under consideration and the term programming means the process of determining a particular course of action.

2.2. Requirement for employing LPP Technique:

1. There must be a well defined objective functions
2. There must be alternative course of action to choose.
3. At least some of the resources must be in limited supply, which give rise to constraints
4. Both the objective function and constraints must be linear equations or inequalities.

2.3 Mathematical Formulation of L.P.P

If x_j $(j=1,2,\ldots n)$ are the n decision variables of the problem and if the system is subject to m constraints the general mathematical model can be written in the form

Optimize $Z = f(x_1, x_2, \ldots x_n)$

Subject to $g_i(x_1, x_2, \ldots x_n) \leq = \geq b_i, (i = 1, 2, \ldots m)$

and $x_1, x_2, \ldots x_n \geq 0$.

Procedure for forming a LPP Model:

Step: 1

Identify the unknown decision variables to be determined and assign symbols to them.

Step: 2 : ~~Ide~~ Identify all the restrictions or constraints in the problem and express them as linear equation or inequalities of decision variables.

Step: 3 Dentify the objective or aim and represent it also as a linear function of decision variables.

Step: 4 Express the complete formulation of LPP as a general mathematical model.

Example: 1 A firm manufactures two types of products A and B and sells them at a profit of Rs 2 on type A and Rs. 3 on type B. Each product is processing on two machines M_1 and M_2. Type A requires 1 minute of processing time on M_1 and 2 minutes on M_2. Type B require 1 minute on M_1 and 1 minute on M_2. Machine M_1 is available for not more than 6 hours 40 minutes while M_2 is available for 10 hours during any working day. Formulate the problem as a LPP so as to maximize the profit.

Solution :

Let the firm decide to produce x_1 units of product A and x_2 units of product B to maximize its profit.

To produce these units of type A and type B products, it requires x_1+x_2 processing minutes on M_1
$2x_1+x_2$ processing minutes on M_2.

Since machine M_1 is available for more than 6 hours and 40 minutes and machine B is available for 10 hours during any working day, the constraints are,

$$x_1 + x_2 \le 400$$
$$2x_1 + x_2 \le 600 \ \&$$
$$x_1, x_2 \ge 0.$$

Since the profit from type A is Rs. 2 and Profit from type B is Rs. 3, the total profit is $2x_1 + 3x_2$.
As the objective is to maximize the profit, the objective function is maximize $Z = 2x_1 + 3x_2$.

∴ The complete formulation of the LPP is

Maximize $Z = 2x_1 + 3x_2$.

subject to the constraints

$$x_1 + x_2 \le 400$$
$$2x_1 + x_2 \le 600 \ \& \ x_1, x_2 \ge 0.$$

Example : 2 (Production Allocation Problems)

A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below.

Machine	Time per unit (minutes)			Machine capacity (minutes/day)
	Product 1	Product 2	Product 3	
M_1	2	3	2	440
M_2	4	—	3	470
M_3	2	5	—	430

It is required to determine the number of units to be manufactures for each product daily. The profit per unit for product 1, 2 and 3 is Rs 4, Rs 3 and Rs. 6 respectively. It is assumed that all the amounts products are consumed in the market. Formulate the mathematical model for the problems.

Solution:

Let x_1, x_2 and x_3 be the number units of products 1, 2 and 3 produced respectively.

To produce these amount of product 1, 2 and 3, it requires:

$2x_1 + 3x_2 + 2x_3$ minutes on M_1,

$4x_1 + 3x_3$ minutes on M_2

$2x_1 + 5x_2$ minutes on M_3.

But the capacity of the machine M_1, M_2 and M_3 are 440, 470 and 430 (minutes / day).

$\therefore$ The constraints are

$$2x_1 + 3x_2 + 2x_3 \leq 440$$
$$4x_1 + 3x_3 \leq 470$$
$$2x_1 + 5x_2 \leq 430$$
and $x_1, x_2, x_3 \geq 0$.

Since the profit per unit product 1, 2 and 3 is Rs. 4, Rs. 3 and Rs. 6 respectively, the total profit is $4x_1 + 3x_2 + 6x_3$. As the objective is to maximize the profit, the objective function is maximize $z = 4x_1 + 3x_2 + 6x_3$.

$\therefore$ The complete formulation of the LPP is

Maximize $z = 4x_1 + 3x_2 + 6x_3$.

$\therefore$ The complete formulation of the LPP is

Maximize $Z = 4x_1 + 3x_2 + 6x_3$.

Subject to the constraints,

$$2x_1 + 3x_2 + 2x_3 \le 440$$

$$4x_1 + 3x_3 \le 470$$

$$2x_1 + 5x_2 \le 430$$

and $x_1, x_2, x_3 \ge 0$.