UNIT - I

STATISTICAL QUALITY CONTROL

Statistical quality control, abbreviated as SQC, is one of the most important applications of the statistical techniques in industry. These techniques are based on the theory of probability and sampling, and are being extensively used in almost all industries. Such as armament, aircraft, automobile, textile, electrical equipment, plastic, rubber, electronics, chemicals, petroleum, transportation, medicine, and so on. In fact, it is impossible to think of any industrial field where SQC is not used.

Def: The most important word in the term "Statistical quality control" is quality. By quality we mean an attribute of the product that determines its fitness for use.

Quality control, therefore, covers all the factors and processes of production which may be broadly classified as follows:-

(i) Quality of Materials:-

Material of good quality will result in smooth processing thereby reducing the waste and increasing the output. It will also give better finish to the end products.

(ii) Quality of Manpower:-

Trained and Qualified personnel will give increased efficiency due to the better quality. production through the application of skill and also reduce production cost and waste.

(iii) Quality of Machines:-

Better quality equipment will result in efficient work due to lack or scarcity of breakdowns and thus reduce the cost of defectives.

(iv) Quality of Management:-

A good management is imperative for increase in efficiency, harmony in relations and growth of business and markets.

Engineering :-

The creation and development of a product is basically engineering the development of quality evaluation through improved inspection producers is also engineering; again, the knowledge of causes of defects and sub-standard products and their rectification is engineering.

Statistical:

The concept of the behaviour of a process, which has brought in the idea of "prevention" and "control". is statistical; building an information system to satisfy the concept of "prevention" and "control" and improving upon product quality, requires statistical thinking.

Managerial:

The competent use of the engineering and statistical technology is managerial; the creation of a climate for quality consciousness in the organisation depends upon the policies and practices of the management; the effective co-ordination of the Quality control function with those of others is managerial.

5M

Basis of Statistical Quality Control.

The basis of statistical quality control is the degree of "variability" in the size or the magnitude of a given characteristic of the product variation in the quality of manufactured product in the repetitive process in industry is inherent and inevitable.

Change causes:-

Some stable pattern of variation or "a constant cause system" is inherent in any particular scheme of production and inspection. This pattern results from many minor causes that behave in a random manner. The variation due to these causes is beyond the control of human hand and cannot be prevented or eliminated under any circumstances. One has got to allow for variation within this stable pattern, usually termed as allowable variation. The range of such variation is known as "natural" tolerance of the process.

Assignable Causes:-

The important, factors of assignable causes of variation are sub-standard or defective raw materials, new techniques or operations, negligence of the operators, wrong or improper handling of machines, faulty equipment, unskilled or inexperienced technical staff, and so on. These causes can be identified and eliminated and are to be discovered in a production process before it goes wrong, i.e, before the production becomes defective.

5m

Chance causes of Variation	Assignable causes of Variation
Consist of many individual causes.	Consist of just a few individual causes.
Anyone chance cause results in only a small amount of variation.	Anyone assignable cause can result in a large amount of variation.
Chance variation cannot economically be eliminated from a process.	The presence of assignable variation can be detected, and action to eliminate the causes is usually economically justified.

Some typical chance causes of Variation are:-	Some typical assignable causes of Variation are:-
- slight vibration of a machine.	- Negligence of operators.
- Lack of human perfection in reading instruments and setting controls	- Defective raw material - Faulty equipment.
- Voltage fluctuations and variation in temperatures.	- Improper handling of machines.

STATISTICAL QUALITY CONTROL

Definition:

"SQC may be broadly defined as that industrial management technique by means of which product of uniform acceptable quality are manufactured. It is mainly concerned with setting things right rather than discovering and rejecting those made wrong".

— Duncan.

Benefits of Statistical Quality Control:

The following are some of the benefits that result when a manufacturing process is operating in a state of statistical control:

1. An obvious advantage of SQC is the control, maintenance and improvement in the Quality standards.

2. The act of getting a process in statistical quality control involves the identification and elimination of assignable causes of variation and possibly the inclusion of good ones, viz., new material or methods. This (a) helps in the detection and correction of many production troubles, and (b) brings about a substantial improvement in the product quality and reduction of spoilage and rework.

3. It tells us when to leave a process alone and when to take action to correct troubles, thus preventing frequent and unwarranted adjustments.

4. If a process in control (which is doing about all we can expect of it) is not good enough, we shall have to make more or less a radical (fundamental) change in the process- just

meddling (tampering) with it won't help.

5. A process in control is predictable. We know what it is going to do and thus we can more safely guarantee the product. In the presence of good statistical control by the supplier, the previous lots supply evidence on the present lots, which is not usually the case if the process is not in control.

6. If testing is destructive (e.g., testing the breaking strength of chalk; proofing of ammunition, explosives, crackers, etc.), a process in control gives confidence in the qualities of untested product which is not the case otherwise.

7. It provides better quality assurance at lower inspection cost.

8. Quality control finds its applications not only in the sphere of production, but also in other areas like packaging, scrap and spoilage, recovery,

advertising . etc. Foreign trade items of developing countries like India are particularly appropriate for every type of quality control in every possible area.

9. The Very presence of a quality control scheme in a plant improves and alerts the personnel. Such a scheme is likely to breed "quality consciousness" throughout the organisation which is of immense long-run Value.

10. S.Q.C reduce waste of time and material to the absolute minimum by giving an early warning about the occurrence of defects. Savings in terms of the factors stated above mean less cost of production and hence may ultimately lead to more profits.

Process control and Product control

5m. The main objective in any production process is to control and maintain a satisfactory quality level of the manufactured product so that it conforms to specified quality standards. In other words, we want to ensure that the proportion of defective items in the manufactured product is not too large. This is termed as "Process control" and is achieved through the technique of "control charts" pioneered by W. A. Shewhart in 1924.

On the other hand, by Product control we mean controlling the quality of the product by critical examination at strategic points and this is achieved through "sampling Inspection plans" pioneered by H. F. Dodge and H. C. Romig. Product control aims as guarenteeing a certain quality level to the consumer regardless of what quality level is being maintained by the producer. In other words, it attempts to ensure that the product marketed by sale department does not contain a large number of defective (unsatisfactory) items

Techniques of S.Q.C

- Process control (by control chart device)
  - Variables
    - $\bar{X}$- chart
    - R- chart
    - σ- chart
  - Attributes
    - C- chart
    - np- chart
    - P- chart
- Product control (by sampling inspection plans)
  - Attributes
  - Variables

Control limits, Specification limits and Tolerance limits:

5m

1. Control Limits:-

These are limits of sampling variation of a statistical measure, (e.g., mean, range, or fraction - defective) such that if the production process is under control, the values of the measure calculated from different rational subgroups will lie within these limits.

2. Specification limits:-

These maximum and minimum limits of variation of individual items, as mentioned in the product design, are known as "specification limits".

3. Tolerance limits:-

These are limits of variation of a quality measure of the product between which at least a specified proportion of the product is expected to lie (with a given probability), provided the process is in a state of statistical quality control. For example, we may claim with a probability of 0.99 that at least 90% of the products will have dimensions between some stated limits. These limits are also known as "statistical tolerance limits".

Control charts:-

Control charts are simple to construct and easy to interpret and tell us at a glance whether the sample point falls within 3-σ control limits (discussed below) or not. Any sample point giving outside the 3-σ control limits is an indication of the lack of statistical control. i.e. presence of some assignable causes of variation which must be traced, identified and eliminated.

A typical control chart consists of the following three horizontal lines:

(i) A central line (C.L) indicating the desired standard or the level of process.

(ii) upper control limit (U.C.L), Indicating the upper limit of tolerance.

(iii) Lower control limit (L.C.L), Indicating the lower limit of tolerance.

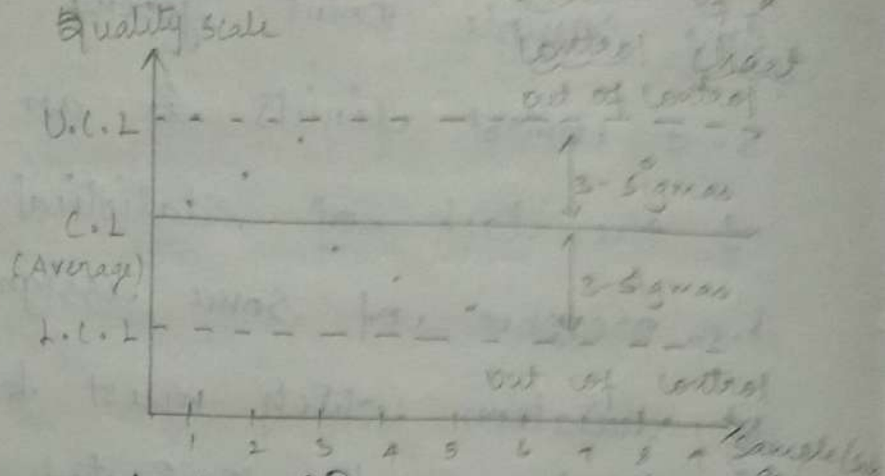
The control line as well as the

upper and lower limits are established by computations based on the past records or current production records.

Major Parts of a Control Chart

10m

The horizontal line:-



The central line represents the average quality of the samples plotted on the chart. The line above the central line shows the upper control limit (UCL) which is commonly obtained by adding 3 sigma's to the average. i.e., mean + 3(S.D). The line below the central line is the lower control limit (L.C.L) which is obtained by subtracting 3 sigmas from the average, i.e., Mean - 3(S.D)

The upper and lower control limits are usually drawn as dotted lines, and the central line is plotted as a bold (dark) line.

$$U.C.L = E(t) + S.E(t).$$
$$L.C.L = E(t) - 3 \cdot S.E(t).$$
$$C.L = E(t).$$

3σ Control limits :-

3.σ limits were proposed by Dr. Shewhart for his control charts from various considerations, the main being probabilistic considerations. Consider the statistic $t = t(x_1, x_2 \ldots x_n)$ a function of the sample observations $x_1, x_2 \ldots x_n$

let $E(t) = \mu_t$ and $Var(t) = \sigma_t^2$.

If the statistic t is normally distributed, than from the fundamental and property of the normal distribution, we have.

$$P[\mu_t - 3\sigma_t < t < \mu_t + 3\sigma_t] = 0.9973$$

$\Rightarrow P[|t - \mu_t| < 3\sigma_t] = 0.9973$ i.e, $P[|t - \mu_t| > 3\sigma_t] = 0.0027$

In other words, the probability that a random value of t goes outside the [illegible] is 0.0027, which is very small. Hence, if t is normally distributed, the limits of variation should be between $\mu_t + 3\sigma_t$ & $\mu_t - 3\sigma_t$ which are termed respectively the upper control limit (U.C.L) and lower control limit (L.C.L). If, for the i^{th} sample, the observed t lies between the upper and lower control limits, there is nothing to worry as in such a case variation between samples is attributed to chance i.e., in this case the process is in statistical control.

Remarks:-

1) If the assumption regarding normality of the statistic t does not hold, then the above argument does not remain strictly valid. In practice, the quality characteristic can seldom be supposed to be exactly normal. For non-normal population, i.e., If the sampling distribution of

statistic t is not normal) we apply Chebychev's Inequality in probability theory which states that for any constant $k > 0$,

$$P\left[|t - E(t)| < k\right] \geq 1 - \frac{Var(t)}{k^2}$$

$$\Rightarrow P\left[|t - \mu_t| < 3\sigma_t\right] \geq 1 - \frac{\sigma_t^2}{9\sigma_t^2} = 8/9 = 0.9,$$

Which is also fairly high for practical purposes and the above argument, holds, more or less. However, in practice, σ_t is not known and is estimated from the sample data and consequently chebychev's inequality does not hold if σ_t is not known.

Tools for SQC:-

Imp. The following four, seperate but related techniques, are the most important statistical tools for data analysis in quality control of the manufactured products:-

1) Shewhart's control chart for variables i.e., for a characteristic which

can be measured quantitatively.
Many quality characteristics of a product are measurable and can be expressed in specific units of measurements such as diameter of a screw, tensile strength of steel pipe, specific resistance of a wire, life of an electric bulb, etc. Such variables are of continuous type and are regarded to follow normal probability law. For quality control charts are used and technically these charts are known as:

(a) Charts for $\bar{x}$ (mean) and R (Range)
and (b) Charts for $\bar{x}$ (mean) and σ (Standard deviation).

2) Shewhart's control chart for fraction defective or p- chart.
This chart is used if we are dealing with attributes in which case the quality characteristics of the product are not amenable to

measurement but can be identified by their absence or presence from the product or by classifying the product as defective or non-defective.

3) Shewhart's control chart for the "Number of Defects" per unit or C-chart. This is usually used with advantage when the characteristic representing the quality of a product is a discrete variable, eg, (i) the number of defective rivets in an aircraft wing, and (ii) the number of surface defects observed in a roll of coated paper or a sheet of photographic film.

4) The portion of the sampling theory which deals with the quality protection given by any specified sampling acceptance procedure.

UNIT-II CONTROL CHARTS FOR VARIABLES

These charts may be applied to any quality characteristic that is measurable. In order to control a measurable characteristic we have to exercise control on the measure of location as well as the measure of dispersion. Usally $\bar{X}$ and R charts are employed to control the mean (location) and standard deviation (dispersion) respectively of the characteristic.

$\bar{X}$ and R charts :-

No production process is perfect enough to produce all the items exactly alike. Some amount of variation, in the product items, is inherent in any production scheme. This variation is the totality of numorous characteristics of the production process viz., raw material, machine setting and handling, operators, etc. As pointed out earlier, this variation is the result of (i) chance causes, (ii) assignable causes.

The control limits in the $\bar{x}$ and R charts are so placed that they reveal the presence or absence of assignable causes of variation in the

a) average - mostly related to machine setting, and

b) range - mostly related to negligence on the part of the operator.

Steps for $\bar{x}$ and R charts:

1. Measurement: Actually the work of a control chart starts first with measurements. Any method of measurement has its own inherent variability. Errors in measurement can enter into the data by:

(i) the use of faulty instruments,

(ii) lack of clear-cut definitions of quality characteristics and the method of taking measurements, and

(iii) lack of experience in the handling or use of instruments, etc.

Since the conclusions drawn from the control chart are broadly based on the variability in the measurements as well as the variability in the quality being measured, it is important that the mistakes in reading measurement instruments or errors in recording data should be minimised so as to draw valid conclusions from control charts.

2. Selection of Samples or Sub-groups: In order to make the control chart analysis effective, it is essential to pay due regard to the rational selection of the samples or sub-groups. The choice of the sample size n and the frequency of sampling, i.e., the time between the selection of two groups, depend upon the process and no hard and fast rules can be laid down for this purpose.

Control limits for $\bar{X}$-charts:

Case 1:-

When standards are given, i.e., both μ and σ are known. The $3-\sigma$ control limits for $\bar{X}$ chart are given by:

$$E(\bar{x}) \pm 3\,S.E(\bar{x}) = \mu \pm 3\sigma/\sqrt{n} = \mu \pm A\sigma, \; (A = 3/\sqrt{n})$$

If μ' and σ' are known or specified values of μ and σ respectively, then

$$UCL_{\bar{x}} = \mu' + A\sigma' \text{ and } LCL_{\bar{x}} = \mu' - A\sigma' \quad \rightarrow (1)$$

where $A (= 3/\sqrt{n})$ is a constant depending on n and its values are tabulated for different values of n from 2 to 25 in Table VIII in the Appendix.

Case 2:-

Standards not given .. If both μ and σ are unknown, then using their estimates $\bar{\bar{x}}$ and $\hat{\sigma}$ given in equations we get the 3-σ control limits for the $\bar{x}$-chart as:

$$\bar{\bar{x}} \pm 3\frac{\bar{R}}{d_2} \cdot \frac{1}{\sqrt{n}} = \bar{\bar{x}} \pm \left(\frac{3}{d_2\sqrt{n}}\right)\bar{R} = \bar{\bar{x}} \pm A_2\bar{R}, \; \left(A_2 = \frac{3}{d_2\sqrt{n}}\right)$$

$$\therefore UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} \text{ and } LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} \rightarrow (2)$$

Since d_2 is a constant depending on n, $A_2 = 3/(d_2\sqrt{n})$ also depends only on n and its values have been computed and tabulated for different values of n from 2 to 25 and are given in

~~The~~ If, on the other hand, the control limits are to be obtained in terms of $\bar{s}$ rather than $\bar{R}$, then an estimate of σ can be obtained from the relation.

$$E(s) = c_2\sigma \Rightarrow \bar{s} = c_2\sigma$$

$$\text{i.e., } \hat{\sigma} = \bar{s}/c_2 \longrightarrow (3)$$

Where, $c_2 = \sqrt{\frac{2}{n}} \cdot \frac{\left(\frac{n-2}{2}\right)!}{\left(\frac{n-3}{2}\right)!}$, is a constant depending on n.

$$\left.\begin{aligned} \therefore UCL_{\bar{x}} &= \bar{\bar{x}} + \left(\frac{3}{\sqrt{n}\,c_2}\right)\bar{s} = \bar{\bar{x}} + A_1\bar{s} \\ \text{and } LCL_{\bar{x}} &= \bar{\bar{x}} - \left(\frac{3}{\sqrt{n}\,c_2}\right)\bar{s} = \bar{\bar{x}} - A_1\bar{s} \end{aligned}\right\} \rightarrow (4)$$

The factor $A_1 = 3/(\sqrt{n}\,c_2)$ has been tabulated for different values of n.

Control limits for the R-chart:

R-chart is constructed for controlling the variation in the dispersion (variability) of the product. The procedure for constructing R-chart is similar to that for the $\bar{x}$-chart and involves the following steps:

1) Compute the range $R_i = \max_j x_{ij} - \min_j x_{ij}, (i = 1, 2 \ldots, n)$ for each sample:

2) Compute the mean of the sample ranges:

$$\bar{R} = \frac{1}{k} \sum_{i=1}^{k} R_i = \frac{1}{k} (R_1 + R_2 + \ldots R_k).$$

3). Computation of control limits:-

The $3-\sigma$ controls limits for R-chart are: $E(R) \pm 3\sigma_R$. $E(R)$ is estimated by $\bar{R}$ and σ_R is estimated from the relation:

$$\sigma_R = d_3 \hat{\sigma} = d_3 \cdot \frac{\bar{R}}{d_2}, \rightarrow (5)$$

Where d_2 and d_3 are constants depending on n.

$$\therefore UCL_R = E(R) + 3\sigma_R = \bar{R} + \frac{3d_3}{d_2}\bar{R} \rightarrow (6)$$

$$\Rightarrow UCL_R = \left(1 + \frac{3d_3}{d_2}\right)\bar{R} = D_4 \bar{R} \rightarrow (7)$$

Similarly $LCL_R = \left(1 - \frac{3d_3}{d_2}\right)\bar{R} = D_3 \bar{R} \rightarrow (8)$

The values of D_4 and D_3 depend only on n and have been computed and tabulated for different values of n from ~~2~~ 1.

However, if σ is known, then

$$UCL_R = E(R) + 3\sigma_R = d_2\sigma + 3d_3\sigma$$
$$= (d_2 + 3d_3)\sigma = D_2\sigma \longrightarrow (9)$$

$$LCL_R = E(R) - 3\sigma_R = d_2\sigma - 3d_3\sigma$$
$$= (d_2 - 3d_3) = D_1\sigma \longrightarrow (10)$$

In each case, (σ known or unknown) the central line is given by:

$$CL_R = E(R) = \bar{R} \longrightarrow (11)$$

Since range can never be negative, LCL_R must be greater than or equal to 0. In case it comes out to be negative, it is taken as zero.

Criterion for Detecting Lack of Control in $\bar{X}$ and R - Charts.

As pointed out earlier, the main object of the control chart is to indicate when a process is not in control. The criteria for detecting lack of control are, therefore, of fundamental and crucial importance. The pattern of the sample points in a control chart is the key to the proper interpretation of the working of the process. The following situations depict lack of control.

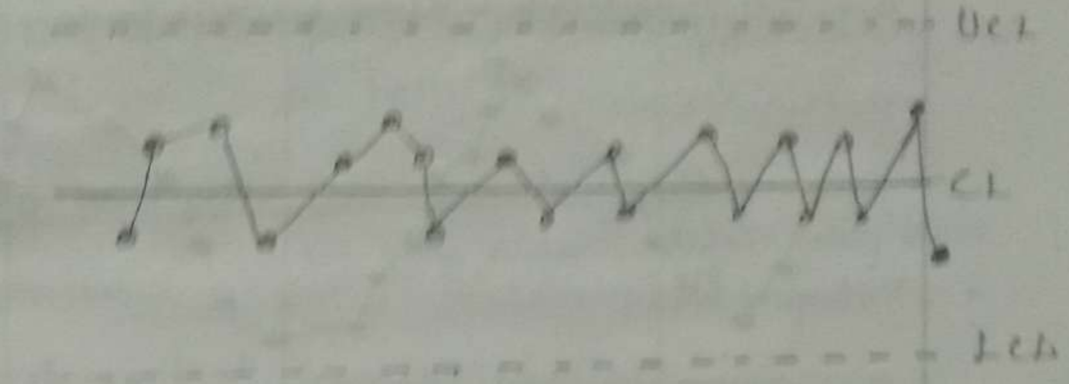
1. A point outside the control limits. The probabilistic considerations provide a basis for hunting for lack of control in such a situation. A point going outside control limits is a clear indication of the presence of assignable causes of variation which must be searched and corrected. A point outside the control limits may result from an increased dispersion or change in level. lack of uniformity may be due to the variation in the quality of raw material, deficiency in the skill of the operators, loss of alignment among machines, change of working conditions, etc. It may be indicated by a point above the upper control limit for ranges. It may also result in points outside the control

limit for means.

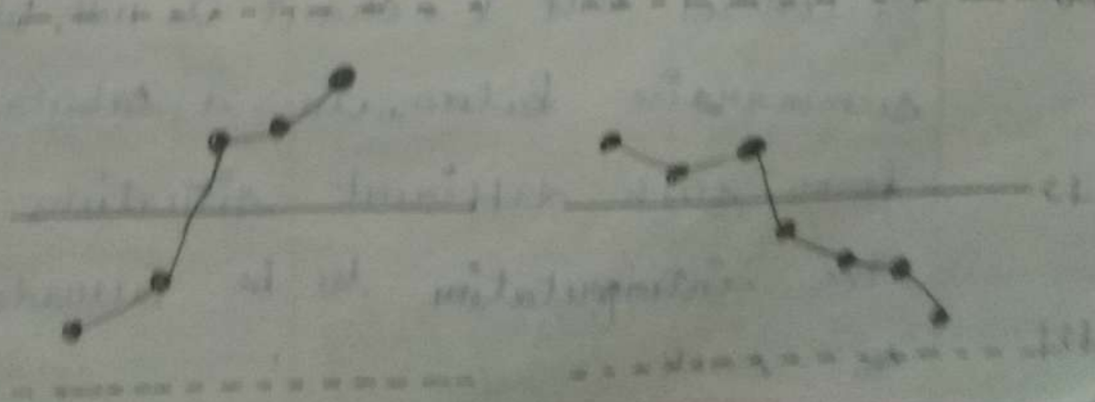
2. A run of seven or more points. Although all the sample points are within control limits, usually the pattern of points in the chart indicates assignable causes. One such situation is a run of 7 or more points above or below the central line in the control chart. Such runs indicate shift in the process level. On R-chart a run of points above the central line is indication of increase in process spread and therefore represents an undesirable situation, while a run below the central line indicates an improvement in the sense that the variability has been reduced, i.e, the process could be held to a closer tolerance.

3) One or more points in the vicinity of control limits. or a run of points beyond some secondary limits, e.g., a run of 2, 3 points beyond 2-σ limits or a run of 4, 5 points beyond 1-σ limits

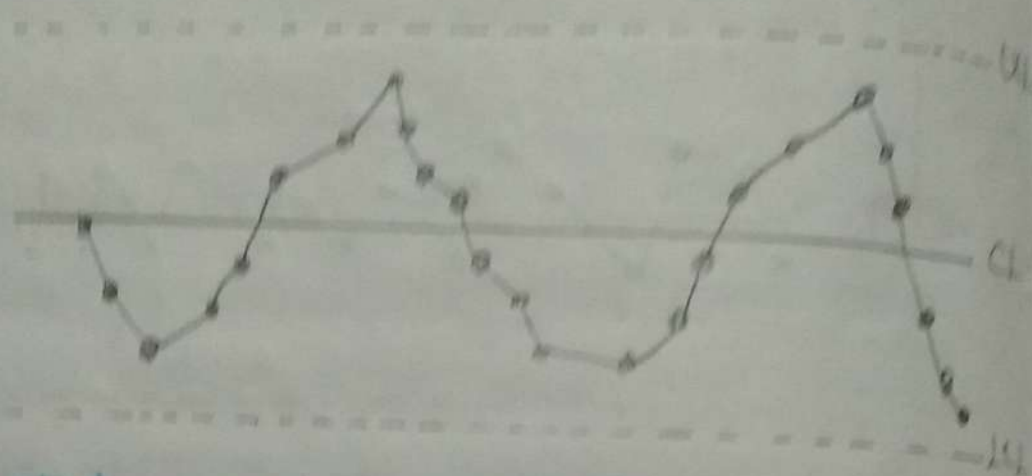
4. The sample points on $\bar{X}$ and R-chart too close to the central line, exhibit another form of assignable-cause. This situation represents systematic differences within samples or sub-groups and result from improper selection of samples and biases in measurements.



5. Presence of Trends. The Trends exhibited by sample points on the control chart are also an indication of assignable cause. Then pattern, a phenomenon usually observed in engineering industry, indicates the gradual shift in the process level.



b) Presence of cycles: In some cases the cyclic pattern of points in the control chart indicates the presence of assignable causes of variation. Such patterns are due to material or/and any mechanical reasons.



Interpretation of $\bar{x}$ and R-Charts:-

In order to judge if a process is in control, $\bar{x}$ and R-charts should be examined together and the process should be deemed in statistical control if both the charts show a state of control. Situations exists where R-chart is in a state of control but $\bar{x}$-chart is not. We summarize below, in a tabular form, such different situations and the interpretation to be accorded to each.

Sl. No	Situations in		Interpretation
	R-chart	$\bar{X}$-chart	
1.	In control	points beyond limits only on one side	Level of process has shifted.
2.	In control	points beyond limits on both sides	Level of process is changing in erratic manner - frequent adjustments.
3.	Out of control	points beyond limits on both sides	Variability has increased.
4.	out of control	Out of control on one side	Both level and variability have changed.
5.	In control	Run of 7 or more points on one side of central line	Shift in process level.
6.	In control	Trend of 7 or more points. No point outside control limits.	Process level is gradually changing.
7.	Runs of 7 or more points above central line	- - -	Variability has increased
8.	points too close to the central line	- - -	Systematic differences within subgroups
9	- - -	points too close to the central	systematic differences within subgroups

Control chart for Standard Deviation.

Since standard deviation is an ideal measure of dispersion, a combination of control chart for mean ($\bar{x}$) and standard deviation (s), known as $\bar{x}$ and s-charts is theoretically more appropriate than a combination of $\bar{x}$ and R-Charts for controlling process average and process variability.

In a sample of size n from normal population with standard deviation σ, we have.

$$E(s^2) = \frac{n-1}{n}\sigma^2 \quad \rightarrow (1)$$

and $E(s) = C_2 . \sigma$, where $C_2 = \sqrt{\frac{2}{n}} \frac{[(n-2)/2]!}{[(n-3)/2]!}$ $\rightarrow$ (2)

The values of C_2 have been tabulated for different values of n

Hence, in sampling from normal population, we have.

$$Var(s) = E(s^2) - [E(s)]^2 = \left(\frac{n-1}{n} - C_2^2\right)\sigma^2$$

$$S.E(s) = C_3 . \sigma, \text{ where } C_3 = \sqrt{\frac{n-1}{n} - C_2^2}$$

$$UCL_s = E(s) + 3\,S.E.(s) = (C_2 + 3C_3)\sigma = B_2\,\sigma$$

$$LCL_s = E(s) - 3\,S.E.(s) = (C_2 - 3C_3)\sigma = B_1\,\sigma$$ → (3)

Central line $= CL_s = C_2 \cdot \sigma$

Values of B_1 and B_2 have been tabulated for different values of n.

If the value of σ is not specified or not known, then we use its estimate, based on $\bar{s}$ and given by $\hat{\sigma} = \bar{s}/C_2$. In this case,

$$UCL_s = E(s) + 3\,S.E.(s) = \bar{s} + 3\frac{C_3}{C_2}\cdot\bar{s}$$

$$= \left(1 + \frac{3C_3}{C_2}\right)\bar{s} = B_4 \cdot \bar{s}$$ → (4)

Similarly, we shall get.

$$LCL_s = \left(1 - \frac{3C_3}{C_2}\right)\bar{s} = B_3 \cdot \bar{s}$$ → (5)

and $CL_s = \bar{s}$ → (6)

where B_3 and B_4 have been tabulated for different values of n. Since s can never be negative, if LCL, as will be the ... taken to be zero.