

**UNIT III:**

Central difference interpolation formula- Gauss forward and backward difference formula- Stirling's, Bessel's central forward formula – simple problems

**Central difference formula:**

**Introduction:**

Newton's forward and backward difference interpolation formula are best suited for interpolating near the beginning and respectively of a difference table. But when we require to interpolate near the middle (center) of a difference table then the following central difference interpolation are most suitable.

If  $f(x)$  takes values  $f(0), f(1), f(2), \dots$  equally spaced having unit intervals (or) it takes values  $f(-3), f(-2), f(-1), f(0), f(1), f(2), \dots$ . Then using the central difference operators  $\delta$ .

The central difference operator  $\delta$  is defined by the operator equation.

$$\delta.f(x) = f(x+h/2) - f(x-h/2)$$

$$\delta = E^{1/2} - E^{-1/2}$$

**Central difference formula:**

1. Gauss forward formula
2. Gauss backward formula
3. Stirling formula
4. Bessel's formula

**1. Gauss forward formula:**

Newton's forward formula is  $y_u = y_0 + \Delta y_0 + U C_1 \Delta^2 y_0 + U C_2 \Delta^3 y_0 + \dots$  (1)

$$\Delta^3 y_{-1} = \Delta^2 y_0 - \Delta^2 y_{-1}$$

$$\Rightarrow \Delta^2 y_0 = \Delta^2 y_{-1} + \Delta^3 y_{-1}$$

$$\Delta^5 y_{-1} = \Delta^4 y_0 - \Delta^4 y_{-1}$$

$$\Rightarrow \Delta^4 y_0 = \Delta^4 y_{-1} + \Delta^5 y_{-1} \text{ Etc.,}$$

Substituting these values in(1)we have

$$y_u = y_0 + U c_1 \Delta y_0 + U c_2 \Delta y_0 [\Delta^2 y_{-1} + \Delta^3 y_{-1}] + U c_3 [\Delta^3 y_{-1} + \Delta^4 y_{-1}] + \dots$$

$$= y_0 + U c_1 \Delta y_0 + U c_2 \Delta y_0 \Delta^2 y_{-1} + U c_2 \Delta^3 y_{-1} + U c_3 \Delta^3 y_{-1} + U c_3 \Delta^4 y_{-1} + \dots$$

$$= y_0 + U c_1 \Delta y_0 + U c_2 \Delta y_0 \Delta^2 y_{-1} + (U c_2 + U c_3) \Delta^3 y_{-1} + \dots$$

$$y_u = y_0 + U c_1 \Delta y_0 + U c_2 \Delta^2 y_{-1} + U c_1 \Delta^3 y_{-1} + \dots$$

It implies the odd difference just below the central line from  $y_0$  and even difference on the central line.

## 2. Gauss backward formula:

Gauss forward formula is

$$y_u = y_0 + U c_1 \Delta y_0 + U c_2 \Delta^2 y_{-1} + U c_1 \Delta^3 y_{-1} + \dots$$

put

$$\Delta^2 y_{-1} = \Delta y_0 - \Delta y_{-1}$$

$$\Rightarrow \Delta y_0 = \Delta^2 y_{-1} + \Delta y_{-1}$$

$$\Delta^4 y_{-2} = \Delta^3 y_{-1} - \Delta^3 y_{-2}$$

$$\Delta^3 y_{-1} = \Delta^3 y_{-2} + \Delta^4 y_{-2}$$

$$y_u = y_0 + U c_1 [\Delta^2 y_{-1} + \Delta y_{-1}] + U c_2 [\Delta^3 y_{-2} + \Delta^4 y_{-2}] + U c_1 \Delta^3 y_{-1} + \dots$$

$$y_u = y_0 + U c_1 \Delta^2 y_{-1} + U c_1 \Delta y_{-1} + U c_2 \Delta^3 y_{-2} + U c_2 \Delta^4 y_{-2} + U c_1 \Delta^3 y_{-1} + \dots$$

$$y_u = y_0 + U c_1 \Delta y_{-1} + U c_2 \Delta^2 y_{-1} + U c_3 \Delta^3 y_{-2} + \dots$$

If implies the odd difference just below the central line from  $y_0$  and even difference on the central line.

### 3. Stirling formula:

Gauss forward formula is

$$y_u = y_0 + U c_1 \Delta y_0 + U c_2 \Delta^2 y_{-1} + U c_3 \Delta^3 y_{-1} + \dots \quad (1)$$

Gauss backward formula is

$$y_u = y_0 + U c_1 \Delta y_{-1} + U c_2 \Delta^2 y_{-1} + U c_3 \Delta^3 y_{-2} + \dots \quad (2)$$

find the mean of (1) and (2)

$$y_u = y_0 + U c_1 \left[ \frac{\Delta y_0 + \Delta y_{-1}}{2} \right] + U^2 / 2! \Delta^2 y_{-1} + U(U^2 - 1) / 3! \left[ \frac{\Delta^2 y_{-1} + \Delta^2 y_{-2}}{2} \right] + \dots$$

### 4. Bessel's formula

Gauss backward formula is

$$y_u = y_0 + U c_1 \Delta y_{-1} + U c_2 \Delta^2 y_{-1} + U c_3 \Delta^3 y_{-2} + \dots$$

Shifting the origin from 0 to 1 we have

$$y_u = y_1 + U c_1 \Delta y_0 + U c_2 \Delta^2 y_0 + U c_3 \Delta^3 y_{-1} + \dots \quad (1)$$

This equation is known as Gauss third formula is

$$y_u = y_0 + U c_1 \Delta y_0 + U c_2 \Delta^2 y_{-1} + U c_3 \Delta^3 y_{-1} + \dots \quad (2)$$

mean of there two equations and we get

$$y_u = \left[ \frac{y_0 + y_1}{2} \right] + \left[ \frac{U c_1 + U c_1}{2} \right] \Delta y_0 + U c_2 \left[ \frac{\Delta^2 y_0 + \Delta^2 y_{-1}}{2} \right] + \dots$$

This is Bessel's formula.

Example:

X	0	4	8	12
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Y(x)	143	158	177	199
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Calculate f(5) by central difference formula.

Solution:

central difference Table

U	X	Y <sub>u</sub>	$\Delta Y_u$	$\Delta^2 Y_u$	$\Delta^3 Y_u$
-1	0	143			
			15		
0	4	158		4	
			19		-1
1	8	177		3	
			22		
2	12	199			

$$U = \frac{x - x_0}{2} = \frac{5 - 4}{2} = 0.25$$

Gauss forward formula is

$$i) \quad y_u = y_0 + U C_1 \Delta y_0 + U C_2 \Delta^2 y_{-1} + U^3 C_3 \Delta^3 y_{-1} + \dots$$

$$y_{0.25} = 158 + 0.25 C_1 (19) + 0.25 C_2 (4) + (0.25 + 1) C_3 (-1)$$

$$y_{0.25} = 162.414$$

ii) Gauss backward formula is

$$y_u = y_0 + U C_1 \Delta y_{-1} + U C_2 \Delta^2 y_{-1} + U C_3 \Delta^3 y_{-2} + \dots$$

$$y_{0.25} = 158 + 0.25 C_1 (15) + 0.25 C_2 (4) + (0.25 + 1) C_3 (-1)$$

$$y_{0.25}=162.375$$

### 3. Stirling formula:

$$y_u = y_0 + U c_1 \left[ \frac{\Delta y_0 \Delta y_{-1}}{2} \right] + U^2/2! \Delta^2 y_{-1} + U(U^2-1)/3! \left[ \frac{\Delta^2 y_{-1} \Delta^3 y_{-2}}{2} \right] + \dots$$

$$y_{0.25} = 158 + 0.25 C_1 (19 + 15/2) + 0.25 C_2 (4)/2$$

$$y_{0.25} = 162.375$$

### Bessel's formula

$$y_u = \left[ \frac{y_0 + y_1}{2} \right] + \left[ \frac{U + c_1 + U c_1}{2} \right] \Delta y_0 + U c_2 \left[ \frac{\Delta^2 y_0 + \Delta^3 y_{-1}}{2} \right] + \dots$$

$$y_{0.25} = (1/2)(158 + 177) + (0.25 - 1/2)(15) + 0.25(0.25 - 1)/2(4 + 3)/2$$

$$y_{0.25} = 162.414$$

### Question:

1. State Gauss's forward central difference formula.
2. State Gauss's back ward central difference formula.
3. Define Bessel's interpolation formula.
4. State the Stirling's central difference formula.
5. Derive Gauss 's forward interpolation formula.
6. Using Gauss's backward interpolation formula, find the population for the year 1936 given that

Year (X)	1901	1911	1921	1931	1941	1951
Y	12	15	20	27	39	52

7. Find  $y(x)$  when  $x=0.5$

X	0	1	2	3	4
Y(x)	1	1	15	40	85

8. Using the suitable method, estimate the population for the year 2005.

Year	1971	1981	1991	2001	2011
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Population	36	66	81	93	101
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9. Using Bessel's formula find  $f(25)$  given  $f(20)=2854, f(24)=3162, f(28)=3544, f(32)=3992$ .

10. Derive Stirling's central difference formula.

11. Using Gauss Forward interpolation formula find the population for the year 1946 given that

X	1901	1911	1921	1931	1941	1951
Y	30	45	53	56	62	70

12. Derive Gauss's Backward interpolation formula.

13. Derive Bessel's interpolation formula.

14. From the data given below find the value  $x$  when  $y=13.5$

X	93.0	96.2	100.0	104.2	108.7
Y	11.38	12.8	14.7	17.07	19.91

#### UNIT IV:

Inverse interpolation: Lagrange's method- Interaction of successive approximation method – simple problems.

#### Inverse interpolation:

#### Introduction:

The technique of determining the value of the argument corresponding to the given value of the function when the function lies between two values is known as inverse interpolation.

1. Lagrange's Method
2. Interaction method (or) Successive approximation method.

## 1. Lagrange's Method

The Lagrange's interpolation formula is given

$$f(x) \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x-x_1)(x_0-x_2)\dots(x_0-x_n)} = f(x_0) + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} f(x_1) + \dots + \frac{(x-x_1)(x-x_2)\dots(x_n-x_{n-1})}{(x-x_1)(x_0-x_2)\dots(x_n-x_{n-1})}$$

The formula for interpolation is obtained from Lagrange's interpolation formula by interchanging the variables X and f(x). This for (n+1) argument  $x_0, x_1, x_2, x_3, \dots, x_n$ , the values of x for then values f(x) is given by,

$$x = \frac{(f(x)-f(x_1))(f(x)-f(x_2))\dots(f(x)-f(x_n))}{(f(x)-f(x_1))(f(x_0)-f(x_2))\dots(f(x_0)-f(x_n))} x_0 + \frac{(f(x)-f(x_0))(f(x)-f(x_2))\dots(f(x)-f(x_n))}{(f(x_1)-f(x_0))(f(x_1)-f(x_2))\dots(f(x_1)-f(x_n))} x_1 + \dots$$

## 2. Interaction method (or) Successive approximation method.

Let  $y = f(x)$  be the polynomial of  $n^{\text{th}}$  degree then to find the value of x corresponding to the given value of the y (function) we make the interpolation formula. Let us consider Newton's forward formula.

$$y_u = y_0 + \Delta y_0 + U C_1 \Delta^2 y_0 + U C_2 \Delta^3 y_0 + U C_3 \Delta^4 y_0 + \dots$$

$$\Rightarrow \frac{y - y_0 - x C_2 \Delta^2 y_0 \pm x C_3 \Delta^3 y_0 \dots}{\Delta y_0}$$

$$X = \frac{y - y_0}{\Delta y_0} - \frac{x C_2 \Delta y_0}{\Delta y_0} - \frac{x C_3 \Delta^3 y_0}{\Delta y_0} - \dots (1)$$

Neglecting 5<sup>th</sup> and higher order difference for first approximation we suppose the polynomial to the first degree only

Show that second and higher order difference are neglected implying  $x^{(1)}$  = first approximation values of  $x \frac{y-y_0}{\Delta y_0}$

For the second approximation we put  $x = x^{(1)}$  is (1) implying  $x^{(2)} \frac{y-y_0}{\Delta y_0} \frac{y-y_0}{\Delta y_0} - \frac{x_{C2} \Delta y_0}{\Delta y_0} - \frac{x_{C3} \Delta^3 y_0}{\Delta y_0}$

For third approximation  $x = x^{(2)}$  is (1) implying  $x^{(2)} \frac{y-y_0}{\Delta y_0} \frac{y-y_0}{\Delta y_0} - \frac{x_{C2} \Delta y_0}{\Delta y_0} - \frac{x_{C3} \Delta^3 y_0}{\Delta y_0}$

similarly further approximation can be determined the process can be continued will be obtained tw Successive approximation to be equal.

**Example:**

Apply Lagrange’s formula inversely to find a root of the equation  $f(x)=0$ , when  $f(30)=-30, f(34)=-13, f(38)=3$  and  $f(42)=18$ .

x	30	34	38	42
F(x)	-30	-13	3	18

Lagrange’s formula inverse interpolation is

$$x = \frac{(f(x)-f(x_1))(f(x)-f(x_2))\dots f(x-x_n)}{(f(x)-f(x_1))(f(x_0)-f(x_2))\dots (f(x_0)-f(x_n))}x_0 + \frac{(f(x)-f(x_0))(f(x)-f(x_2))\dots (f(x)-f(x_n))}{(f(x_1)-f(x_0))(f(x_1)-f(x_2))\dots (f(x_1)-f(x_n))}x_1 + \dots$$

given that

$$= \frac{(0+13)(0-3)(0-18)}{(-30+13)(30-3)(-30-18)}(30) + \frac{(0+13)(0-3)(0-13)}{(-13+30)(-13-3)(-13-13)}(34) + \frac{(0+30)(0-13)(0-18)}{(3+30)(3+13)(3-18)}(38) +$$

$$\frac{(0+30)(0-13)(0-3)}{(18+30)(18+13)(18-3)}(42)$$

$X = 37.2305$

Example: Interaction method

The following values of  $y=f(x)$  are given



X	10	15	20
F(x)	1754	2648	3564

Find the value of x for  $f(x) = 3000$  by successive approximation method.

Solution:

The difference table is given by

X	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$
10	1754		
		894	
15	2648		22
		916	
20	3564		

For successive approximation method the formula is given by,

$$X = \frac{y-y_0}{\Delta y_0} - \frac{x_{c2} \Delta y_0}{\Delta y_0} - \frac{x_{c3} \Delta^2 y_0}{\Delta y_0} - \dots (1)$$

Since higher order difference not existing where

$$f(0)=1754, \Delta f(0) = 894, \Delta^2 f(0) = 22, f(x)=3000$$

The first approximation( $x_1$ )

$$X_1 = \frac{f(x)-f(x_0)}{\Delta f(x_0)} = \frac{3000-1754}{894} = 1.394$$

The Second approximation( $x_2$ )

$$X_2 = \frac{f(x)-f(x_0)}{\Delta f(x_0)} - \frac{x_1(x_1-1)}{2!} \frac{\Delta^2 f(x_0)}{\Delta y_0}$$

$$= \frac{3000-1754}{894} - \frac{1.394-(1.394-1)}{2} \frac{22}{894}$$

$$X^2=1.387$$

The Third approximation( $x_3$ )

$$X_3 = \frac{f(x) - f(x_0)}{\Delta f(x_0)} - \frac{x_1(x_1-1)}{2!} \frac{\Delta^2 f(x_0)}{\Delta y_0}$$

$$= 1.394 - \frac{1.394-(1.394-1)}{2} \frac{22}{894}$$

$$= 1.387$$

So  $x_2$  and  $x_3$  are coincide hence the value of  $x$  is  $x = x_0 + h/(x_3)$

$$X = 10 + 5(1.387)$$

$$X = 16.935$$

### Questions:

1. What is inverse interpolation?
2. State lagrange's inverse interpolation formula.
3. Write any two methods of obtaining inverse interpolation.
4. Briefly explain the iteration of successive approximation method.

5. From the data given below find the value of  $X$  when  $Y=13$ .

X	93.0	96.2	100.0	104.2	108.7
Y	11.38	12.8	14.7	17.07	19.91

6. The Values of  $x$  and  $y(x)$  are given below

X	5	6	9	11
Y	12	13	14	16

Find the value of  $x$  when  $Y= 15$  using Lagrange's inverse interpolation formula.

7. Find the value of x when y=19 using Lagrange's inverse interpolation formula.

X	0	1	2
Y=f(x)	0	1	20

8. Obtain Lagrange's inverse interpolation formula.

9. Find the value of x corresponding to y= 100 from the following table.

X	3	5	7	9	11
Y	6	24	58	108	174

10. Apply Lagrange's formula inversely to find a root of the equation  $f(x)=0$ , when  $f(30)=-30$ ,  $f(34)=-13$ ,  $f(38)=3$  and  $f(42)=18$ .

## UNIT :V

### Numerical Differentiation:

Numerical differentiation is the process by which we can find the derivative of a function at some values of the independent variable when we are given a set of values of function.

### THE GENERAL FORMULA OF TRAPEZOIDAL RULE

In numerical analysis, the trapezoidal rule or method is a idea for approximating the definite integral, the average of the left and right sums as well as usually imparts a better approximation than either does individually.

$$I = \int_{x_0}^{x_k} f(x) dx$$

Also, we know from Newton-Cotes general quadrature formula that Applied Mathematics and Sciences:

$$I = h [ky_0 + k^2/2 \Delta y_0 + (k^3/3 - k^2/2) \Delta^2 y_0/2! + (k^4/4 - k^3 + k^2) \Delta^3 y_0/3! + (k^5/5 - 3k^4/2 + 11k^3/3 - 3k^2) \Delta^4 y_0/4! + \dots]$$

Now, putting k = 1 in the above formula and neglecting the second and higher difference

$$\text{we get, } \int y dx_{x_0+h}^{x_0} = h[y_0 + 1/2 \Delta y_0] = h[y_0 + 1/2 (y_1 - y_0)] = 1/2 h [(y_0 + y_1)]$$

$$\text{Similarly, } \int y dx_{x_0+2h}^{x_0+h} = 1/2 h (y_1 + y_2) \dots \dots \dots \int y dx_{x_0+kh}^{x_0+(k-1)h} = 1/2 h (y_{k-1} + y_k)$$

Adding these all integrals, we get,

$$\int y dx_{x_0+kh}^{x_0} = h[ 1/2 (Y_0 + Y_n) + (y_1 + y_2 + y_3 + \dots + y_{k-1}) + y_k]$$

This rule is acquainted as the trapezoidal rule.

**THE GENERAL FORMULA OF SIMPSON’S ONE- THIRD RULE**

In numerical integration, the Simpson’s 1/3 rule is a numerical scheme for discovering the integral  $\int y dx$  b a within some finite limits a and b. Simpson’s 1/3 rule approximates f(x) with a polynomial of degree two p(x), i.e a parabola between the two limits a and b, and then searches the integral of that bounded parabola which is applied to exhibit the approximate integral  $\int y dx$  b . Besides, Simpson’s one-third rule is a tract of trapezoidal rule therein the integrand is approximated through a second-order polynomial.

$$I = h [ky_0 + k^2/2 \Delta y_0 + (k^3/3 - k^2/2) \Delta^2 y_0/2! + (k^4/4 - k^3 + k^2) \Delta^3 y_0/3! + (k^5/5 - 3k^4/2 + 11k^3/3 - 3k^2) \Delta^4 y_0/4! + k]$$

Putting  $k = 2$  in the formula and neglecting the third and higher difference

$$\text{we get, } \int_{y_0}^{y_0+2h} y = h [2y_0 + 2\Delta y_0 + y_0 (8/3 - 2) / 2 \Delta^2 y_0]$$

$$= h [2y_0 + 2(y_1 - y_0) + 1/3 (2 - 2\Delta y_0)]$$

$$= 1/3 h (y_0 + 4y_1 + y_2)$$

$$\text{Similarly, } \int_{y_2}^{y_2+4h} y = 1/3 h (y_2 + 4y_3 + y_4)$$

$$\int_{y_{k-2}}^{y_{k-2}+2h} y = 1/3 h (y_{k-2} + 4y_{k-1} + y_k)$$

When  $k$  is even.

Adding these all integrals, we obtain,

$$\begin{aligned} & \int_{y_0}^{y_0+2h} y + \int_{y_2}^{y_2+4h} y + \dots + \int_{y_{k-2}}^{y_{k-2}+2h} y \\ &= 1/3 h [(y_0 + y_k) + 4(y_1 + y_3 + \dots + y_{k-1}) + 2(y_2 + y_4 + \dots + y_{k-2})] \text{ Or, } \int_{y_0}^{y_0+kh} y \\ &= h/3 [(y_0 + y_k) + 4(y_1 + y_3 + \dots + y_{k-1}) + 2(y_2 + y_4 + \dots + y_{k-2})]. \end{aligned}$$

This formula is known as Simpson's one-third rule. If the number of sub-divisions of the interval is even then this method is only applied.

### THE GENERAL FORMULA OF SIMPSON'S THIRD-EIGHT RULE :

Simpson's three-eighth rule is a process for approximating a definite integral by evaluating the integrand at finitely many points and based upon a cubic interpolation rather than a quadratic interpolation. The difference is Simpson's 3/8 method applies a third-degree polynomial (cubic) to calculate the curve. Further, we know from Newton-Cotes general quadrature formula that

$$I = h [y_0 + k^2/2 \Delta y_0 + (3/3 - k^2/2) \Delta^2 y_0 / 2! + (4/4 - k^3 + k^2) \Delta^3 y_0 / 3! + (5/5 - 3k^4/2 + 11k^3/3 - 3k^2) \Delta^4 y_0 / 4! + \dots]$$

Putting  $k=3$  in the formula and neglecting all differences above the third, we get,

$$\int x^{0+3h} = h [3y_0 + 9/2 \Delta y_0 + (27/3 - 9/2) \Delta^2 y_0 / 2! + (81/4 - 27 + 9) \Delta^3 y_0 / 3! ]$$

$$= h [3y_0 + 9/2 (y_1 - y_0) + 9/4 (y_2 - 2y_1 + y_0) + 8/3 (y_3 - 3y_2 + 3y_1 - y_0)]$$

$$= \int x^{0+3h} = 3/8 h (y_0 + 3y_1 + 3y_2 + y_3)$$

Similarly,  $\int x^{0+6h} = 3/8 h (y_3 + 3y_4 + 3y_5 + y_6)$

$\int x^{0+yh} = 3/8 h (y_{k-3} + 3y_{k-2} + 3y_{k-1} + y_k)$  Adding these all integrals, we get,

$$\int x^{0+3h} + \int x^{0+6h} + y + \int x^{0+yh}$$

$$= 3h/8 [(y_0 + y_k) + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{k-1}) + 2(y_3 + y_6 + \dots + y_{k-3})]$$

This formula is known as Simpson's three-eighths rule.

### Weddle's Rule

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Let the values of a function  $f(x)$  be tabulated at points  $x_i$  equally spaced by  $h = x_{i+1} - x_i$ , so  $f_1 = f(x_1)$ ,  $f_2 = f(x_2)$ , .... Then Weddle's rule approximating the integral of  $f(x)$  is given by

$$I = 3h/10 [(y_0 + y_k) + 5(y_1 + y_5 + y_7 + y_{11}) + (y_2 + y_4 + y_6 + y_8 + y_{10} - 2y_6 + 6[y_3 + y_9])]$$

### Example:

Evaluate  $I = \int_0^6 \frac{1}{(1+x)} dx$  using

- i) Trapezoidal rule
- ii) Simpsons rule (both)
- iii) Weddle's rule

also check up by the direct integration.

Solution:

i) Trapezoidal rule

X	0	1	2	3	4	5	6
Y	1	0.5	0.33	0.25	0.2	0.167	0.143

$$I = h \left[ \frac{1}{2} (Y_0 + Y_n) + (y_1 + y_2 + y_3 + \dots + y_{k-1}) + y_k \right]$$

$$h = 1$$

$$= 1 \left[ \frac{1}{2} (1 + 0.143) + (0.5 + 0.33 + 0.25 + 0.2 + 0.167) \right]$$

$$I = 2.0185.$$

ii) a) Simpsons one- third rule

$$I = \frac{h}{3} [(y_0 + y_k) + 4(y_1 + y_3 + \dots + y_{k-1}) + 2(y_2 + y_4 + \dots + y_{k-2})].$$

$$I = \frac{1}{3} (1 + 0.143) + 4(0.5 + 0.25 + 0.167)].$$

$$I = 1.957$$

b) Simpson's three-eighths rule

$$I = \frac{3h}{8} [(y_0 + y_k) + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{k-1}) + 2(y_3 + y_6 + \dots + y_{k-3})]$$

$$I = \frac{3(1)}{8} [(1 + 0.143) + 3(0.5 + 0.33 + 0.2 + 0.167) + 2(0.25)]$$

$$I = 1.963$$

iii) Weddle's rule

$$I = \frac{3h}{10} [(y_0 + y_k) + 5(y_1 + y_5 + y_7 + y_{11}) + (y_2 + y_4 + y_6 + y_8 + y_{10} - 2y_9) + 6(y_3 + y_9)]$$

$$I = \frac{3(1)}{10} [(1 + 0.143) + 5(0.5 + 0.167) + 0.33 + 0.2 + 6(0.25)]$$

$$I = 1.9524$$

**Question:**

1. What is numerical integration?
2. State Trapezoidal rule.
3. State Simpson's three-eighth rule.
4. Write the formula for Simpson's one-third rule.
5. Derive Simpson's one-third rule.
6. Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  Using Trapezoidal rule with  $h=0.2$ .
7. Evaluate  $\log_e 7$  by i) Simpson's one-third rule  
ii) Weddle's Rule.
8. Evaluate  $\int_1^2 \frac{dx}{1+x^2}$  Using Trapezoidal rule with two sub intervals.
9. Using Simpson's one-third rule evaluate  $\int_0^1 x e^x dx$  taking 4 intervals.
10. Compute the values of defined integrals  $Y = \int_{0.2}^{1.4} (\sin x - \log e^x + e^x) dx$ 
  - i) Trapezoidal rule
  - ii) Simpson's rule (both)
  - iii) Weddle's rulealso check up by the direct integration
11. Derive Simpson's three-eighth rule.
12. Derive Weddle's Rule.