SEMESTER : V CORE COURSE : IX

Inst Hour	:	6
Credit	:	5
Code	:	18K5M09

REAL ANALYSIS

UNIT 1.

Real Number system - Field axioms -Order relation in R. Absolute value of a real number & its properties -Supremum & Infimum of a set- order completeness property - countable & (Chapter 1: Sections 2-7&10 of Text Book 1)

UNIT 2:

Continuous functions - Limit of a Function - Algebra of Limits - Infinite Limits - Continuity of a function - Types of discontinuities - Elementary properties of continuous functions -Uniform (Chapter 5: of Text Book 1)

UNIT 3:

Differentiability of a function - Derivability & continuity - Algebra of derivatives - Inverse Function (Chapter 6: Sections1-5 of Text Book 1)

Rolle's Theorem -Mean Value Theorems on derivatives- Taylor's Theorem with remainder-(Chapter 7: Sections 1-6 of Text Book 1)

UNIT 5:

Riemann integrability and integral of a bounded function over finite domain - Darboux's theorem -Another equivalent definition of Integrability and Integral - conditions for integrability - Particular classes of bounded Integrable functions - Properties of Integrable functions -Functions defined by Definite Integrals - First Mean Value Theorem of Integral Calculus -(Chapter 6 of Text Book 2)

Text Book(s)

- [1] M.K.,Singhal & Asha Rani Singhal, A First Course in Real Analysis , R. Chand & Co June
- [2] Shanthi Narayan, A Course of Mathematical Analysis. 1964

Books for Reference

- [1] Tom.M.Apostol, Mathematical Analysis ,II Edition.
- [2] S.C.Malik , Elements of Real Analysis.

Question Pattern (Both in English & Tamil Version)

Section A: 10 x 2 = 20 Marks, 2 Questions from each Unit.

Section B: 5 x 5 = 25 Marks, EITHER OR (a or b) Pattern, One question from each Unit.

Section C: $3 \times 10 = 30$ Marks, 3 out of 5, One Question from each Unit.

Newsitied

மேற்றும் : 3.1

சார்பு f என்பது கடைவைன் I எர் பிறு வறையறுக்கப்பட்டுள்ளது என்றும் I எர் கடு பிறு சால்றுக்கு வனக்கையூ உண்டு வன்றும் க என்பது சுடு எம்மி எய்ன என்றும் கொண்டால் Cf என்பனையும் வனக்கைய காணர் தக்களையை என கிடுப் $(fc)^{-1}(n_0) = C[f^{-1}(n_0)]$.

Bynnd:

lem
$$f(n) - f(n\omega) = f(n\omega)$$
 [$ponpwonpwon . uq)$

lim $f(n) - f(n\omega) = f(n\omega)$ [$ponpwonpwon . uq)$

lim $f(n) - f(n\omega) = f(n\omega)$ [$f(n) - f(n\omega)$]

 $f(n) - f(n\omega)$
 $f(n) - f(n\omega)$

 $G_{9}^{m}m_{6}^{m}:3.2$.

வாழ்கள் f,g வகையாவ திடை வைன் I என் மிறு வறைய ருக்கியட்டுள்ளது எனி ரும் I என் ஒரு முன்ன வை இடிற்று செவர்றுக்கு வகைக்கைக்கோள் உண்வுள்ளும் வகையையை f+g வண்யனவடிய இடிற்று வகைக்கைக் கொணர்றுக்கமையை மேவும் (f+g)'(no) = f'(no) + g'(no) என கிறுவுக. Blanur:

lim
$$(f+g)(n) - (f+g)(no)$$
 $n \rightarrow no$
 $n - no$

lim $[f(n) - f(no)] + [g(n) - g(no)]$
 $n \rightarrow no$
 $n \rightarrow no$

- + (no) +g'(no).

-: (f+g)'(no) = f(no)+g(no). oron mon solverg.

Boy ourony (fag) no=f'(no)-g'(no) oron

8 g ny 16 3.3 1

சார்புகள் fog orional திடைவை நன் பிரு வறையருக்கப் படுகள்ளன என்றும் I ன் வடு புள்ள கடிற்று அவற்றுக்க வகைக் வக டுக்கள் உண் வென்றும் கூ சுத சுதாண்டால் சூற வகைக் வகயும் கடர்ர வகைக் வகம் காண நீரக்கலையை . கடிற (†g)'(no) = f(no) g'(no) + g(no) f'(no) என கிற வுக .

Film mid :-

lim fg(n)-fg(no) n-no.

dim frangin - timos ginos

finigin) - finiginus + finiginus - finoiginus

 $\lim_{n\to n_0} \left[f(n) \left(\frac{\partial(n) - g(n_0)}{n - n_0} \right) + g(n_0) \left(\frac{f(n) - f(n_0)}{n - n_0} \right) \right]$ f Jong no Bugg agricio Dwad 2 Mmgn A no Bugg

fro sommande sommy

··· lim $f(n) = f(n_0)$ Bogi lim $g(n_0) = f(n_0)$?

= $\lim_{n\to n_0} \left[f(n) \left(\frac{g(n) - g(n\omega)}{n - n_0} \right) + g(n\omega) \left(\frac{f(n) - f(n\omega)}{n - n_0} \right) \right]$

= $\lim_{n\to n_0} f(n) \left(\frac{g(n) - g(n_0)}{n-n_0} \right) + \lim_{n\to n_0} g(n_0) \left(\frac{f(n) - f(n_0)}{n-n_0} \right) \right)$

= $\lim_{n\to n_0} f(n) \lim_{n\to n_0} \left(\frac{g(n) - g(n_0)}{n-n_0} \right) + \lim_{n\to n_0} g(n_0) \lim_{n\to n_0} \left(\frac{f(n) - f(n_0)}{n-n_0} \right)$

= fino) glino) +gino) flino).

 $\frac{1}{2} \left(\frac{1}{2} \frac{1}{2} \left(\frac{1}{2} n_0 \right) = \frac{1}{2} \left(\frac{1}{2} n_0 \right) \frac{1}{2} \left(\frac{1}{2} n_$ あかかいしょ

89776: 3-4.

र ठालांनु ठामंतु मण ठालांतु प्रतानीकी यामसार्थित्रित्र orange foros to orange oranan Lord sning 1/1 stong no orang Ymmwn Dansungspieg owing Engola Bonto ((4)(no) = -fino)) {f(no)32 oringi Ringos:-Enound:

रमां में रेडकार माठ जाका मुलामी भी का कार मार्ट में देश में जाका. Ligno fino) +0 oringi orania.

 $\frac{y_{frn} - y_{frno}}{y_{-no}} = -\frac{f_{rno} - f_{rno}}{y_{-no}} \cdot \frac{1}{f_{rno}} \cdot \frac{1}{f_{rno}}$

 $\lim_{n\to n_0} \frac{f(n) - f(n_0)}{n-n_0} = f(n_0) \longrightarrow (2)$

formy miny nynchisonwnon miny orosugnos

lin nano fin) = fino) to -> (3)

DNG OLOBANABANON ONON DIJON ONY (1), (2XX3) lim 1/fenos - 1/fenos confor oungi sonji

Ding contry song - f'(no) f(no)? ormayi orugoving.

57 mg6: 3.5

Jungung in Je g oron vone I orong 36 pring Don new W.

Den sung in Suc Eminey orong a. Gung Denniyam I orong

Bog Em nom Mund no orong ymmund amauligging

Orong & g(no) = 0 orong & nanminan f/g seng wordely

Sing oron BON.

ក្សាញ្ជាល់ — ខេត្តកំពុល់ 3.3 A 3.4 D ប ឃុំ បញ្ចុំខ្លួន. ឧត្តកុំពុលំ 3.6 មូខាំងសាលា ១៤ភ្

#niyam f, wind g soons noninishwam aniyam

Jobi Conji oniny for Sonorion g oringi, f, no ordery

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yind yind will workwish gray ordings ordination

gof, and no ording yindwind workwish gray ording songer

Conji (guf)'(no) = g'(f(nu)), f(no) ordings songya.

gof on organis f sous.

lim $(g \circ f) (n_0 th) - (g \circ f)(n_0) = g(f(n_0)) \cdot f(n_0)$



കാമ്പ് പ്രദേശ വര്യായും വിഷ് വരു വരുവെന്നു പ്രവാധനുക്കാവും വേഷ്

 $F(h) = \begin{cases} g(f(n_0+h) - g(f(n_0)) & f(n_0+h) - f(n_0) \neq 0 \\ f(n_0+h) - f(n_0) & (if f(n_0+h) - f(n_0) = 0 \end{cases}$

Loggio G(h) = (gof)(noth) - (gof)(no) h.

F(h). $f(n_0+h) - f(n_0)$ $h \to c_1 \quad h \to 0.$ $f(n_0+h) - f(n_0) = 0.$

lim $f(n_0+h)-f(n_0) = f'(n_0)$.

Digeography for them with f(h) = g'(f(n))The happenson of the fine f(h) = g'(f(n))Elongen of word them g'(f(n)) + h - g'(f(n)) = g'(f(n)) $k \to 0$ $k \to 0$

orm myn couming. Endo 6>0, 8>0 ound oraniment

19 (teno)+k)-9 (teno) - 9 (teno) (16 -> (2)

Printy forming no ormy yandwar womawhighing doll.

Oroslew 8'so order of many organishwang toll no order

Yandward; 14thoths - fines 108 -7133

fineth) = fines orang /=(1)-9'(4mes) = ore

-79

f(h) = g(f(notk) - g(f(no)) f(noth) - f(no)

= g (f(no) + k) - g (f(no))

(=(h)) - g'(+mo)) < E & mo (1k) < S -> (S)

(+(no+h) - f(no)) | < S. Gogi 1h) < S' ormung

(F(h) - g'+(no)) | < E

ie, lim F(h) = g'(f(no)). 87976 Bly viving.

4. Banne 4.1 NO onnum orgnomy womanne

Anong is Forming on Donchown son son son works during son of the s

Blogio, Diogin Minio Digin esona inales /f(no) DeBC.

Byno !-

オラターショイリース

no -> yo -> g (yo) = no

noth -> york -> g (yotk) = noth

cofloure asserve.

ordy onium ofoure nowne sign Bisac y Ormingio Orajya nanmound.

(. . g ormung for yoursig oning)

g (no) - 1/fmo)

· Josi vollsundayis ywnanyis dry in Boin n Jusqu's f(n)= y =) g(y) = or or or or or or or or or.

+ (no) = yo oromas.

9 (40) = 20 26 6.

Donungg yn y+k ormung y maning samour as yning ร อาอร์บอา ารอักทูล กลาอภ์ ทุกอก สกกับ อาอร์บอุกลับ อาจุดที่ ๆ อาณร์บูง miy gyen ginginanonyna oniungi.

ord noth owny Bombonine anwing yound -wrong signowing conquir yindwas fonoth) = yoth

 $= \int f(n_0) = y_0$

=> f(no+h) = yo+k

=) g(40) = no 24000°

=) g (yotk) = noth.

formung to oring yimmund workensiges, ening

I orang to orang yamman ngninishwan sang to orang yamman ngninish - wron sanguang or orang yo orang yamman ngninish was to orangwangulang to orangwangulang to orangwangulang to orangwangulang to orangwangulang

16 g (yo+k) = lb g'(yo) =) lb (no+h-no)=0.

the how is known how our feno, own monwours.

= g (70+k) - g (70) = g (70+k) - g (70) = 70+k-y0.

 $= \frac{n_0 + h - n_0}{\gamma_0 + k - \gamma_0} = \frac{1}{(\gamma_0 + k - \gamma_0)}$ $= \frac{1}{k + 0} \left[\frac{g(\gamma_0 + k) - g(\gamma_0)}{k} \right] = \underbrace{\int_{k \to 0}^{k} (f(n_0 + k) - f(n_0))}_{k}$

· · · g(no) = 1 (no) Quyny oron.

onn un () com un lo oroning ym nund Domawagesige onn un () cog o oroning ym nund Domawagesige on un oroning ym nund Domawagesige of the oroning of the oroning ym nund Domawagesige of the oroning of the oroning ym nund Domawagesige of the oroning oroning or oroning oronin

5. LARIUNIMON GYMYLE

59776 5.1

यामकारी मुलंक Landunkamon हम्मेम् (aibsa) में काळापकी DIMON MARQUEL DONA PLEGGE DANY. BLOGE f'(a), f'(b) 10 orallo a & Bi b & Bi Domeon Cowing No abwawsin Jemy fl(c) =0 orange uy siano yu.

கிறுவல்! வகை (i) \$1(6) >0 வக்க.

ULG (i):- f(m) <0 00081n0, h, >0 3: f(m) < f(m) f x Ja, a+h(f NORML 398 B sning soli a stony you mula.

Lim $n \to \alpha + 0 \qquad \frac{f(\alpha) - f(\alpha)}{n - \alpha} = f(\alpha) , \epsilon = -f(\alpha) , h_1 > 0 3$

 $\alpha < n < \alpha + h \operatorname{orosol} n$ $\left| \frac{f(n) - f(n)}{n - \alpha} - f(n) \right| < \epsilon \operatorname{oroson} n$.

i., $f(ra) - \epsilon < \frac{f(ra) - f(ra)}{r - a} < f(ra) + \epsilon$.

flaste = 0 & nsa, ... frastas

<u>пор сіі)</u>; + (сы) о опой в. Э h2 >0 э: frm) < f(ы) н ∈]6-h2, Ы I some of borong you mann simawing of any ordi.

 $\lim_{n\to b-0} \frac{f(n)-f(b)}{n-b} = f'(b). \in =f'(b) = \int_{-\infty}^{\infty} h_{2}(0.3.1)$

b-hz < n < b , oi on no / fen > - + (16) < 6.

 $ii \ f'(6) - \epsilon < f(n) - f(6) < f'(6) + \epsilon$.

· · · f'(b) -6 =0 & n < b, i. f(n) < f(b).

[9,5] owing in Agw Don (norman onny of drawny warmy ging oromo, forong organishwange 09:(iv) f'(c) \$0, f'(c) so orona (f'(c) so origi uning 2 mmg ound hard d: franktes the eJc-harel Eugi Ding on my tres infimum ownering Gronns 2 ming. ormore + (ce) bo.

Un(v):- flee) Ko Hoorond Rflee) KO A Uy IN e ming ound haso, fins & fees & n &] c, c+h[. 0100102 f 100) 40.

Un (vi): Qames 270 Juy my 480452 flee) = 0.

noma(ii) f'(a) >0 :. f((6) <0, ... f((a) f'(6) <0.

(a,b) β g $\sigma \circ \delta \circ \gamma$ $\sigma \circ \delta$ $\sigma \circ \delta \circ \delta$ $\sigma \circ \delta \circ \delta$.

· · · g (a) = - f (a)

· · g(b) =-f(6)

· · · g(a) - g(b) < 0.

) yeung g(n) oloning (a,6) nd sonswy is gue NMAMLJERS ONRY GUYL & (a) - & (5) to orazivy disnumber f'(c) = 0, f'(c) = 0.

Eggy i Bour andvice 7.

दिवामी दिन्ती १-

Jaish as masulle of griss painty flas = flas ormally form of or 1618 Sopring: 4.1 of orasives (a,6) in oppositional oring forming C story orce alowning of the files = o other buy somenyis. - A summy and a not ball someon

Big w Som normand Francis rainance normais - wrong drings of its of some menders and some of your Blosing agring. (M, m) ovak vana (a, b) of Fin Bonce, the soly work orash.

Erign grap ging frances converses acides (0,6)2 -mm orden namagoù R(n)=0 200. Bylons DOBCI):- M=m Dang. -: (a15)m + or in confin mings. DON: (ii) Mfm own A. fe (a) =f(6) own ugnor m, m a,600 Francy commy trimina Ficuso 2500. Olonow F(a) + M Olonia.

Ame noumled grips rowande conveyored (4,6) of coming Ogneria Dluna aniyi organ Gones copone orizy dow Gyw Som owning 2-in soone con normally fees = M charis up someye. => f(a) + f(c) + f(b) => a + c + 5. fa) + m + f(b)

C=0, C= 5.

and a, bid dunishing. (4,6) à omure escare Thou finst (1) de Oi Biflours nec-n owing yours of

7 (c-h) = fres - fres-fres=0.

1(r-h)-f(e) =0. lim f(r-h)-f(e) ≥0

L'Jas 20 - 0 Macthoris A.

700) = 700)

=> f(c+h) = f(c) => f(c+h) - f(c) =0

 $\lim_{h\to m} f(c-h) - f(c) = 0. = Rf'(c) < 0 \rightarrow (2)$

Osci Moning forming en Domanigase onivo Qui.

L'fles = Rflees = fles)->3

0,0 80 MOBY 7'CO =0 3506.

M + 7(6) our oragga nanonLnyi 7/10 = 0 orany Dronbyu.

தேற்றும் : 2.) வலக்றாக்கியின் கிடையரிய தேற்றம்

By my ": (i) [a, 6] of orion on on ognine Dunon oning (ii) Ja, b [of of oron un as in Dona Wing of is soning oronivy JaibEN coming and only down down for +(b)-+(a) = +'(c)

18) BUON 6 1-

[a,b]ne = mm n 2 mg & @ i +(n) = f(n) + An = I own or Bygin or misons. BIEG A and ung 2 And a Amyula off of marken modul on minder on my ->0

- (i) AAN ON ON LAISA OPNING EA, B) A APRING DINNER DANY.

 TON +AN ONE CONSTRUCT EA, BJA OPNING DINNER DANY.

 J(N) OTHUR EA, BJA OPNING DINNER DANY.
 - (ii) Manimonsoning f(m) ording Jaistin Domawieghia aniy

 Man ording Jaistin Domawieghia shing

 f(m) + An ording i Jaistin Domawie 9" gha Doniy

 f(m) ording Jaistin Domawie gosia dniy
 - (iii) PANM MAWIND Up

 A oroning wongled from the oron or Biggs our mount.

 (iii)

 DDA3 mivy of oroning snay son Man dronount.

 ALBOUNCEM (Mis Almongy nowally ?.

Jaib [α cowing about now f(c) = 0 or f(c) = 0 or

=) f(a) = f(b) =) $f(a) + Aa \pm f(b) + Ab$ Aa + Ab = f(b) - f(a) -A(b-a) = f(b) - f(a) -A = f(b) - f(a) b-a(iii)

CAMADIWOOD DONE LO DAY BARRE!

69776 : 3.1

() [a16] fig ordivona ogniris Duna onny

(i) Ja, b [& f, g ownown worawingsis sniy.

(iii) Ja, b [2 im oring yes ni g(n) = 0 oroge uy 7, g oronoma [a16] N Domowy ABOULL BO Innyan orost N Ja16 Coton of account of the flat = $\frac{f'(c)}{g'(c)}$ oranging

18/00 now ?;-

Bi Gover i) on voy $g(b) \neq g(a)$ as G on m g(b) = g(a)ठाळ्ती में वु(n) ठाळां म नागें भोलंड हिनामां हिनां मुं की कां 3 हाँ छ छ -UNGBOSE OUNGING BOOMSON.

Olosian Ja, 6 Ca 2 mm or orosing our orong song g(n) =0 ologiug somboling - Dy nanGBBOULL nanimonaum 2 mm boingner a E Bounder n's Aronun La G.

7(6) = g(a) oran oraning germ. $g(6) \Rightarrow g(9), g(6) - g(9) \neq 0.$

[a,b] ~ 2 mm dromorgy namboli fin) = fin) + Agin -) 1 oran evanguagiouns. DEDO A ovaring 2 sing Gangwin orenjage oranim owning under

(i) nann mawoing fon, fon) doing so soniyani Ja, 6 [2 DMRWLJEBB Mirnor.

f(n) = Ag(n) own of Do aniyani Jais In Donaweggis annungs.

The owner of Jailer womaning gam mitings.

(iii) nather manificity in a strong uniquina of the miles of months of a desired of the miles of the miles of the miles of the original of the committee of the miles of the original of the committee of the comm

compile up processing.

\$ '(1) + ng'(1) = 0. = 3 \$ (1) = \$ (1) + ng'(1) }

\$ '(1) + ng'(1) = 0. = 3 \$ (1) = -ng'(1) }

-n = \$ (1) = 5 (1) = 0.

f(n) = f(b) f(a) + Pg(a) = f(b) + Pg(b) Ag(a) + Pg(b) = f(b) - f(a) -P = f(b) - f(a) $\frac{1}{6-a} \rightarrow (iii)$

(ii) 8 (iii) MO is $g = \frac{1}{f(b)} - \frac{1}{g(a)} = \frac{1}{g$

52770: 4:1

ALWNAM Oggge DINNA ALWNAM Ogggeng ANA gradum Def ARNAML DISTIBLED STORED La, 63 to obosing son unones

(i) + n-1 ovoring [a16] of orcio summer oning

OTOOTA JA, 5 [N c orony or 600 or mom orong,

 $f(b) = f(a) + \frac{b-a}{1} f'(a) + \frac{(b-a)^2}{2!} f''(a) + \cdots \frac{(b-a)^{n-1}f^{n-1}}{(n-y)!}$ + (b-a) f (c). orguly diamentaly.

Boumis :-

BL 60 un 6 () on vy 4, +1, +1'... + n-1 oron uma [n/6]~ DIMO WATE ROUL OGNERIBOTWOOD Anny Olonon Fon, orong Animu [a,6] ก กล้อ เมืองกฎ ณฑายพฤษอยกษ์.

 $f(n) = f(b) - f(n) - \frac{b-n}{1!} f(n) + (\frac{b-n}{2!} f''(n) + \cdots + (\frac{b-n}{n!})^n A \cdots$ BASB A oronvey 2 Aring Flory Wid OTB jaga MANIMONUL Broming w conmin.

e) nanmanma vy f, f', f".... f n-1 ovorvane [a, b] of organised when anizami Buyi $n \rightarrow (b-n)k$. (k=1,2,...n) oranivare $[a_1b]$ N' Ogninishwam onnyami.

i. fin) oronung [9,6]n Ogninisonunon onning.

(ii) Nanmanmin uy f,f,f",...f7-1 oran vane Ja15 [2) UMAWLIGAB onny Evogi.

dany. i. f(n) olong Ja, b[n asmawigga's anny

(ii) f(n) owing mn'y vanguangun siaser wangen

vy [a16]N' Coming anownwammong flin)=0 090 otopi drombyi.

f(n) = f(b) - f(n) - b-n f(n) = (n) (b-n) n

 $F(n) = 0 - F(n) - (\frac{b-n}{1!})f(n) = "(n) - F(n) - 1 \cdot (\frac{b-n}{1!})^{\frac{1}{2}} = "(n).$ - (F"(n) & (b-n) (-1) --- - (6-20) (-1).

 $F(n) = -\frac{(b-n)^{n-1}}{(n-y)} = \frac{1}{(c)} + Dn(b-n)^{n-1}$

F(c) = 0. =) $-(6-n)^{n-1}$ $F^{n}(c) + An(6-c)^{n-1} = 0.$

An (b-c)n-1 = (b-c)n-1 Fn(c)

 $A = \frac{f'(c)}{r(n-i)!} = A = \frac{f'(c)}{n!} = ---- > (ii)$

During in n=6 oranno

 $F(b) = F(b) - F(b) - \frac{b-b}{1!} F(cb) - (6-b)^{1} Q$

F(6) =0 OSOMAN F(a) = F(6)

F(a) =0, A= Fhce, oron ON 19 func

 $(D =) = (6) - f(a) - \frac{b-a}{1!} f(a) \cdots \frac{(b-a)^{n}}{n!} f(a) = 0.$ $f(b) - f(a) \cdot \frac{(b-a)}{m!} f'(a) + \cdots + \frac{(b-a)^{n-1}}{(b-a)^n} f^{n-1}(a) +$

(6-a) f (n) +.

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in nogligai iled supostavnos ortinalos o 2226.

- (i) for asoury [aisIn organisis Dwood soning
- (ii) ford doorg Ja,6EN DIMAMLGABB Anny Gomes Caib JN of doorg DIMONNAROULL Dning Orand Corning

 $F(b) = F(a) + (b-a)f(a) + \dots + (b-a)^{n-1}f^{n-1}(a) + (b-a)^{n}f^{n}(c)$ (6] a_1b] oroginal organization

Boumis !-

- (i) Al Grouni Gran ung

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- (ii) 2° GWUNCYM VY

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9 (m) 40.

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Jail En c orosing orbidorwood desorty

= f(b) -f(m) = f'(c) -> @ otrogivy som by i.

 $f(b) = f(b) - f(b) - (b-b)f((b) - (b-b)^{n-1} f^{n-1}(b)$. f(b) = f(b) = 0.

g(b) = b - b = 0

 $f(b) = f(b) - f(a) - (b-a)f(b) - \cdots - (b-b) \frac{n-1}{(n-1)!} f^{n-1}(a) \rightarrow (3)$

 $f'(n) = f'(n) - f'(n) - 1 - (b-n) f''(n) - \cdots -$

 $(n-1)(b-n)^{n-2}(-1)f(n)^{n-1}$ $(b-n)^{n-1}$ f(n)

 $f'(n) = -(b-n)^{n-1}g^{n}(n)$ (n-1)!

9 (m) = -1

 $f(c) = -\frac{(b-c)^{n-1}}{n-1} + \frac{n}{(n)}$ by g(c) = y

 $-f(b) + f(a) + (b-a) f'(a) + \dots + (b-a)^{n-1} (b-a) f'(a)$ $+(b) = f(a) + (b-a) f'(a) + \dots + (b-a)^{n-1} \cdot f'(a)$ $+ \frac{(b-a)^{n-1}}{(n-1)!} \cdot \frac{(b-a)^{n-1}}{(b-a)!} \cdot \frac{f'(a)}{(b-a)!} \cdot \frac{f'(a)}{(b-a)!}$

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them the remarks, the Rigormanic [aib] on order without some of U(p, f), L(p,f) out y agramami Des amorribamnols. Int UCP. +) ownung Laibin I am of them areas agramments ormovous. Dong Jem on owni organous.

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dignary Sup L (Pit) = Ston.

Danman, yroni disumenge :-

[a,b] N agricy on w oning f & of findn = findn บารา อาอาเรียวทาง ป บาราบร หือสำ กฎกลผาเลิดเมพ thing starbubis. Disg oung unity too Russi organs orangi. f को Kuri nyrans

Stinian oversi Orginavogi.

Elday: formung [a15] no Aum ngnmawis angung otalis ferlais) otani Orfianis.

5 4 WJasello,

[0,1] Growing Some normally of soul of surduron of Alend AGRABINIA OTON BIRDAS. BOYC / finsdx = 1/2 20 8 BromB.

Ling! - f orong enrione [011] doing DonLower May f(n) =0. n allegen vioironne Dade i ouneg = 1. अ विकिष्ण जन्न अन्तर जन्मीर जन्म प्राम NMJW JAKOVOU.

n own vg [villy 2 mm as ymin oroot is frn-on volgy oboi 1200 Doncom Dasob. Jignoug OS fin) SI. NE [OII] By Maky of Jamy essignens onny orong ogalang.

Ding regulys many Alori - Agromantal Dolora oton simplosings smounder.

p={0=n0, n1, ... nn-, nn, ... nn=1} orong [0,1] nong Bookone oggazi este vita visione.

Iniy for songworn words of $M_{n=1, M_n=0}$ (n=1,2,3...n) orong and and orday). $\int f(n) dn = \inf \left(s(D) \right) = \lim_{n \to \infty} \left[\frac{1}{n} \left(\frac{1+\frac{1}{n}}{1} \right) \right] = \frac{1}{2} \to 0$ $\int \int f(n) dn = \sup \left[s(D) \right] = \lim_{n \to \infty} \frac{1}{2} \left(\frac{1-\frac{1}{n}}{1} \right) = \frac{1}{2} \to 0$ $\int s(D) MS \stackrel{\circ}{n} y \quad \int \int dn = \int \int f(n) = \frac{1}{2}.$

6.3 Lani Haiso Canalo!

STIME! [a16] owing Gyw Done own will foroing aning while is a may and in deal of the grand of a may shoot is decorated of the first of the standard of the sta

(i) $S(D) < \int_{a}^{b} f(n) dn + e + D with <math>|D| \leq \delta$ ii) $S(D) > \int_{a}^{b} f(n) dn - e + D = |D| \leq \delta$.

Blevone!

I find = k + n E[a,b] owns.

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.. D. SD, SACDLE SCD.).

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-! E(D,)-S(D) = MASA - (MA'SA'+MA"SA") = (MA-MAI) SA'+ (MA-MA") SA"

I fonotek, the [aib] . => -kemmi emmek.

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eg our agrague magn oronsuas.

 $SCDI) \leq \int_{a}^{b} f(n) dn \star \epsilon \ell_{2}$ $SCDI) = a(p-1) k \delta \leq SCDI) \leq SCDI$

Bugü D_ 2D, ⇒ SCD2) ≤ SCD,

 $S(D) - 2(p-1)k\delta \leq S(D_1) = S(D) \leq 2(p-1)k\delta + S(D_1)$ $\langle E_2 + \int_0^{\pi} f(n)dn + E_2 = \int_0^{\pi} f(n)dn + e$.

Ggigo Mandiaducia.

(5)

தேற்றம்: விறையாடுற்றுக்கான கியற்றனை இறிறம்!

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 $\int_{a}^{b} f(n) dn = \int_{a}^{b} f(n) dn = \int_{a}^{b} f(n) dn$

(1) 11 2 8.

 $\int_{a}^{b} S(D) = \int_{a}^{b} f(n) dn + \epsilon/2 = \int_{a}^{b} f(n) dn + \epsilon/2$ $\int_{a}^{b} f(n) dn - \epsilon/2 = \int_{a}^{b} f(n) dn - \epsilon/2$ $\int_{a}^{b} f(n) dn - \epsilon/2 < S(D) = \int_{a}^{b} f(n) dn + \epsilon/2$ $\int_{a}^{b} f(n) dn - \epsilon/2 < S(D) = \int_{a}^{b} f(n) dn + \epsilon/2$

=) w(D) = S(D) - S(D) < E + 11 D11 < S.

envarand over yours

 $S(D) - A(D) = [S(D) - \int f(n)dn] + [\int f(n)dn - \int f(n)dn] + [\int f(n$

Ifondn-Itoudn Zo since eso - I fonsoln = I fonsoln. No ognonowneggån snig.

Organowall gris borrigamloi vooryam'-

- (i) thing of assign surveyable of ognomawing passens [a15] N 2 mmg - Buggi [aic] stomg i Lc. 6] oring Dome on mandagi Day vimilar 2 ming stools cowing young [a19] dising Doncord Will 2 ming oborovoré. Elegré Ossesses oning + Journ Dougs QUULLY, Ognomawiggingna Cares BC (16) storing Don one mand 2 in may storing (a16) oring Don normany i drog viory in 2 on mynz 2 minos oros unis.
- ம் விற்ற விட்டிற்கான கூடுற்ற மற்றும் குழிற்ற பண்டுகள்: I [tin) ± gin)] dn = Stensoln ± Sqin)dn.
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- (V) Modulus allo
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