DIGITAL DESIGN

Subject Code:18K5CS08

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UNIT-I

Number System:Review of Decimal Number System –Binary Number System-Binary to Decimal Conversion-Decimal to Binary Conversion-Hexadecimal Number System-Hexadecimal to decimal Conversion-Decimal to Hexa Decimal

To Binary Conversion-Binary to Hexa Decimal Conversion-Octal Number System-Octal to Decimal Coversion-Decimal to Octal Conversion-Octal to Binary Conversion –Binary to octal Conversion.

UNIT-II

Binary Arithmetic: Binary Addtion-Binary Subtraction – Binary Multiplication-Binary Division-1's Complement and 2's Complements- Subtraction using Complements-Signed Binary Numbers-Binary Code. BCD Codes-8421 code -2421 and 4221 codes-Excess 3 Codes- Gray Codes-ASCII Code.

NUMBER SYSTEM

- A computer programm is process the binary data represented in binary system
- To used binary system in for a long time.
- The concept to binary system to decimal system later to hexadecimal system.
- Convert one number system to one number system to corresponding another number system.
- Decimal Number System:
- This is decimal number use ten digits namely 0,1,2,3,4,5,6,7,8,9,0
- This base are base or radix of ten.
- All the digits in decimal Number system expressed in $10^{0,}$ 10^1,10^2,10^3
- 10^{-1},10^{-2},10^{-3} fractional Part
- The Quantities 10^0,10^1,10^2,10^3 and 10^{-1},10^{-2},10^{-3} are called weights.
- Ex:25=20+5 Ex : 438 = 400 + 30 + 8 Ex 46.25 = 40+6+0.2+0.05
- 2 x 10 + 5 x 1 4 x100 + 3 x10 + 8 x 1 =4 x10 +6x1+2x0.1+5x0.01
- $2 \times 10^1 + 5 \times 10^0$ $4 \times 10^2 + 3 \times 10^1 + 8 \text{ X } 10^0$ $^{=}4 \times 10^1 + 6 \times 10^1 + 2 \times 10^1 + 5 \times 10^{-2}$

Binary Number System

- Binary number system used only two symbols or digits 0 and 1
- A 0 and 1 is called a bit
- The bits have a power of $2^0, 2^1, 2^2, 2^3$ and fractional part $2^{-1}, 2^{-2}, 2^{-3}$
- A four bit binary number is called as a nibble
- A 8 bit binary called a byte
- A 16 bit word called as word
- 32 bit binary called as a double word
- Binary to Decimal Conversion
- i) $(101)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
- $= 4 + 0 + 1$
- $=(5)_{10}$
- Ii)$(10011)_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^1 + 1 \times 2^0$
- $= 16+0+0+2+1$
- $=(19)_{10}$

Decimal to Binary Conversion

- A decimal number converted to binary dividing by 2
- And Collecting the remainders (Double dabble method)
- 2| 19 ↑ (LSB)
- |9
- |4
- |2
- |1
- 0 (MSB)
- Collecting the remainders in the reverse $(19)_{10}$ =$(10011)_{10}$
- Decimal in fraction
- To multiplied by 2 suceesively and collecting the carries from top to bottom
- 0.625 X 2 =1.250 (MSB)
- 0.250 x 2 = 0.500
- 0.500 x 2 =1.000 ↓ (LSB)
- Collecting Carries: $(0.625)_{10}$ = $(0.101)_2$

Hexadecimal Number System

- The hexadecimal number system base is 16
- The base digits are 0,1,2,3,4,5,6,7,8,9,0,A,B,C,D,E,F
- The base of the hexadecimal number system is 16 and then binary number system is 2
- A hex digit represents the group of four bit binary sequence

Hex	Dec	Hex	Dec
0	0	8	8
1	1	9	9
2	2	A	10
3	3	B	11
4	4	C	12
5	5	D	13
6	6	E	14
7	7	F	15

Decimal	**Hexa decimal**	**Binary $2^3 + 2^2 + 2^1 + 2^0$ 8+4+2+1**
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

The hexadecimal four bit binary equivalence decimal number 0 to 15

HEXADECIMAL TO DECIMAL CONVERSION

- The hex to decimal conversion in similar binary to decimal conversion
- The weights used for $16^0, 16^1, 16^2$
- For the fractional part $16^{-1}, 16^{-2}, 16^{-3}$
- Hexadecimal number system have a hexadecimal point
- Convert a hexadecimal number $D5_H$
- $(D5)_{16} = (13 \times 16^1 + 5 \times 16^0)$
- $= (13 \times 16 + 5 \times 1)_{10}$
- $= (208 + 5)_{10}$
- $= (213)_{15}$
- EX:
- $(3FC.8)_H = 3 \times 16^2 x\ 15 x\ 16^1 x 12 x 16^0 + 8 x 16^{-1}$
- $= 3x256 + 15x16 + 12x1 + 8x1/16$
- $= 768 + 240 + 12 + 0.5$
- $= (1020.5)10$
- $(E9)_H = 14x16^1 + 9x16^0$
- $= 14 \times 16 + 9x1$
- $= 224 + 9$
- $= (233)_{10}$

Decimal to Hexadecimal Conversion

- Decimal number ton hex
- Divide the decimal Number by 16
- Collect the remainder to top to the bottom.
- Ex:
- 16 |213
- |13
- 0   - 13 =>D
- $(213)_{10}=(D5)_H$
- Convert the fractional part of decimal number multiply 16 repeatedly and collect carries
- 0.5 x 16 = 8.0
- $(0.5)_{10} = (0.8)_H$
- Convert to decimal to hex equivalents
- EX :65,534 EX:98.625
- 16|16,534
- |4095 - 14 =>E (LSB)
- |255 - 15=>F
- |15 - 15=>F
- 0 - 15 =>F (MSB)

- The hex number 98.625 as separated as the integer part 98 and fractional part
- The integer 98 is converted into hex by repeated division by 16
- 16|98
- |6
- 0
- $(98)_{10}$ = $(62_{)H}$
- The fraction 0.625 is converted in to hex multiplying 0.625 by 16 and collecting the carry
- 0.625 x 16
- 10.000
- The carry obtained is 10 to hex as ‘A’
- Therefore,
- $(98.625)_{10}$ = (62.A)

HEXADECIMAL TO BINARY CONVERSION

- Convert a decimal number to binary dividing given decimal number 2
- The hexadecimal number system is 16 is equal to 2^4 convert a hexadecimal number to binary .
- Replace each hex digit with is equivalent 4-bit binary.

He x	Dec	Binary ($2^3+2^2+2^1+2^0$)
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111

Hex	Dec	Binary ($2^3+2^2+2^1+2^0$))
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

- Convert the hexadecimal numbers to binary
- Ex:$(3A.7)_H$
- Integer part taken 3A . 3 and A.
- Convert to binary equivalent
- $(3A)_H = (0011\ 1010)_2$
- The fractional part $(.7)_H$ converted to binary the four bit equivalent for 7
- $(.7)_H = (.0111)_2$
- $(3A.7)_H = (0011\ 1010\ 0111)_2$

- Ex:$(CD:E8)_H$
- $(CD)_H = (1100\ 1101)_2$
- $(.E8)_H = (.1110\ 1000)_2$

- $(CD.E8)_H = (1100\ 1101\ 1110\ \ 1000)_2$

BINARY TO HEXA DECIMAL CONVERSION

- To Convert a binary number to hex
- Arrange the bits into group of 4 bits starting from LSB
- If the final group has less than 4 bits,just inculde zeros to make it group of 4 bits.
- Integer part or fractional part must be represented by a group of 4 bits
- Integer 4-bits group are fromed starting from hexadecimal point and moving towards left
- Fractional part 4 bit groups are formed starting from the hexadecimal point moving toward right.
- Ex:
- $(1010.1101)_2 = (A.D)_H$
- $(110.101)_2 = (0110\ .0101)_2 = (6.A)_H$
- $(1110.11)_2 = (1110.1100)_2 = (E.C)_H$
- $(1110111.1)_2 = (111\ 0111.1000)_2$
  $= (7 \quad 7 \quad .8)_H$
- $=(77.8)_H$

Octal number System

- The Octal number system base is 8
- The basic digits are used are 0,1,2,3,4,5,6,7
- Octal number system is 8 and base binary system is 2 since $2^3 = 8$

Decimal	octal	Binary($2^2+2^1+2^0$)
0	0	000
1	1	001
2	2	010
3	3	011
4	4	100
5	5	101
6	6	110
7	7	111

Octal to Decimal Conversion

- The octal to decimal conversion is similar to the binary conversion
- The weights used are integer part $8^0, 8^1, 8^2$ and fractional part $8^{-1}, 8^{-2}$
- Octal point to spearte the integer and fractional parts.
- Ex: octal number to decimal
- $(75)_8 = (7 \times 8^1 + 5 \times 8^0)_{10}$
- $= (7 \times 8 + 5 \times 1)_{10}$
- $= (56+5)_{10}$
- $= (61)_{10}$

- EX: $(45.6)_8 = 4 \times 8^1 + 5 \times 8^0 + 6 \times 8^{-1}$
- $= 4 \times 8 + 5 \times 1 + 6 \times 1/8$
- $= 32+5+0.75$
- $= (37.75)_{10}$

Decimal to Octal Conversion

- Convert a decimal number to Octal
- Divide the decimal number 8 repeatedly and the collect remainders from top to bottom
- The remainders also must be taken in octal.
- Ex:

8|68 (LSB)

|8 - 4

|1 -0

0 (MSB)

$(68)_{10} = (104)_8$

To convert the fractional part decimal number multiply by 8 repeatedly and collect the carries

0.5 x 8 = 4.0

$(0.5)_{10} = (0.4)_8$

- The octal number 98.625 is separated as integer part 98 and fractional part.625
- The integer 98 is converted into octal by repeated division by 8
- Ex:98.625
- 8|98 ↑ (LSB)
- |12
- |1
- 0 (MSB)
- The fraction 0.625 is conveterd into octal by multiplying 0.625 by 8 collecting the carry
- 0.625 x 8 =5.000
- The carry obtained is 5
- $(98.625)_{10} = (142.5)_8$

Octal to Binary Conversion

- To convert a decimal number to binary to dividing the given decimal number 2.
- The base of octal number system is 8 which is equal to 2^8 to covert the octal number to binary
- Each Octal digit with its equivalent 3-bit binary
- 3 bit binary combination the weights are required only 2^2 ,2^1,2^0
- Ex:

Octal	Binary	Decimal
27	010111	23
135	001 011 101	93
45.5	100 101 101	37.625

Binary to Octal Conversion

- To Convert to binary number to octal
- To arrange the bits into group of 3 bits starting from LSB
- Final group has less than 3 bits, just include zero to make it group 3 bits.
- Ex:(10101) into octal
- Arrange the bits as (10101)
- Include a zero for the first group at the front
- The binary combination now becomes(010 101)
- Replace each 3-bit binary group its equivalent to octal digit.
- i.e.010= 2 and 101=5
- Therefore $(10101)_2 = (010\ 101)_2 = (25)_8$
- Integer part or fractional part by a group of 3 bits. Groups are formed starting from the octal point and moving toward right.

Ex:

$(110.101)_2 = (\ 101 \quad 011)_2 = (6.5)_8$

$(10111.1)_2 = (010 \quad 111.100)_2$

$= (\ 2 \quad 7 \quad 4)_8 = (27.4)_8$

Ex:

$(11011011.1111)2 = (011\ 011\ 011\ 111\ 100)_2$

$= (\ 3 \quad 3 \quad 3 \quad 7 \quad 4) = (333.74)_8$

UNIT-II

- Binary Addition
- Binary Subtraction
- Binary Multiplication
- Binary Division
- 1’s and 2’Complement
- Subtraction using Complements
- Signed Binary Numbers

Binary Arthimetic

- The **arithmetic of binary numbers** means the operation of addition, subtraction, multiplication and division.
- **Binary arithmetic** operation starts from the least significant bit i.e. from the right most side.

- **Binary Addition**
- Rules
- 0 + 0 = 0
- 0 + 1 = 1
- 1 + 0 = 1
- 1 + 1 = 10 (Read as 0 with a Carry 1)
- 1+1+1=11(Read as 1 with a Carry 1)

- Example
- 10001001 and 10010101

```
        1
  10001001
  10010101
----------
 100011110
```

- **Binary Subtraction**
- 0 – 0 = 0
- 0 – 1 = 11 (1 result 1 carry) , borrow 1 from the next more significant bit
- 1 – 0 = 1
- 1 – 1 = 0

Binary Subtract

- Ex : 3- 13 (1 Cannot be Subtracted from 0)

   1

1 1 0 1

0 0 1 1   0 – 1 = 11

---

1 0 1 0

- A binary arithmetic example is given to understand the operation more clearly

10101010

10100010

00001000

- **Binary Multiplication**
- **Rules**
- **0x0=0**
- **0x1=0**
- **1x0=0**
- **1x1=1**

- 0×0=0
- 1×0=0
- 0×1=0
- 1×1=1 (there is no carry or borrow for this)

```
    1 0 0 1
  ×   1 0 1
  _________
    1 0 0 1
  0 0 0 0
1 0 0 1
___________
1 0 1 1 0 1
```

Binary Arithmetic

- **Binary Division**

Two rules

1) 0 ÷ 1 = 0

2)1 ÷ 1= 1

Binary Subtraction:

0-0=0

0-1=11

1-0=1

1-1=0

Example 2.11

Divide 6 by 3.

Solution:

```
   2
3 |6
   6
   -
   0
   -
```

Here, the dividend is 6 and the divisor is 3. After the division, the quotient is remainder is 0. Repeating this in binary,

```
        1 0
11   | 1 1 0
       1 1
      -----
         0 0
      -----
```

Example 2.12

Divide 10 by 4.

Solution:

```
    2
4 | 10
     8
   ---
     2
   ---
```

```
   2.5
4 | 10
     8
   ---
    20
    20
   ---
     0
   ---
```

Here, the quotient is 2 and the remainder is also 2.

Including the fractional part, the quotient is 2.5.

The same example is repeated in binary.

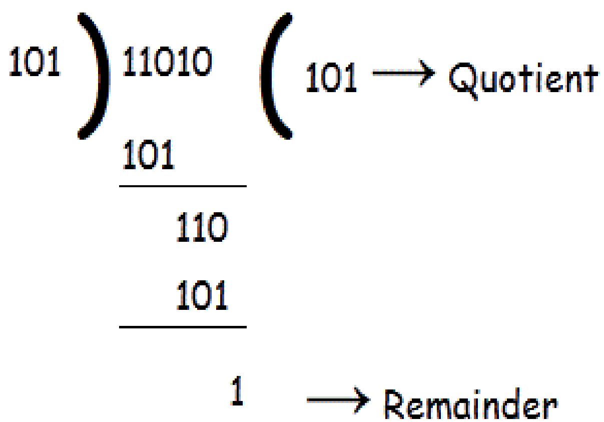
```
       10                    10.1
100 | 1010           100 | 1010

      100                  100
      ----                 -----
       10                    100
      ----
                             100
                             ----
                               0
                             ----
```

Here, the quotient is 10 (2) and the remainder is also 10 (2).

Including the fractional part, the quotient is 10.1 (2.5).

The method explained can be extended for bigger numbers also.

- **Binary Division**



1's and 2's Complements

- 1's Complement
- To used to Perform Subtraction using adder Circuit
- 1's Complement of 1 is 1-1=0
- 1's Complement of 0 is 1-0=1
- Remember (Binary number Changing)
- 1's Complement of 0 is 1
- 1's Complement of 1 is 0

EXAMPLES

Binary Number	1's Complement
1011	0100
110001	001110
11001110	00110001

2’S Complement

- 2’ complement of a binary number
- Adding 1 to the 1’s Complement
- Ex 1 0 1 0
- First Convert to 1’s Complement
- Second Add 1
- 0 1 0 1 +
- Add 1 1
- 2’Complement 0 1 1 0

SUBTRACTION USING COMPLEMENTS

- 1’S and 2’s Complement representation of
  Binary numbers.
- To performing subtraction using the process of addition(adder circuit)

1's Complement Subtraction

- Steps Followed
- Take the minuend in Binary
- Take the 1's Complement of Subtrahend and
- Add it to the minuend
- If carry is produced ,add the carry to the sum
- Subtraction rule (Remember)

- 0-0=0
- 0-1=11
- 1-0=1
- 1-1=0
- Ex: Subtract 4 from 13 Apply the rules binary subtraction
- 1 1 0 1 13
- 0 1 0 0 4
- 1 0 0 1 9

- Repeating the above process in 1’ Complement Subtraction

```
 1 1 0 1   +
 1 0 1 1
---------
 1 0 0 0   +
       1
---------
 1 0 0 1
```

Addition

0 + 0 = 0
0 + 1 = 1
1 + 0 = 1
1 + 1 = 10 (Read as 0 with a Carry 1)
1+1+1=11(Read as 1 with a Carry 1)

- Add 4 Digit Binary Number
- Easy way: 1 + 1 + 1 + 1 (in base 10) = 4, which in binary is 100.
- Added in binary:
- 1 + 1 = 10
- 10 + 1 = 11
- 11 + 1 = 100

2's Complement Subtraction

- Take the minuend in binary .
- Take the 2's Complement of subtrahend and add it to the minuend.
- If a carry is produced. Ignore the Carry
- If carry is not produced. it indicates that the result is negative and the result is in 2's complement form

- Ex: subtract 4 from 13

1 1 0 1	13
0 1 0 0	4
1 0 0 1	9

Repeating the above process in 2's complement subtraction

1's Complement of subtrahend = 1 0 1 1

2' s Complement of Subtrahend = 1 0 1 1

+ 1

= 1 1 0 0

Minuend =	1 1 0 1	+
2’ Complement	1 1 0 0	
	1 1 0 0 1	

Igonre the 1 Carry result as 1 0 0 1

Signed Binary Numbers

- + Sign to denote Positive Numbers
- - Sign to denote the Negative Numbers
- 0 is used to represent (+) Positive Numbers
- 1 is used to represent (-) Negative Numbers
- The bit used to represent the sign of the number is
- Called the sign bit

Ex:	Sign	Magnitude Bit
• +5	0	000 0101
• -5	1	000 0101
• +14	0	000 1110
• -14	1	000 1110

- Magnitude part is the same for both positive and negative numbers
- + 0 0 000 0000
- - 0 1 000 0000
- The positive numbers are represented in their true binary form with 0 as sign bit in MSB position

- Sign 1’s Complement representation
- 1. Write the positive binary from of the number,inculding sign bit
- 2 Invert each bit, including the sign bit
- Ex +12 ,-12
- 1’s complement representation
- + 12 = 0 000 1100
- -12 = 1 111 0011

Sign 2' Complement representation

1.Write the postive binary form of the number, inculding the sign bit

2. Invert each bit, inculding sign bit

Ex

+ 12	=	0	000	1100
1's Complement	=	1	111	0011
Add 1				1
		1	111	0100

Binary Codes

Types of Binary Code

1. BCD Codes – 8421
2. 2421 and 4221 codes
3. Excess-3 code
4. Gray code
5. ASCII code

- A group of bits which are used to represent decimal numbers 0 to 9 are called Binary coded Decimal codes.
- BCD Code is 8421
- Indicates the binary weights of the four bits$2^3,2^2,2^1,2^0$
- 1001 9

BCD (8421) Addition

- 1.Add the two numbers,using the rules of binary addition
- 2.if the 4 bit sum is equal to or less then 9,it is a vaild BCD number
- 3.if the 4 bit sum is greater then 9 or if a carry out of a BCD group is generated the result is invalid BCD.
- EX:
- 5 0101
- 2 0010
- 7 0111

- To get the correct BCD result add 6 (i.e 0110)
- To the first group and take the carry next group.

- 7 + 0111
- 6 0110
- 13 1101 (Invalid BCD its exceeds 9)
- 0110
- 1 0011 valid BCD number(0001 0011)

2421 and 4221 CODES

- This code are called as Self Complementing Codes
- Two such codes are 2421 and 4221 binary combination of decimal digits
- 2421
- eg: Binary code for 2 = 0010
- 4221
- Eg: Binary code for 4= 0001

Decimal	2421	4221
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0110
5	1011	1001
6	1100	1100
7	1101	1101
8	1110	1110
9	1111	1111

1’s Complement of Codes

- 2421
- Binary code 2 = 0010
- 1’s Complement 1101 and this code represents 7 ,which is 9 complement of 2
- Eg:4
- Binary Code for 4 = 0100
- I’s Complement = 1011 represents 5,which 9 complement of 4

- 4221
- Binary code for 1 =0001
- 1’s Complments = 1110
- and this code represents 8 ,which is 9’s complement of 1

Excess -3 code

- Another BCD Code
- Decimal digit adding 0011(3) to the 8421 BCD
- Code
- The six invalid codes are 0000,0001,0010
- 1110,and 1111

dec	8421	Execss
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

Decimal	Excess
26	0101 1001
97	1100 1010
853	1011 1000 0110

- Excess -3 Addition
- Sum Less than 9 an Excess -6 number result
- To return to excess-3 from to subract 3
- 2+ 0101
- 3 0110
- 5 1011
- 0011
- 1000

- A carry is produced
- Excess-3 code exceeds 9,there will be carry from one group into the next.
- The group that produced the carry will be in 8421 from and the next group will be in Excess -3 code

- 29 0101 1100
- 49 0111 1100

- 78 1101 1000
- 0011 0011

- 1010 1011
- First group in excess-6 and second group in 8421 subract 3 from the first group and add3
- To the second correct result in excess-3

Gray Code

- To used for sequence counting
- To count advanced by one , to reduce error
- Binary to Gray conversion
- 1.the MSB of the Gray code is the same as the MSB of the binary
- 2.Coding from left to right,add each adjancent pair of bits to get the next bit of the gray code
- Omit the carries if occurs

- Convert the binary to Gray
- Step 1:
- The left most bit(MSB) in Gray code is the same as the MSB of the binary
- 1 0 1 1 (Binary)
- 1
- Step 2
- Add the left most bit to the adjacent one
- 1 + 0 1 1
- 1 1

- Step 3
- Add the next adjacent pair
- 1 0 + 1 1
- 1 1 1
- Step 4
- Add the next adjacent pair and omit the carry
- 1 0 1 + 1
- 1 1 1 0
- $(1011)_2 = (1110)_G$
-

Gray to Binary Conversion

- 1.The MSB of the Binary is the same as the MSB of the gray
- 2.Coding from left to right, add the binary digit generated to the adjacent Gray bit to get the next bit of the binary
- Omit the Carries if Occurs
- Step1: The left most bit in binary is the same as the MSB of the Gray
- 1 1 1 0 (Gray)
- 1
- Step2
- Add the binary digit generated to the adjacent bit of the gray code
- 1 1 1 0
- 1 0

Step 3

Add the binary digit generated to the adjacent bit of the gray code

1 1 1 0

1 0 1

Step 4.

Add the binary digit generated to the adjacent bit of the Gray code

1 1 1 0

1 0 1 1

The conversion $(1110)_G = (1011)_2$

ASCII CODE

- American code for information Interchange
- The 7 bit code to represent decimal digits
- 0 to 9,
- Alphabets A to Z
- Both lower case and upper case and special characters

ASCII	Decimal	HEX	7-Binary
0	48	30	011 0000
1	49	31	011 0001
2	50	32	0110010
9	57	39	0111001
:	58	3A	011 1010