POPULATION STUDIES

UNIT- III

MORTALITY:

Mortality refers to the number of deaths in a given time or a place or a proportion of deaths in relation to a population.

Measurement of Mortality:

A measure of the frequency of occurence of death in a defined population during a Specified interval.

Mortality measures are often the Same. it is a matter of what we choose to measure, illness or death.

Death Rate:

the ratio of deaths to the population of a particular area or during a particular Period of time.

usually calculated as the number of death per one thousand people.

there are different methods of finding out death rates. Some Such rates are,

- Crude Death Rate
- Age Specific Death Rate
- Standard or Adjusted Death Rate
- Infant Death Rate

Crude Death Rate:

David - M. Heer in his Society and the Populatio has said that "Crude death rame may be defined as the ratio of the number of deaths which occur within a given population during a specific year, to the size of death populatio at a mid year".

According to Thompson and Lewis "This is calculated in exactly the same manner as the crude birth rate".

the formula is,

$$\frac{D}{P} \times K$$

where

D = Deaths registered in a year

P = Population of an area in that year

K = Constant 1000.

for eg:

$$\frac{\text{Deaths (300)}}{\text{Population (12,000)}} = 0.025 \text{ death Persons.}$$

$$0.025 \times 1000 = 25 \text{ deaths per thousand.}$$

Age Specific Death Rate.

This is another method of finding our death rate. We know that mortality rate among the children is usually high. thereafter pressure of death decreases.

But when the old age comes then the pressure of death again increases.

In order to find out the death rate correctly, the peoples should be classified under different categories according to age groups.

Thereafter Age Specific Death rate is found with the help of following formula.

$$\text{Age Specific Death rate} = \frac{\text{No. of. deaths in a specific section of population on an area in a given period.}}{\text{Total Population of that specific area in the same period.}} \times 1000$$

With the help of Specific death rate, it is also to find out crude death rate as well. On the same basis it is also possible to find out sex specific death rate.

In otherwords, there is no consideration about the number of days or month which a child or person lived.

Standardised or Adjusted Death Rate.

Every death rate must relate to either two places or time and is thus always comparative.

As regards standard or adjusted death rate Barclay while comparing it with crude death rate has said that "An age standardised death rate fills either of the two needs. One is simply to summarise a set of age specific rates independently of the ago composition of the population".

The other is to show the probable influence of the population age, composition on its crude death rate when its actual age specific death rates are not known.

In other words of Thompson and Lewis, " the standardis death rates, based on age specific death rates, supplies a simple and accurate basis for comparing the death rates of different populations.

Standardisation can be of two types.

a) Direct Standardisation

b) Indirect Standardisation

a) Direct Standardisation.

In Direct Standardisation Different Age, Specific Rates are applied to Standard population, whereas whereas in the other, a standardised set of rates to different population by age are applied.

this can be applied by finding out five yearly age specific death rates of a population which is independent of age structure of population.

In this population or town or place is supposed to be standard. Barclay has said that. " Thus there are two common types of producers for computing standard rates.

one consists of applying different age specific rates to standard population. this is called in English and American usage as direct standardisation.
this is called that direct standardisation.

under direct standardisation distribution pattern of population of an area or country is taken as standard one and thereafter standardised death rate is found out on the basis of following formula.

$$\text{Standardised Death Rate} = \frac{P_S \times D_I}{\Sigma EP}$$

where,

P_S = Standard population of the age group.

D_1 = Age Specific death Rate of local population.

b) Indirect Standardisation:

All Standardised death rate is decided and population of a particular territory and also death rate for the local population is calculated with the help of following formula:

$$\text{Index death rate of local Population} = E \frac{PL \times DS}{\Sigma PL}$$

PL = Local population for various age groups.

DS = Age Specific death rates taken as standard.

In this it is essential to find out crude death rate of standard population. which is done with the help of following formula:

$$\frac{\Sigma D}{\Sigma P} \times 1000$$

this also needs correction factor.

$$\text{Correction factor} = \frac{\text{CDR of Standardised Population}}{\text{Index death rate of local people}}$$

SDRL = CDRL x Correction factor.

$$CDR = \frac{\Sigma (P \times DS)}{\Sigma P_S}$$

Specific Death Rate.

Death can occur due to more the one specific reason then another problem is that the apparent cause of death may note be real cause of death.

this is can be found out with the help of following formula.

$$CSDR = \frac{Di}{P} \times K$$

where,

CSDR = Cause specific death rate

Di = Number of deaths due to a particular cause in a year

P = Mid year population in that year

K = Constant i.e. 1000.

But this type of death rate has its own problems.

Infant Mortality.

One of the serious problem of our society is the problem of Infant mortality.

In some societies rates of infant mortality is very high whereas in other it is low, but there is no society which is free from this.

In fact the pressure of death is maximum on the infants. It decreases there after but again increases on the person who attain age of 55 or above.

this rate can be found out with the help of the following formula.

$$IMR = \frac{do}{Births} \times 1000$$

do = Number of children who die before completing early years of their life.

Birth = Number of live children ~~who die before~~ in same year.

UNIT - IV

Life table: (Mortality table)

the life table gives the life history of a hypothetical group or Cohort as it is gradually diminished by deaths. It is conventional method of expressing the most fundamental and essential facts about the age distribution of mortality in a tabular form.

It is a useful tool for measuring the probability of life and death of various age sectors.

the life table gives a summary of the mortality experien of any population group during a given period and is a very effective and comprehensive method for providing concise measures of the longevity of that population.

The data for constructing a life table are the census data and death registration data. life tables are generally constructed for various sections of the people which as experience shows, have sharply different patterns of mortality.

Thus there are life tables constructed for different races, occupational groups and sex.

Life tables are as well constructed on regional basis and other factors accounting differential mortality.

Curate Expectation and Complete Expectation.

The Curate Expectation of life, usually denoted by e_x gives the average number of Complete Years of life lived by the Cohort L_0 after the ax x. by each of L_x Persons attaining that age.

the Complete Expectation of life, usually denoted by e_x^o measures the average number of Years a Person of given age x Can be expected to live under the prevailing mortality Conditions.

It gives the number of years of life entirely Completed and included the fraction of the year Survived in the Year in which death occurs, which on the average can be taken to be (1/2) year.

Thus We have.

$$e_x^o = e_x + 1/2 \rightarrow (1)$$

Since total number of years lived by L_x Persons of age x is given by

$$T_x = \int_0^\infty L_{x+t}\, dt,$$

the Complete expectations of life of a person atteing age x is

$$e_x^o = \frac{T_x}{L_x} \rightarrow (2)$$

e_x^o, the expectation of life at age 0, is the average age at death or a average longevity of a person belonging for a given Community.

Assumption. Description and Construction of life tables.

Assumption:

the following are a few simplified assumption which are used in the Construction of the life table.

i) The Cohort is closed for emigration or Immigration. In other words, there is no change in the census expect the losses due to deaths.

ii) Individuals die at each age according to pre-determined Schedule which is fixed and does not change.

iii) The cohort originates from some standard number of births. Say 10,000 or 1,00,000 which is called the radix of the table

iv) the deaths are distributed uniformly over the period $(x, x+1)$ for each x.

Description :

A typical life table has generally the following Columns.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
x	L_x	d_x	q_x	p_x	m_x	μ_x	l_x	e_x^0	e_x

Columns of life table.

The various symbols entioling the table have already been defined in the preceding sections.

Here we shall very briefly outline the various with the second column steps required for computing the tables.

1. $d_x = L_x - L_{x+1}$

2. $Q_x = \frac{d_x}{L_x}$

3. $P_x = 1 - Q_x = \frac{L_{x+1}}{L_x}$

4. $M_x = \frac{2Q_x}{2 - Q_x}$

5. $H_{x+(1/2)} = M_x$

6. $L_x = L_x - 1/2\, d_x = \frac{1}{2}(L_x + L_{x+1})$

7. $T_x = L_x + L_{x+i} = \frac{1}{2}(L_x + L_{x+1})$

8. $e_x^0 = \frac{T_x}{L_x}$

9. $e^x = e_x^0 - 1/2$

The columns (5) (6) and (7) do not occur in all the life table. However the remaining columns are must in any tables.

Construction :

If we can complete the quantities Q_x or (P_x) for all $x \geq 0$. the only other data which is needed is the radix L_0 and the Q_x columns is thus called the pivotal column of the life tables.

Starting with radix L_0 and Q_x, $(x = 0, 1, 2 \ldots)$ We have

$$d_0 = L_0 Q_0 \Rightarrow L_1 = L_0 - d_0$$

$$d_1 = L_1 Q_1 \text{ (or)}$$

$$L_2 = L_1 - d_1 .$$

and so on. from these values of L_x, $(x = 0, 1, 2 \ldots)$ the columns L_x, T_x and e_x^0 of the tables can now be computed by using the relations

$$L_x = \frac{1}{2} (L_x + L_{x+1})$$

$$T_x = \sum_{i=x}^{\infty} L_i ,$$

$$e_x^0 = (T_x / L_x)$$

We have seen that the complete life table can be constructed if we know the values of L_x and Q_x $(x = 0, 1, 2 \ldots)$

The values $m_x = d_x / L_x$ are computed, where the corresponding values are in basis of census records and deaths constraction data. It should be equal to the age of oldest survivor.

Uses of life table

Although the basic objective of the life tables is to give a clear picture of the age distribution of mortality is a given population group.

It has been used widely in a large numbee of Spheres today life tables is widely accepted as Important basic marital in demographic and public health Studies.

In the words of William Farr, life table is the "Biometer" of the population. We enumerate below some of the population Important application of life tables.

1. For Actuaries in insurance:

life tables are indispensable for the solution of all questions concerning the duration of human life.

these ~~tables~~ tables based on the scientific use of statistical methods are the key stone or the pivot on which the whole Science of life assurance hinges.

2. For Population projection:

life tables are used by demographers to devise measures each as "Net reproduction Rate" to study the rate of growth of population, also used in projection by age, Sex, etc.

Expectation of Life : Definition

It is thus Statistically determined average number of years of life remaining after the Specific age for a given group of individuals is Called expectation of life.

Life expectancy is based on an estimate of the average age that members of a particular population group will be when they die.

Life expectancy tables are Calculated based on death probabilities according to Farr's death rate method :

$$Q_x = M_x / (B_x + (M_x/2))$$

Where

M_x = the number of deaths at the age of x to under $x+1$ years in the reported period ;

B_x = Average Population aged x to under $x+1$ in the basis period.

Q_x = Death probability from age x.

UNIT - V

Reproduction Rate :

the growth rate of a Population Per generation; equivalent to the number of female offspring that each female produces over her lifetime. Hence this is known as Reproduction rate.

Gross reproduction Rates and Net reproduction Rates:

In Population, the Net Reproduction Rate, Ro is the average number of offspring (often Specially daughters) that would be born to a female if she passed through her lifetime Conforming to the age - Specific fertility and mortality rates of a given year.

this rate is Similar to Gross Reproduction rate but it takes into account that Some females will die before Completing their Childbearing age.

An Ro of one means that each generations of mothers is having exactly enough daughters to replace themselves in the population. If the Ro is less than one, the Reproductive performance of the population is below replacement level.

GROSS Reproduction Rate (GRR) :

This indicates the number of daughters of a hypothetical Cohort of women by the end of the reproductiv life, if she bears the births according to a given Schedule of age specific fertility rate without experiencing any mortality till the end of reproductive life.

$$GRR = 5 \sum_{x=15} \frac{{}_5B^f{}_x}{{}_5W_x}$$

${}_5B^f_x$ = no. of. female birth to women aged x to x+5.

$$GRR = \frac{f_B}{B} \times TFR, \quad \frac{f_B}{B} = \frac{\text{No. of. female births}}{\text{Total. No. of Births.}}$$

NET Reproduction Rate (NRR) :

NRR is GRR adjusted for mortality Schedule of Cohort of women.

Number of daughter that would be born to a Cohort of women during their lifetime if they experience a fixed Schedules of ASFR and ASMR, therefore

$$NRR = 5 \sum_{x=15} \frac{{}_nB_x}{{}_5W_x} \times {}_5^f\Pi_x$$

${}_5^f\Pi_x$ = Survival rate.

$$\text{Survival rate} = \frac{\text{Mean Size of the Cohort of the women age x to x+5}}{\text{Initial Size of Cohort}}$$

Construction of Mortality Rates.

A Mortality rate is a measure of the frequency of occurence of death in a defined population during a specific interval.

Mortality measures are often the same and its just the matter of the measure of illness or death.

The relative frequency of deaths in a specific population during a specified time, often cited as the percentage of human death during a public death crisis or health issues or of wildlife deaths due to environmental perils. Patients over the age of 80 had the highest mortality rate during the pandemic situations. This is termed as the higher mortality rate.

Mortality tables are based on characteristics, such as gender and age., Life insurance companies use mortality tables to help determine premiums and to make sure the insurance company remains solvent.

Mortality tables typically cover from birth through age 100 in one-year increments.

Gompertz - Makeham Law of mortality :-

the Gompertz - makeham Law States that the human death rate is the Sum of an age Independent component and a age dependent Component which increase exponentially with age. In a protected environment where external Cause of death are rare.

The age Independent mortality Components is often negligible. In this Case the formula simplifies to a Gompertz law of mortality.

In 1825 Benjamin Gompertz proposed an exponential increase in death rates with age.

Gompertz makehame Parameters:

$\alpha > 0$ (real)

$\beta > 0$ (real)

$\lambda > 0$ (real)

Support

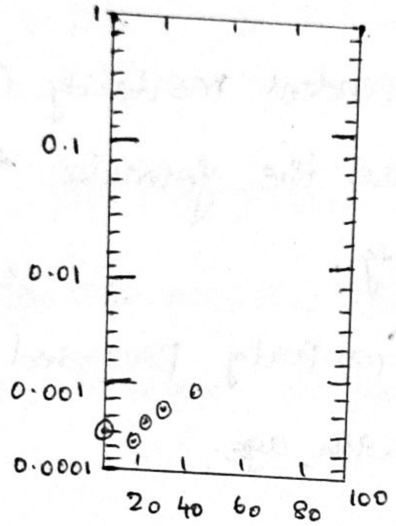
$x \in R^+$

PDF $(\alpha e^{\beta x} + \lambda) \cdot \exp\left(-\lambda x - \frac{\alpha}{\beta}(e^{\beta x}\right)$

$1 - \exp\left(-\lambda x - \frac{\alpha}{\beta}(e^{\beta x} - 1)\right)$

the Gompertz makeham law of Mortality describes the age dynamic of human mortality rather accurately in the age At advanced ages.

Some Studies have found that death rates increase more Slowly a Phenomenon known as the late-life mortality deceleration.



The decline in the human mortality rate before the 1950's was mostly due to a decrease in the age Independent mortality Component.

While the age dependent mortality Component was surprisingly Stable. Since the 1950's a new mortality trend has Started in the term of an expected decline in mortality rates at advanced ages and "rectangularization" of the Survival Curve.

The hazard function for the Gompertz makeham distribution is the most often characterized as

$h(x) = \alpha R^{\beta x} + \lambda$. the empirical magnitude of the beta parameter is about 0.85, implying a doubling of mortality every. $69/0.85 = 8$ years.

The Quantice function can be expressed in a closed form expressing using the lambert w function.

$$Q(u) = \frac{\alpha}{\beta \lambda} - \frac{1}{\lambda} \ln(1-u) - \frac{1}{\beta} W_0 \left(\frac{\alpha R^{\alpha/\lambda} (1-u)^{-(\beta/\lambda)}}{\lambda} \right)$$

the Gompertz law is the same a fisher-tippet distribution for the negative of age, restricted to negative values for the random variables.

Makeham's law :

Law of mortality a mathematical formula depending on age mortality that makeham Law

$$M_x = A + BC^x.$$