STATISTICAL QUALITY CONTROL

CODE: 18K5S10

UNIT - III

Control charts for Attributes :-

In spite of wide applications of $\bar{x}$ and R charts as a powerful tool of diagnosis of sources of trouple in a production processes, their use is restricted because of the following limitations.

1. They are charts for variables only ie., for quality characteristics which can be measured and expressed in numbers.

2. In certain situations they are impracticable and un-economical, eg, if the number of measurable characteristics, each of which could be a possible candidate for $\bar{x}$ and R charts is too large, say 30 000, or so then obviously there can't be 30,000 control charts.

As an alternative to $\bar{x}$ and R-charts, we have the control chart for attributes which can be used for quality characteristics.

(i) which can be observed only as attributes by classifying an item as defective or non-defective ie., conforming to specifications or not and

(ii) which are actually observed as attributes even through they could be measured as variables e.g., go and no-go gauge test results

They are two control charts for attributes:

(a) control chart for fraction defective (p-chart) or the number of defectives (np or d chart) and (b) c-chart

Control chart for fraction defective (p-chart):-

If 'd' is the number of defectives in a sample of size n, then the sample proportion defective is $p = d/n$. Hence, d is a binomial variate with parameters n and P.

$E(d) = nP$ and $Var(d) = nPQ$, $Q = 1-P$

Thus $E(p) = E(d/n) = \frac{1}{n} E(d) = P$ and

$V(p) = Var(d/n) = \frac{1}{n^2} Var(d) = \frac{PQ}{n}$

Thus 3-σ control limits for p-chart are given by

$E(p) \pm 3\ S.E(p) = P \pm 3\sqrt{\frac{PQ}{n}} = P \pm A\sqrt{PQ}$

Where $A = 3/\sqrt{n}$ has been tabulated for different values of n

Case-(i) Standards specified :-

If P' is the given (or) known value of P

then $UCL_p = P' + A\sqrt{P'(1-P')}$; $LCL_p = P' - A\sqrt{P'(1-P')}$;

$CL_p = P'$

Case - II Standards not specified :-

Let d_i be the number of defectives and p_i the fraction defective for the i^{th} sample ($i = 1, 2, \ldots k$) of size n, then the population proportion P is estimated by the statistic

$\bar{p}$ is given by $\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{\sum n_i p_i}{\sum n_i}$

It may be remarked here that $\bar{p}$ is an unbiased estimate of P since

$$E(\bar{p}) = \sum E(d_i)/\sum n_i = \left[\sum (n_i P)/\sum n_i\right] = P$$

In this case $UCL_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})}$

$LCL_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})}$; $CL_p = \bar{p}$

Control chart for Number of Defectives (d-chart) :-

If instead of p, the sample proportion defective, we use of d, the number of defectives in the sample, then the 3-σ control limits for d-chart are given by

$$E(d) \pm 3 \cdot SE(d) = np \pm 3\sqrt{np(1-p)}$$

Case - I Standard specified

If p' is the given value of p then

$$UCL_d = np' + 3\sqrt{(np')(1-p')} \; ; \; LCL_d = np' - 3\sqrt{np'(1-p')}$$

$$CL_d = np'$$

Case - II Standards not specified :-

Using $\bar{p}$ an estimate of p as in we get

$$UCL_d = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \,;\; LCL_d = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$CL_d = n\bar{p}$$

Interpretation of p-chart:-

1. The p-chart a process is judged to be in statistical control in the same way as is done for $\bar{x}$ and R charts. If all the sample points fall within the control limits without exhibiting any specific pattern, the process is said to be in control

2. points outside the UCL are termed as high spots. These suggest deterioration in the quality and should be regularly reported to the production engineers. The reasons for such deterioration could possibly the known and removed if the details of conditions under which data were collected.

3. Points below LCL are called low spots. Such points represent a situation showing improvement in the product quality.

4. When a number of points fall outside the control a revised estimate of p should be obtained by eliminating all the point that fall above UCL. The standard fraction defective p should be revised periodically in this way.

Example : 1

The following are the figures of defectives in 22 lots each containing 2000 rubber belts.
425, 430, 216, 341, 225, 322, 280, 306, 337, 305, 356
402, 216, 264, 126, 409, 193, 326, 280, 389, 451, 420

Draw control chart for fraction defective and Comment on the state of control of the process.

Solution :-

Here we have a fixed sample size $n = 2000$ for each lot. If d_i and p_i are respectively the number of defectives and sample fraction defective for the the ith lot, then $p_i = \frac{d_i}{2000}$ $(i = 1, 2, \ldots 22)$

S. NO	d	p =(d/2000)	S. NO	d	p =(d/2000)
1	425	0.2125	12	402	0.2010
2	430	0.2150	13	216	0.1080
3	216	0.1080	14	264	0.1320
4	341	0.1705	15	126	0.0630
5	225	0.1125	16	409	0.2045
6	322	0.1610	17	193	0.0965
7	280	0.1400	18	326	0.1630
8	306	0.1530	19	280	0.1400
9	337	0.1685	20	389	0.1945
10	305	0.1525	21	451	0.2255
11	356	0.1780	22	420	0.2100
	3,543	1.7715		3476	1.7380

In the usual notations, we have

$$\bar{p} = \frac{\Sigma p_i}{k} = \frac{1.7715 + 1.7380}{22} = \frac{3.5095}{22} = 0.1595$$

$$\Rightarrow \bar{q} = 1 - \bar{p} = 1 - 0.1595 = 0.8405$$

3-σ control limits for p-chart are given by (b)

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.1595 \pm 3\sqrt{0.1595 \times 0.8405/2000}$$

$$= 0.1595 \pm 3\sqrt{0.0000672} = 0.1595 \pm 0.0246$$

$$\therefore UCL_p = 0.1595 + 0.0246$$

$$= 0.1841$$

$$LCL_p = 0.1595 - 0.0246 = 0.1349$$

$$CL_p = \bar{p} = 0.1595.$$

Example : 2

20 samples each of size 10 were inspected the number of defectives detected in each of them is given below.

Samples NO :	1	2	3	4	5	6	7	8	9	10
No. of defectives :	0	1	0	3	9	2	0	7	0	1
sample No :	11	12	13	14	15	16	17	18	19	20
No. of defectives :	1	0	0	5	1	0	0	2	1	0

Construct the 'number of defectives' chart and establish quality standard for the future

Solution :-

Here we have samples of fixed size $n=10$. The total number of defectives in all the 20 samples is

$$\Sigma d = 0+1+0+3+9+\cdots+2+1+0 = 31$$

An estimate of the process fraction defectives is given by

$$\bar{p} = \frac{\Sigma d}{nk} = \frac{31}{10 \times 20} = 0.155 \Rightarrow \bar{q} = 1-\bar{p}$$

$$= 0.845$$

The 3-σ control limits for number defectives chart (np-chart) are given by

$$CL_{np} = n\bar{p} = 1.55$$

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 1.55 + 3\sqrt{1.55 \times 0.845} = 1.55 + 3.43$$

$$= 4.98$$

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 1.55 - 3\sqrt{1.55 \times 0.845} = 0$$

The control chart for the 'number of defective' is obtained on plotting the number of defectives against the corresponding sample number

UNIT-IV

Control chart for Number of Defects Per Unit (c-chart) :-

The field of application of c-chart is much more restricted as compared to $\bar{x}$ and R charts (or) p-chart.

np-chart which applies to the number of defectives in a sample, c-chart applied to the number of defects per unit. sample size for c-chart may be a single unit like a radio, (or) group of units or it may be a unit of fixed time, length, are, etc.

For example: In case of surface defects, area of the surface is the sample size; in case of casting defects, a single part is the sample size. However, defined sample size should be constant in the sense that different samples have essentially equal opportunity for the occurence of defects.

Control Limits for c-chart :-

The sample size n ie., the area of opportunity is very large and the probability p of the occurence of a defect in any one spot is very small such that np is finite. In such situations from statistical theory we know that the pattern of variations in data cab be represented by poisson distribution, and consequently 3-σ control limits based on poisson distribution are used.

If we assume that c is poisson variate with parameter, λ we get

$$E(c) = \lambda \text{ and } Var(c) = \lambda$$

Thus 3-σ control limits for c-chart are given by

$$UCL_c = E(c) + 3\sqrt{Var(c)} = \lambda + 3\sqrt{\lambda}$$

$$LCL_c = E(c) - 3\sqrt{Var(c)} = \lambda - 3\sqrt{\lambda}$$

$$CL_c = \lambda$$

Case (i) standards specified :-

If λ' is the specified value of λ.

Then $UCL_c = \lambda' + 3\sqrt{\lambda'}$; $LCL_c = \lambda' - 3\sqrt{\lambda'}$; $CL_c = \lambda'$

Case (ii) standards not specified :-

If the value of λ is not known, it is estimated by the mean number of defects per unit. Thus if c_i is the number of defects observed on the ith $(i = 1, 2 \ldots k)$ inspected unit, then an estimate of λ is given by

$$\hat{\lambda} = \bar{c} = \sum_{i=1}^{k} c_i / k$$

It can be easily that $\bar{c}$ is an unbiased estimate of λ. The control limits in this case are given by

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} \; ; \; LCL_c = \bar{c} - 3\sqrt{\bar{c}} \; ; \; CL_c = \bar{c}$$

Since c can't be negative, if LCL is given by above formulae comes out to be negative, it is regarded as zero.

The observed number of defects on the inspected units are then plotted on the control chart. The interpretations for c-chart are similar to these of p-chart.

C-chart for variable sample size (or) U-chart :-

In this case instead of plotting c, the statistic $u = c/n$ is plotted, n being the sample size which is varying. If n_i is the sample size and c_i the total number of defects observed in the i^{th} sample, then

$$u_i = c_i / n_i \quad (i = 1, 2, \cdots k)$$

gives the average number of defects per unit for the i^{th} sample.

In this case an estimate of λ, the mean number of defects per unit in the lot, based on all the k-samples is the given by

$$\hat{\lambda} = \bar{u} = \frac{1}{k} \sum_{i=1}^{k} u_i$$

We know that if $\bar{x}$ is the mean of a random sample of size n then $S.E(\bar{x}) = \sigma/\sqrt{n}$. Hence, the standard error of the average number of defect per unit is given by

$$S.E.(u) = \sqrt{\lambda/n} = \sqrt{\bar{u}/n}$$

Hence, 3-σ control limits for u-chart

$$UCL_u = \bar{u} + 3\sqrt{\bar{u}/n} \; ; \; LCL_u = \bar{u} - 3\sqrt{\bar{u}/n} \; ;$$

$$CL_u = \bar{u}$$

The central line however, will be same. The interpretation of these charts is similar to the p-chart or d-chart.

Applications of C-chart :-

The limited field of application of c-chart (compared to $\bar{x}$, R, p charts) there do exist situations in industry where c-chart is definitely needed. Some of the representative types of defects to which c-chart can be applied and with advantage

(1) C is number of imperfections observed in a bale of cloth.

(2) c is the number surface defects observed in roll of coated paper or a sheet of Photographic film and (ii) a galvanised sheet or a plated or enamelled surface of given area

(3) C is the number of defects of all types observed in aircraft sub-assemblies or final assembly.

(4) C is the number of breakdowns at weak spots in insulation in a given length of insulated wire subject to a specified test voltage.

(5) C is the number of defects observed in stains (or) blemishes on a Surface

(6) C is the number of soiled packages in a given consignment

(7). C chart has been applied to sampling acceptance procedures based on number of defects per unit, e.g., in case of inspection of fairly complex assembled units such as T.V. sets, aircraft engines, tanks, machine guns, etc.

Natural Tolerance limits and specification Limits :-

A process in statistical control implies that the control charts for both the mean and range show complete homogeneity and in such a case, a measure of the variation of the individual products is given by the S.D (σ) estimate by $\bar{R}/d_2$ or $\bar{s}/c_2$ from control data. If μ and σ are the process average and process standard deviation respectively. Then the limits $\mu \pm 3\sigma$ are called the Natural Tolerance limits.

If μ and σ are not known then $\hat{\mu} \pm 3\hat{\sigma}$ are the estimates of the natural tolerance limits where $\hat{\mu} = \bar{\bar{x}}$ and $\hat{\sigma} = \bar{R}/d_2$ or $\hat{\sigma} = \bar{s}/c_2$

The process is in statistical control as exihubited by control charts, the customer may not be statified with the product. This happens when the process does not conform to specification limits for that item.

These specification limits are generally given in terms of sw upper and lower tolerance limit.

Let x_{mas} and x_{min} denote the upper specification limits and lower specification limit respectively. when both these limits are specified, a comparison of these with the natural tolerance limits.

Interpretations of Natural tolerance limits and specification limits:-

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A process in such a case almost all the manufactured items will conform to specifications as long as the process is in statistical control and is appropriately centered as in position A, B and C

If the process is operating under one of these conditions, $\bar{x}$ may be permitted to go out of control, provide it does not go too far; in other words the distribution of x may be allowed to fluctuate between positions B and C.

This will save the time and money for frequent machine setting and delays due to looking for assignable causes of variation which will not be responsible for unsatisfactory product

(ii) In such a situation since, even considerable shifts in the level of working may not result in the items falling outside specification limits the time interval between taking successive samples for control chart inspection can be appreciably increased.

(iii) The larger the ratio $x_{max} - x_{min}$ to the natural tolerance 6σ, the greater is the likelihood of getting good product without assistance from any controls chart.

(b) Specification limits coincide with tolerance

limits ie., $X_{max} - X_{min} = 6\sigma$

(C). Natural tolerance is greater than specified tolerance. ie.

$$X_{max} - X_{min} < 6\sigma.$$

The upper and lower edges of the central band are given respectively by

$$USL - 3\sigma, \; LSL + 3\sigma$$

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UNIT – V

Acceptable Quality Level (AQL) :-

This is the quality level of a good lot. It is the percent defective that can be considered satisfactory as a process average, and represents a level of quality which the producer wants accepted with a high probability of acceptance. In other words if α is the producer's risk then the level of quality which results in $100(1-\alpha)\%$ acceptance of the good lots submitted for inspection is called the acceptable quality level.

$$P\text{ (Rejecting a lot of quality } p_1) = 0.05$$

$$P_\alpha = P\text{ Accepting of a lot of quality } p_1) = 0.95$$

'p_1' is known as the "Acceptance Quality Level" and a lot of this quality is considered as satisfactory by the consumer.

Lot Tolerance proportion or Percentage Defective (LTPD) :-

The lot tolerance proportion defective usually denoted by p_t is the lot quality which is considered to be bad by the consumer.

The consumer is not willing to accept lots having proportion defective p_1 or greater.

$100 P_t$ is called Lot Tolerance Percentage Defective.

In other words this is the quality level which the consumer regards as rejectable and is usually abbreviated as RQL (Rejecting Quality Level). A lot of quality P_t stands to be accepted some arbitrary and small fraction of time usually 10%

Consumer's Risk :-

Any sampling scheme would involve certain risk on the part of the consumer in the sense that he has to accept certain percentage of undesirably bad lots.

More precisely the probability of accepting a lot with fraction defect i.e. P_t is termed as consumer's risk is written as P_c. Usually it is denoted by β. This is taken by dodge and Roming as 10% or 0.10

Consumer's risk $= P_c = P$ [accepting a lot of quality P_t) $= \beta$

Producer's Risk :-

The producer has also to face the situation that some good lots will be rejected. He ought demand adequate protection against such contingencies happening too frequently just as the consumer can claim reasonable protection against accepting too many bad lots

The probability of rejecting a lot with [illegible] process average percentage defective is called the producer's risk P_p and is usually denoted by α.

Thus producer's risk $= P_p = P($ of rejecting a lot of quality $p) = \alpha$.

Rectifying Inspection Plans :-

The inspection of the rejected lots and replacing the defective pieces found in the rejected lots by the good ones, eliminates the number of defectives in the lot to a great extent thus improving the lot quality. These plans are called "rectifying inspection plans"

Average outgoing Quality Limit :-

Let the producer's fraction defectives ie., lot quality before inspection be 'p'. This is termed as 'incoming quality'. The fraction defective of the lot after inspection is known as outgoing quality of the lot. The expected fraction defective remaining in the lot after the application of the sampling inspection plan is termed as Average outgoing Quality (AOQ) $\bar{p}$. obviously it is a function of the incoming quality 'p'.

OC curve :

Operating characteristic (OC) curve of a sampling plan is a graphic representation of the relationship between the probability of acceptance $P_a(p)$ or generally denoted by $L(p)$, for variations in the incoming lot quality 'p', for the general points on the curve.

Average sample number (ASN) and Average Amount of Total Inspection :

The average sample number (ASN) is the expected value of the sample size required for coming to a decision about the acceptance or rejection of the lot in an acceptance-rejection sampling plan.

We observe that

ATI = ASN + (Average size of inspection of the remainder in the rejected lots)

Thus, if the lot is accepted on the basis of the sampling inspection plan then ATI = ASN otherwise ATI > ASN. In other words ASN gives the average number of units inspected per accepted lot.

For example :-

If a single sampling acceptance-rejection plan is used, the number of items inspected from each lot will be the corresponding sample size n ie.,

$$ASN = n$$

and this will be true, independently of the quality of the submitted lots.

However, for an acceptance-rectification single sampling plan calling for 100% inspection of the rejected lots, additional $(N-n)$ items, will have to be inspected for each rejected lot. where N is the lot size. Thus, in this case, the number of items inspected per lot varies from lot to lot and is equal to n if the lot is accepted and equal to N if the lot is rejected on the basis of the sampling inspection plan.

$$ATI = nL(P) + N[1-L(P)]$$

Where $L(P) = P_a(P)$ is the probability of acceptance of the lot quality p on the basis of the sampling inspection.

$$\begin{aligned} ATI &= nL(P) + (N-n+n)[1-L(P)] \\ &= nL(P) + (N-n)\,1-L(P) + n[1-L(P)] \\ &= n + (N-n)[1-L(P)] \end{aligned}$$

Sampling Inspection plan for Attributes:-

(i) Single sampling plan (ii) Double Sampling plan and (iii) Sequential sampling Plan.

(i) Single Sampling plan:-

If the decision about accepting or rejecting a lot is taken on the basis of one sampling only, the acceptance plan is described as single Sampling Plan. It is completely speified by three numbers N, n and c. where

N is the lot size,
n is the sample size and
c is the acceptance number, ie., maximum allowable number of defectives in the sample.

The single sampling plan may be described as follows.

1. Select a random sample of size n from a lot of size N

2. Inspect all the articles included in the sample. Let d be the number of defectives in the sample.

3. If $d \leq c$, accept the lot, replacing defective Pieces found in the sample by one non-defective ones.

4. If $d > c$ rejected the lot. In this case we inspect the entire lot and replace all the defectives ~~Pts~~ Pieces by standard ones.

Single sampling plan is very simple to understand. design and carry out.

Determination of n and c for sampling inspection:

The Lot size N is invariably known. Thus the two unknown quantities that need to be determined in the Sampling Plan n and c.

In a lot of incoming quality p, the number of defective pieces is Np and non-defective pieces is $N - NP = N(1-P)$.

$$g(x, P) = \left[{}^{Np}C_x \times {}^{N-Np}C_{n-x} \,/\, {}^{N}C_n \right]$$

Hence the consumer's risk is given by

$$P_c = P = \sum_{x=0}^{c} g(x_i\, p_i) = \sum_{x=0}^{c} \left[\frac{{}^{NP}C_x \times {}^{N-Np}C_{n-x}}{{}^{N}C_n} \right]$$