18KP3SELS4

ACTUARIAL SCIENCE

UNIT-III

1. What is IRR?

The Internal Rate of Return (IRR) is the rate of which an investment is repaid by proceeds from a project.

It is the rate that regulates the present value of the cash inflows with the present value of cash outflow of an investment.

$$\sum_{t=1}^{n} \frac{A_t}{(1+r)^t} = C$$

2. What is NPV method?

Net present value (NPV) method is one which the method takes into account the time value of money. It correctly postulates that cash flow arising at different time period is in value and are comparable only.

3. Explain Discounted payback period method:

The payback period is defined as the number of years required to recover the original investment. In the cash of discounted present value of future cash inflows and and find out the number of years required to recover the initial investment. If discounted payback period is less than the desired payback period, the project is accepted.

Let us Consider the following Investment proposal with an Initial Investment of Rs. 24,000

Year	Cash inflow (Rs)	PV factor @ 10%	Discounted PV (DPV) (Rs)	Cumulation DPV (Rs)
1	8,000	0.909	7,272	7,272
2	10,000	0.826	8,260	16,530
3	10,000	0.751	7,510	24,040
4	12,000	0.683	8,196	32,236
5	10,000	0.621	6,210	38,441

The number of years required to recover the Initial ammount is 3 years.

Hence the discounted Payback period is 3 years.

4. Write about Time weighted rate of return and money weighted rate of Return.

Time weighted rate of return.

The Time weighted rate of return (TWR) is a measure of the compound rate of growth.

The TWR measure is often used to compare the returns of Investments managers because it eliminates the distoring effects on growth rates created by inflows and outflows of money.

Money weighted rate of return.

Money weighted rate of return is a measure of the performance of an investment. the money-weighted rate of return is calculated by finding the rate of return that will set the present values of all cash flows equal to the value of the initial investment.

5. Derive the Individual risk models.

the Individual risk model

$$(B_j, x_j)\ 1 \leq j \leq n$$

describes a portfilio of $n \in N$ insurance policis cover a given fixed short time period, ignore any effects on the interest). For each policy $j=1, \ldots n$ the number of claims $B_j \in \{0,1\}$ and if $(B_j=1)$ the amount of the claim $x_j \in (0,\infty)$ are random variables. independent but not necessarily identically distributed for different j.

We denote by $Y_j = B_j x_j$ the payment under the j^{th} policy and by $S = Y_1 + \ldots + Y_n$ the aggregate total claim amount of all policies.

Net premiums for the policies are $E(Y_i)$, We will now study the risk associated, more precisely the insurer's risk that the benefits are higher than the premium, individually, or for a portfilo.

6. Discounted Cash Flow (DCF) methods.

This include, Net present Value method, internal rate of return (IRR) method, Discounted payback period method.

a) Payback period method:

Payback Period is the number of years required to recover the cash invested in a project. If the annual cash inflows are same, the Payback Period can be computed dividing the cash invested by the annual cash inflows. If unequal, It can be calculated by adding up the cash inflows until the total is equal to the initial cost invested.

b) Average rate of return method:

The Average rate of return is an account method. It is calculated as the ratio of the average annual profit. after taxes, to the average investment in the profit for instance.

c) Net present value (NPV) method.

this method takes into account the time value of money. It correctly postulates that cash flows arising at different time periods differ in value and are comparable only when their equivalent. Present values are found.

The NPV proposal is computed by the following relation

NPV = PV of cash inflows - PV of cash outflows

the proposal is accepted, if NPV > C. Otherwise it is rejected. the PV of all cash outflows is determined using some pre-determined discounting rate, which generally equals the cost of capital.

d) Internal rate of return (IRR) method.

The internal rate of return is the rate of which an investment is repaid by proceeds from a project. In otherwords it is the rate that equates the present value of the cash inflows with the present value of cash outflows of an investment, It is determined by using the formula,

$$\sum_{t=1}^{n} \frac{A_t}{(1+r)^t} = C$$

where $A_1, A_2 \ldots A_n$ represent cash inflows,
C is the cost of Investment project.

e) Discounted Payback period method.

The payback period is defined as the number of years required to recover the original investment. In the case of discounted Payback period. We consider the discounted present value of future cash inflows and the number of years required to recover the Initial Investment.

7. Time Value of Money

One of the basic concepts of finance is the notation that money has time value.

this is be money today is more valuable then the same amount at some future date. We can always put available funds to some use and make them in higher sums. So that a larger decision maker would not value the opportunity to receive some amount of money equally.

With the decision makers, time presence of money is generally expressed by means of an Internet rate.

Questions:

UNIT- III

1. Define IRR.
2. What is NPV Method?
3. Explain Discounted Payback Payment method.
4. Time weighted rate of return.
5. Money weighted rate of return.
6. Derive the Individual risk mode.
7. Explain about Discounted cash flow method.
8. Time value of money.

UNIT - IV

1. Stochastic Interest Rate Models.

Stochastic interest rate models is a form of financial model that is used to help make investment decision.

This type of modeling forecasts the probability of various outcomes under different conditions, using random variables.

Notations and Terminologies.

Consider a pure discount (zero coupon bond that pays one dollar at time T, Let $P(t,T)$, $t \leq T$ denote the price of bond at time t. thus price of the bond, denote by (t,T) is defined as $-\log P(t,T) \cdot (T,t)$.

A plot of the yield - to - maturity against the time - to maturity $t = T - t$ is called the term structure of interest rates. As we shall only consider default free bonds such as Treasury Securities, $R(t,T)$ is the risk free rate of interest A time t.

the notation $T_2 = R(t,T)$ As such T_2 is unobservable and should be regarded as a state (latent) variable many term structure models postulates that T_1 is the only determinant of the term structure, these models are called one. factor models.

2. Long term State of Interest :

The characteristics that interest rate of all maturities are perfectly corrected, "the number of state variables driving the term structure may be determined by long term SARVr rate of Interest". Defined as

$$l_t = \lim_{T \to \infty} R(t,T).$$

3. Annual Percentage Rate :

The actual Annual percentage rate or APR is defined as the amount to which a unit compounds after a single year.

respectively as

$$i_{APR} = \left(1 + \frac{i^{(m)}}{m}\right)^{M} - 1 \quad \text{or} \quad e^{\delta} - 1$$

the amount to which a unit invested at time 0 accumulates at the effective interest rates i_{APR} over a time duration T.

$$(1 + i_{APR})^{T} = \left(1 + \frac{i^{(m)}}{m}\right)^{MT} = e^{\delta T}.$$

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4. Outstanding Loan Balance

In the amortization method part of each payment pays interest on the loan and part of each payment repays some of the principal of the loan. (The total amount borrowed). At a point in the repayment process we may need to ascertain the outstanding loan balance.

For example,

If the loan needs to be refinanced or if the loan is to be purchased by another lender, it is vital to know how much of the original loan currently remains unpaid.

The outstanding loan balance can be determined in two ways

- Prospectively
- Retrosepectively.

Prospectively - The outstanding loan balance is the present value.

Retrospectively - The outstanding loan balance is the original amount of the loan accumulated to the present date minus the accumulated to the present date minus the accumulated value of all the loan payments that have already been paid.

5. Amortization.

the borrower repays the lender by means of instalment payments at regularly aspected time points. the present value of the installment payments equals the

$$\text{Loan principal } L = (\text{Payment Amount}) \times a_{\overline{n}|}$$

6. Generic Amortization Schedule.

Payment Index	Payment amount	Interest Paid	principal repaid	Outstanding loan balance
t	1	iB_{t-1}	$t - iB_{t-1}$	B_t
0				$a_{\overline{n}|}$
1	1	$1-v^n$	v^n	$a_{\overline{n-1}|}$
2	1	$1-v^{n-1}$	v^{n-1}	$a_{\overline{n-2}|}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t^{th}	1	$1-v^{n-1+1}$	v^{n-1+1}	$a_{\overline{n-t}|}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n-1$	1	$1-v^2$	v^2	$a_{\overline{1}|}$
n	1	$1-v$	v	0
Total	n	$n - a_{\overline{n}|}$	$a_{\overline{n}|}$	

We see from this table that the total principal paid over all n payments is $a_{\overline{n}|}$, the amount of the original loan.

<u>Simple problems</u>.

1. Construct an amortization table for a loan of \$1000 to be paid in 4 annual payments at 10% annual effective rate.

First note that the payment amount is

$$L / a_{\overline{4}|} = 1000 / 3.169865 = 315.47.$$

the amortization table is then

Payment index	Payment amount	Interest Paid	principal repaid	outstanding loan balance
0				1000.00
1	315.47	100.00	215.47	784.53
2	315.47	78.45	237.02	547.51
3	315.47	54.75	260.72	286.79
4	315.47	28.68	286.79	0.00
Total	1,261.88	261.88	1,000.00	

2. Consider a home mortage loan for \$100,000 at 6% nominal annual rate with equal monthly payments for 30 years. What are the characteristics of this loan.

The effective monthly interest is $i = .06/12 = .005$

The number of payments is $n = 12(30) = 360$

Each monthly payment is $\frac{100000}{a_{\overline{360}|.005}} = \frac{100000}{166.7916} = 599.53$

At the end of 30 years,

Total paid on the loan is \$599.55 (360) = \$216,838 with

Total principal paid \$100,000 and

Total interest paid \$115,838.

the amortization table :

Payment Index	Payment amount	Interest paid	principal repaid	outstanding loan balance
0				100,000.00
1	599.55	500.00	99.55	99,900.45
2	599.55	499.50	100.05	99,800.40
3	599.55	499.00	100.55	99,699.85
4	599.55	498.50	101.05	99,598.80
⋮	⋮	⋮	⋮	⋮
357	599.55	11.84	587.71	1,780.82
358	599.55	8.90	590.65	1,190.17
359	599.55	5.95	593.60	596.57
360	599.55	2.98	596.57	0.00
Total	215,838	115,838	100,000	

It is apparent in this example that to borrow \$100,000, the borrower must pay \$115,838. the cost of the loan can be reduced by, for example, cutting the term (time length) of the loan is half. this requires much larger monthly payment amounts.

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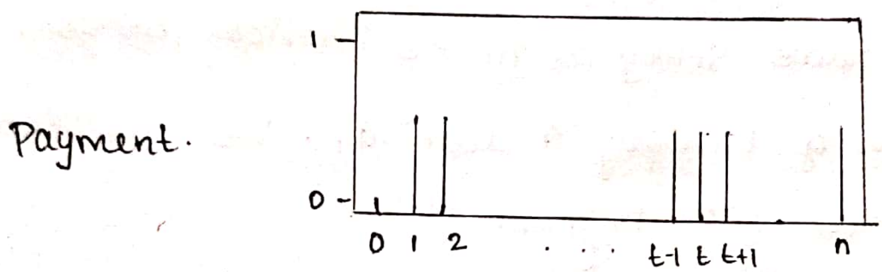
7)



Suppose the payments are each 1 and the loan requires n payments.

Let 'i' denote the effective interest rate for each payment period.

the loan amount is the present value at $t=0$, namely

We seek the outstanding loan balance, denote B_t, right after the t^{th} payment is made.

Retrospective:

$$B_t = a_{\overline{n}|}(1+i)^t - s_{\overline{t}|}$$

$$= \frac{(1-v^n)}{i}(1+i)^t - \frac{(1+i)^t - 1}{i}$$

$$= \frac{1-v^{n-t}}{i} = a_{\overline{n-t}|}$$

thus either approach to this computation yeilds the Same outstanding loan balance. If the Loan is for L dollars, then the equal payment amount should be

$$\frac{L}{a_{\overline{n}|}} \text{ dollars.}$$

Therefore, the outstanding loan balance right after the t^{th} payment is.

A loan is created with 10 annual payments of \$500 at an effective annual rate of 6%, However after 4 years, the borrower needs an additional \$2000 and must restructure all outstanding debts over the remaining 6 years at 7% effective. What is the payment amount during those 6 years?

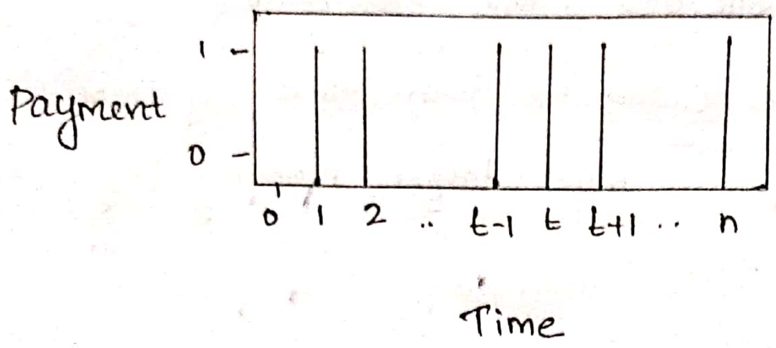
At 4 years the outstanding loan balance is $500\, a_{\overline{6}|0.6}$ the refinanced loan with payments of x dollars will have 6 payments and a present value at its beggining of

$500\, a_{\overline{6}|0.6} + 2000 = x\, a_{\overline{6}|0.7}$ therefore,

$$x = \frac{500\, a_{\overline{6}|0.6} + 2000}{a_{\overline{6}|0.7}}$$

$$= \$935.41$$

8. Amortization Schedule.



In the Same Setting as in the Previous Section, n total payments of 1 repay a loan $\overline{a}|$. We now examine in greater detail the t^{th} payment.

Just after the $(t-1)^{th}$ payment, the outstanding loan balance is

$$B_{t-1} = a_{\overline{n-t+1|}}$$

So, the interest due at the time of the t^{th} payment is

the remainder of this t^{th} payment of 1, namely

is applied to the Principal, reducing the outstanding loan balance to

$$a_{\overline{n-t+1|}} - v^{n-t+1} = \frac{(1-v^{n-t+1})}{i} - v^{n-t+1}$$

$$= \frac{1-(1+i)v^{-t+1}}{i} - \frac{1-v^{n-t}}{i}$$

$$= a_{\overline{n-t|}}$$

which (as we saw in previous section) is the outstanding loan balance right after the t^{th} payment.

9. Varying Payments.

Let R_i denote the t^{th} loan Payment amount made at the end of Period t and i denote the effective interest rate on the loan per payment period.

the amount of Loan L is then

$$L = \sum_{t=1}^{n} v^t R_t$$

the outstanding loan balance Just after the t^{th} payment is B_t.

the t^{th} Payment of R_t is divided between Interest.

and principal : $P_t = R_t - I_t$.

UNIT - IV

1. Stochastic Intereset rate models.

2. Annual Percentage rate (APR).

3. Long term State of interest.

4. Explain the Concept outstanding Loan Balance.

5. Define Amortization.

6. Varying payments.

UNIT - V

1. Discounted Mean Term.

Discounted mean term (DMT) is a measure of sensitivity of Cash flows, to interest rates.

this is simply the weighted average of the terms of the Cash flows, the weights being the present values of the Cash flows.

2. Volatility :

- Volatility is a Central to many applied issues in finance and financial engineering.
- It is a rate at which the price of a Security increases or decreases for a given Set of returns.
- Volatility is measured by calculating the Standard deviation of the annualized returns over a given period of time.
- the Speed or degree of Change in prices is Called Volatility.

3. Matching of Assets and Liabilities.

Risk Control involves calculation of the possible movements of assets and liabilities on an insurer's balance sheet.

Again volatility is a key parameter and in this case it is internal risk managers and possibly the regulator who should be asking tough questions about valuation techniques and particularly about volatility parameters.

4. Underlying Volatility

The price of financial instrument can be thought of as a random variable. In order to describe how much the price might vary over a particular time period, we would look for some approximate statistic

A natural place to start would be to consider the average of the price movements, measured at some time frequency,

$$\gamma = \frac{1}{N_d} \sum_{k=1}^{N_d} |\Delta S_k| \rightarrow (1)$$

Where Nd is the number of time periods observed and Δ_{SK} is the price change in the K^{th} time period.

$$\bar{\sigma} = \sqrt{\frac{1}{N} \Sigma \left(\ln \frac{S_K}{S_{K-1}} \right)^2} \rightarrow (2)$$

Statistics are almost inevitably quoted as RMS or sample variance, which is useful, given that the latter is an unbiased estimator for the population variance.

Black and Scholes proposed the following dynamics for asset prices

$$dS_1 = \mu S_1 dt + a S_1 dW_1 \rightarrow (3)$$

this equation says that the instantaneous change in the price of an asset is driven by the deterministic average component.

5. Zero Coupon bonds.

Assume throughout this section that the model $c(c)$ complete and arbitrage free and let $d> = d(d1;\because;dT)$ be the unique vector of discount factors. Since there must be at least T securities to have a complete model, c must have atleast T rows.

Example. (Zero-Coupon bond).

usually short term investment with interest paid at the end of the term. eg invest \$1000 for 90 days for a return of \$1010.

j	t_j	c_j
1	0	-1000
2	90	+1010

Definition:

The payment stream of a zero coupon bond with maturity t is given by the to th unit vector e_t of R^T:

the price of zero coupon bond with maturity is d_t.

6. Simple fixed - interest Securities.

An investment into the simplest type of a interest security pays interest at rate j at the end of each time unit of over an integer term n and repays the invested money at the end of the term.

The cashflow representation (SCO = C0; □C); C1; Cj);:::: (n □ 1; Cj) : (n; C + Cj)].

In practise, a security is a piece of paper. It is therefore useful to split CO = ((0; □C); C1) into the inows C and their purchase price at time 0.

The intrinsic model for the security is the compound interest model with constant rate i = j. The trading price of the security is then DV att (C), the discounted value of all post-ows.

At any integer time t after the interest payment this value is C whereas the value increases.

And also the interest may be paid on a third, so - called nominal amount N. In any given security model

C = ((1; Nj1); (2; Nj2); : : : ; (n □ 1; Nj n □ 1); (n; R + Nj n))

Interest payments are always calculated from the nominal amount.

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7. DMT of Zero coupon bond.

The Discounted mean term (DMT) of a zero-coupon is simply the term of bond, n. This is because, the average term of a series of cash flows consisting for only one cash flow (The final redemption payment) is obviously the term of that single cash flow.

8. Life table:

A life table presents the proportion surviving, the cumulative hazard function, and the hazards rates of a large group of subjects followed over time.

Life tables are used to examine the mortality changes in the social security population over time.

9. MORTALITY: Definition.

Mortality is the numbers of deaths occuring in a defined population during a selected time interval or survival rates from birth to death.

10 Concept of Risk analysis.

Actuarial risk analysis is not just based on the Short-term horizons but may extend many decades into the future when neccessary.

The training and experience actuaries receive provides them with a uniquely broad-based Combination of Skills suited to risk management allowing them:

- To explore the full ranges of risk that migh affects an organisation
- To Quantify risks and their implications in the Short and long terms.
- To Quantify the Values of any mitigation versus the cost of undertaking it.
- To illustrate the range of Possible outcomes.
- To Integrate risk analysis into the wider economic business management process and
- To Communicate the risks to decision makers in a balanced and effective way.

11 Uses of life table.

1. Life table is used to project future population on the basis of the present death rate.

2. It helps in determining the average expectation of life based on age specific death rates.

3. the method of constructing a life table can be followed to estimate the cause of specific death rates, males and female death rates etc.

4. the survival rates in a life table can be used to calculate the net migration rate on the basis of age distribution at 5 or 10 year interval.

5. Life tables can be used to compare population trends at national and international levels.

6. By constructing a life table based on the age at marriage, marriage patterns and changes in them can be estimated.

12 Assumptions of Life table.

1. A hypothetical Cohort of life table usually Comprise of 1,000 or 10,000 or 1,00,000 births.

2. The deaths are equally distributed throughout the year

3. The Cohort of people diminish gradually by death only.

4. The Cohort is Closed to the in-migration and out-migration

5. The death rate is related to a pre-determined age Specific death rate.

6. The Cohort of persons die at a fixed age which does not change.

7. There is no Changes in the death overtime.

8. The Cohort of life tables are generally constructed Separately for males and females.

13 Definition of Cohort.

A Cohort is a group of Subjects who Share a defining characteristics. (Typically Subjects who experienced a common event in a Selected time period, such as birth or graduation).

Cohort data can oftentimes be made advantageous to demographers than Period data.

14 Redington's theory of Immunization.

In finance, interest rate Immunization as devoloped by Frank Redington is a Strategy that ensures that a change in interest rates will not affect the Value of a portfilo.

- Actuaries are typically concerned with assets held to cover future liabilities.
- For example, a pension scheme holds assets to cover the future Pension payments when they ~~d~~ become due.
- It is therefore useful to consider the Surplus which is simply defined as,

Value of assets - Value of liabilities.

SURPLUS:

The Surplus at a particular time can be defined in terms of the net present values of future cash flow arising from the assets and liabilities, NPV_A and NPV_L.

In the simple case that the NPVs depend on the constituent cash flows discounted at a constant force of interest δ, We can define surplus to be

$$S(\delta) = NPV_A(\delta) - NPV_L(\delta)$$

We could equally have defined this in terms of constant interest rate 'i' but using δ will make more straightforward to illustrate.

Recall that for fixed interest rate quantities $i = e^{\delta} - 1$

IMMUNIZATION:

By assuming that the surplus is zero at the prevailling δ_0 and are concerned with the behaviour of the surplus for small changes in δ, i.e. $\delta_0 \to \delta_0 + \epsilon$.

We require that the surplus remains non-negative to all small changes in δ_0, positive or negative.

Mathematically, We require,

$$S(\delta_0) = 0$$

$$S(\delta_0 + \epsilon) \geq 0$$

When this is true, the surplus is said to be immunized against changes in δ.

REDINGTON IMMUNIZATION:

Redington derived three Conditions for a Surplus to be immunized against Small Changes in δ, as follows.

We consider the Taylor expansion about δ_0 and consider each term

$$S(\delta_0+\epsilon) = \underbrace{S(\delta_0)}_{\text{defined} = 0} + \underbrace{\epsilon}_{\pm} \times \underbrace{\frac{ds}{d\delta}\Big|_{\delta_0}}_{\text{fix}=0} + \underbrace{\frac{\epsilon^2}{2!}}_{>0} \times \underbrace{\frac{d^2s}{d\delta^2}\Big|_{\delta_0}}_{\text{fix}>0} + \underbrace{O(\delta^3)}_{\approx 0}$$

As ϵ could be positive or negative, we arrive at that above Conditions for the derivatives of $S(\delta)$.

These are Redington's Condition for Immunization of a Surplus against Small Changes in δ.

$$S(\delta_0) = 0 \; ; \; S'(\delta_0) = 0 \; ; \; S''(\delta_0) > 0$$

UNIT - V

1. Discounted Mean Term (DMT)
2. Define Volatility.
3. Explain about underlying Volatility
4. What is Zero Coupon bond?
5. Write about Simple fixed - interest Securities.
6. Define DMT of life table
7. Explain about life table
8. What is Mortality.
9. Explain the Concept of Risk analysis
10. Write the uses of life table.
11. Explain the assumptions of life table
12. Derive Redington's theory of Immunization.